Geometrical Seismics
(Seismic phenomenology)

- *P*- and *S*-waves, typical velocities;
- Wavefronts;
- Rays;
- Reflection, Refraction, Conversion;
- Head wave (critical refraction);
- Huygens' principle;
- Fermat principle;
- Snell's law of refraction;
- Seismic wave nomenclature.
- Reading:
  - Reynolds, Sections 5.1-3, 6.1-6.2.2
  - Shearer, 4.1-4.3, 4.9
 - Telford et al., Sections 4.3-4.

Seismic Method
Basic Principles

- Generate a mechanical elastic *signal*.
  - "Controlled source": Locations and times are known with precision.
  - "Passive source": Locations and times need to be determined.
- Several types of seismic waves are generated.
  - So we have to be able to recognize them!
- Signal travels through the subsurface.
- At boundaries between different media, the energy is *reflected*, *transmitted*, or *refracted.*
- The transformed signal is recorded by the receivers on the surface (or borehole, *etc*.)
  - Locations of all detectors are known with precision.
- *Arrivals* are identified and their *times* (and amplitudes) are determined.
- Travel-times are used to determine the subsurface *velocities* and the *positions of boundaries.*

Forward and Inverse Seismic Problems

- Forward problem
  - Layer thicknesses and velocities are known.
  - *Calculate arrival times* (easy to do).
- Inverse problem
  - Arrivals are identified (where possible).
  - Arrival times are known.
  - Find *velocities* and *depths* (not so easy to do).
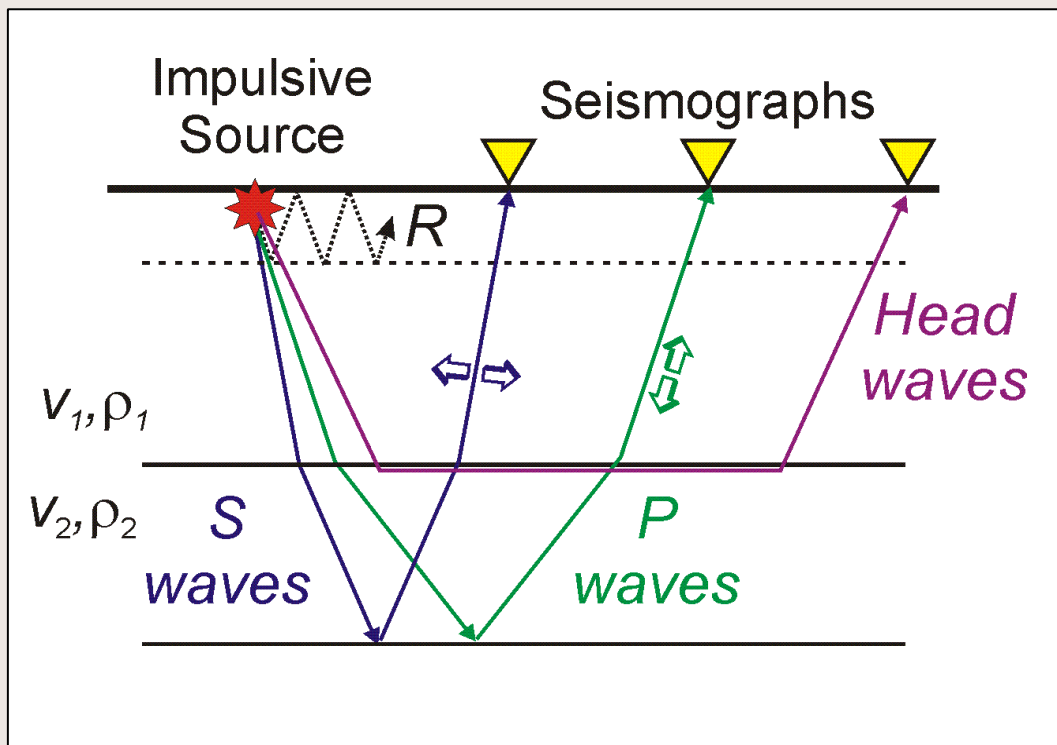
Seismic Properties

- *P*- (*compressional*, faster) and *S*- (*shear*, slower) waves are of primary importance.

- Their propagation and reflection depend on *elastic velocities* (V_P, V_S) of the medium and its *density*.

Material	Vp (m/s)	Vp (ft/s)	Vs (m/s)	Vs (ft/s)	$\rho(g!cm^3)$
Air	332	1090			0.0038
Water	1,400 - 1,600	4,600 - 5,250			1.0
Soil	300 - 900	980 - 2,950	120 - 360	390 - 1,180	1.7 - 2.4
Sandstone	2,000 - 4,300	6,560 - 14,100	700 - 2,800	2,300 - 9,190	2.1 - 2.4
Chalk	2,200 - 2,600	7,220 - 8,530	1,100 - 1,300	3,610 - 4,270	1.8 - 3.1
Limestone	3,500 - 6,100	11,490 - 20,000	2,000 - 3,300	6,560 - 10,830	2.4 - 2.7
Dolomite	3,500 - 6,500	11,490 - 21,330	1,900 - 3,600	6,240-11,810	2.5 - 2.9
Salt	4,450 - 5,500	14,600 - 18,050	2,500 - 3,100	8,200 - 10,170	2.1 - 2.3
Granite	4,500 - 6,000	14,770 - 19,690	2,500 - 3,300	8,200 - 10,830	2.5 - 2.7
Basalt	5,000 - 6,400	16,400 - 21,000	2,800 - 3,400	9,190 - 11,160	2.7 - 3.1
Quartz	6,049	19,846	4,089	13,415	2.65
Calite	6,640	21,783	3,436	11,273	2.71

- Velocities are sensitive to multiple factors:
  - Lithology,
  - Pressure, depth of burial (increase)
  - Temperature (decrease)
  - Fractures, porosity, fluid content (decrease)
  - Anisotropy,...

Wavefronts and Rays

- Vibrations originate at the source and propagate away from it

- **Wavefronts** are defined as surfaces of constant propagation time

- **Rays** are lines that are perpendicular to the wavefronts at every point.

- Wavefronts propagate along the rays at the local wave velocity within the medium.

- Rays generally indicate wave propagation direction and energy flux.

  - However, only in relatively simple cases free of 'caustics' and 'diffractions'.

- In a uniform medium, wavefronts are spheres of progressively increasing radii.

 - At greater distances, spherical wavefronts approach planar shapes:
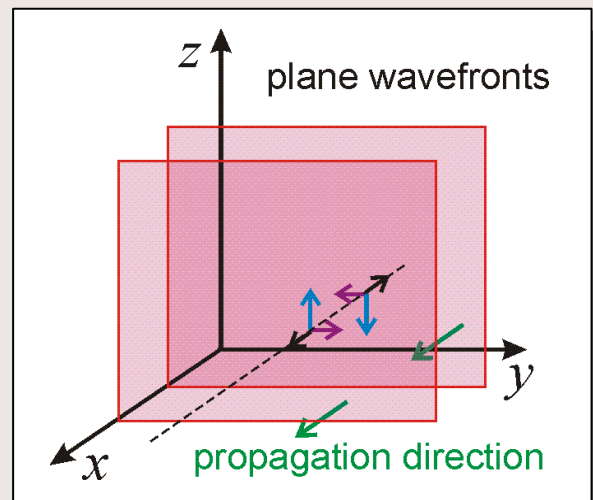
Spherical divergence

- **Waveform/Ray picture is very commonly used in the seismic method.**

- Ray diagrams also allow estimation of the *wave amplitude decay* due to geometrical (spherical) spreading:

The amplitude progressively decreases so that the energy flux across the shaded spherical shell remains constant: $E \sim 1/R^2$.

Therefore, *for spherical wavefronts* (and straight rays), the amplitude decreases with distance, R, as $A \propto 1/R$.
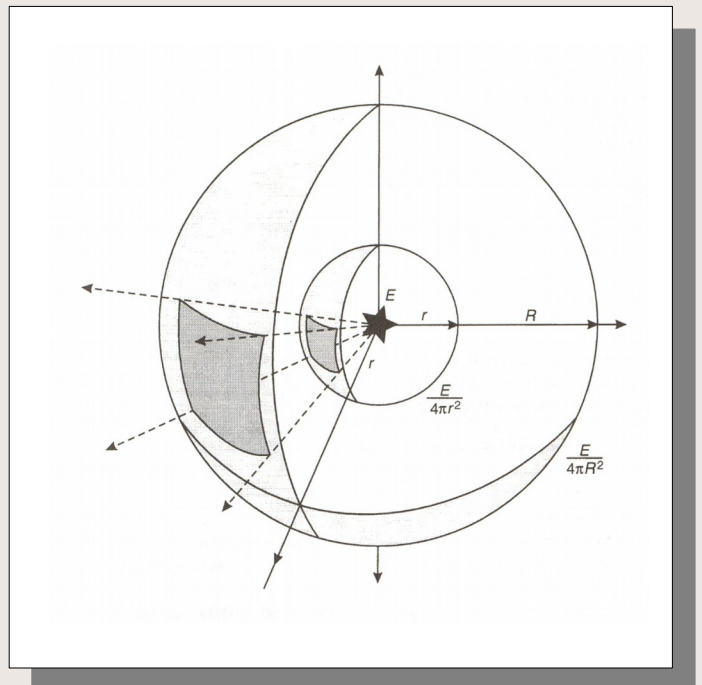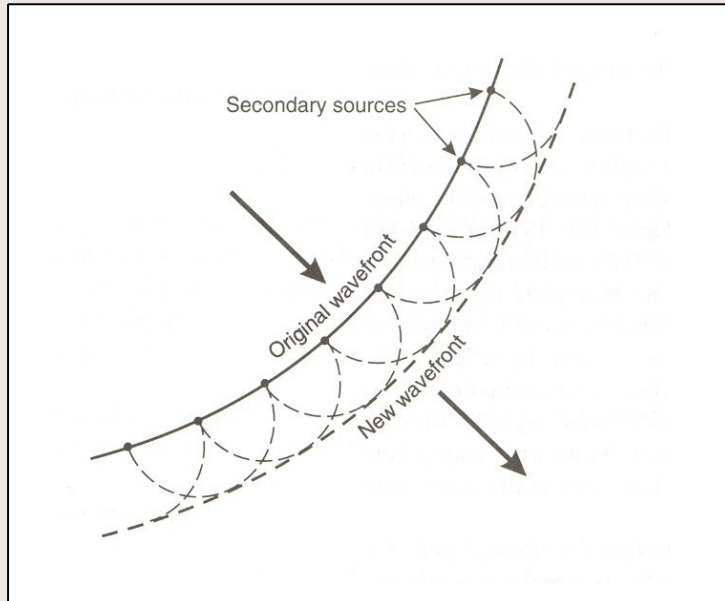

Exercise: show that for cylindrical waves, spreading is
$$A \propto 1/\sqrt{R}$$

Huygens' Principle

- Energy spreads from a point source in a spherical manner. So, near the source, the wavefronts are spherical (circular in 2-D).

- Every point on a wavefront can be viewed as a source of secondary waves that spread put in spheres (circles). Envelope of these spheres is the new wavefront.

- In Lab 1, you will use this principle to work out the head wave propagation problem.

- A more rigorous treatment of this principle is known as the Kirchhoff theory.

Diffraction

- Secondary wavefronts can penetrate into '*shadow zones*' into which the normal, '*specular*' rays from the source cannot enter.

- This is a fundamental effect of wave propagation called *diffraction*.
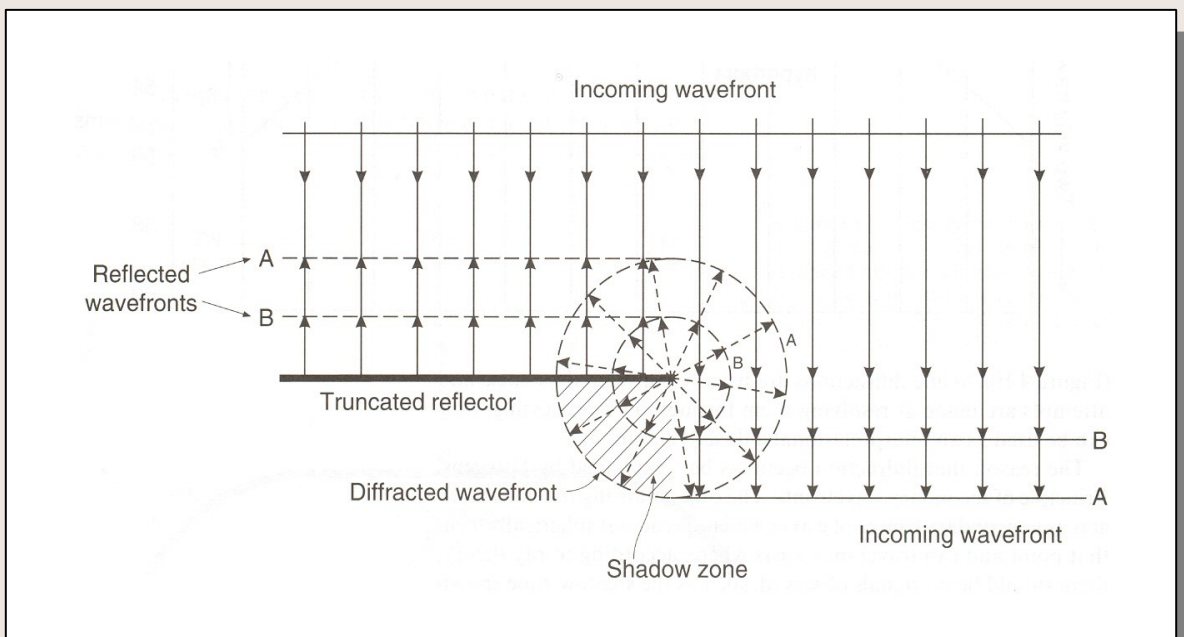
*From Reynolds, 1997*

Fermat Principle
(Least-time path, *brachistochrone*)

- A wave will take the path for which the travel time is *stationary* with respect to minor variations of the ray path.

  - Stationary means when the ray path is slightly perturbed, variation of its travel time is zero (to the first order of perturbation).

- Usually, the ray path has the minimum travel time among its small perturbations.

$$t - t_{specular} = o(\varepsilon)$$

$$t_{specular}$$

**B**

A
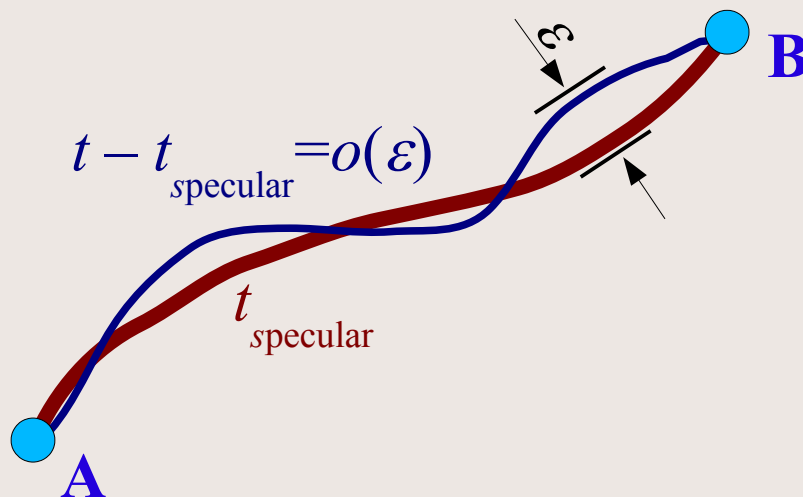
Example of using Fermat principle: refraction

- Show that the travel time is stationary ($\delta t = 0$) for a ray bending at the velocity interface; so that: $\dfrac{\sin \alpha}{V_1} = \dfrac{\sin \beta}{V_2}$

- This relation is called the Snell's law or refraction

α

V_1

β

V_2
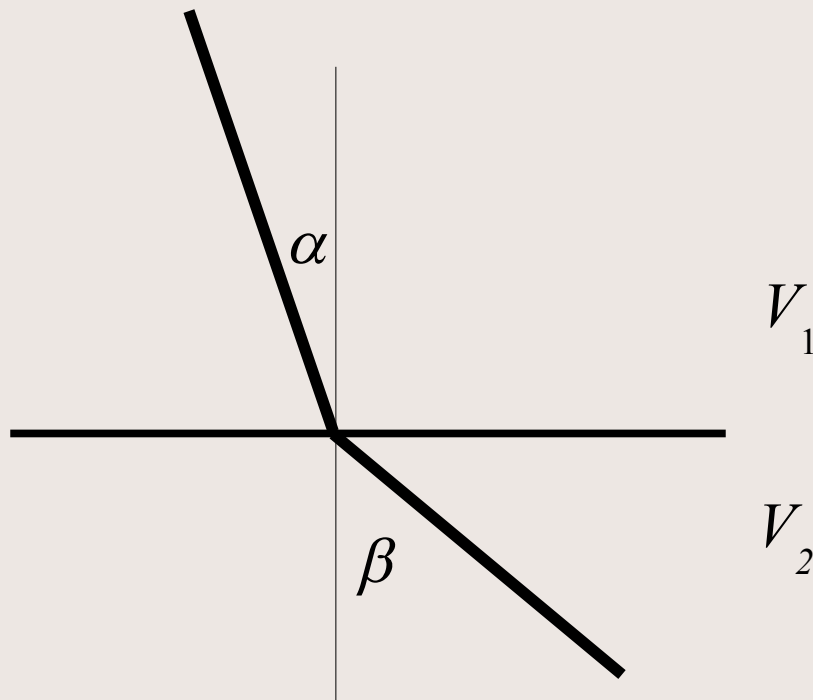
Seismic Phases
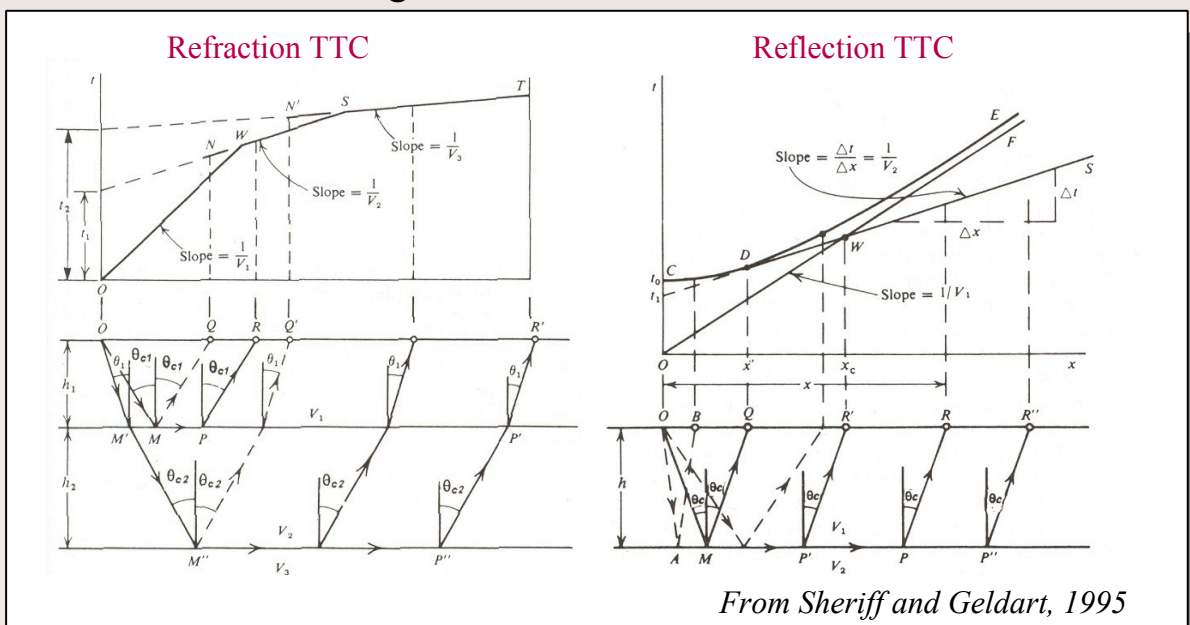## used in refraction/reflection seismics

- *P*- and *S*- body waves, sometimes surface (particularly '*Rayleigh*') waves:
  - Refracted (bending) across velocity interfaces.
  - 'Head waves' traveling along velocity discontinuities.
  - Reflected from velocity and/or density contrasts.

Impulsive Source

Seismographs

R

V_1, ρ_1

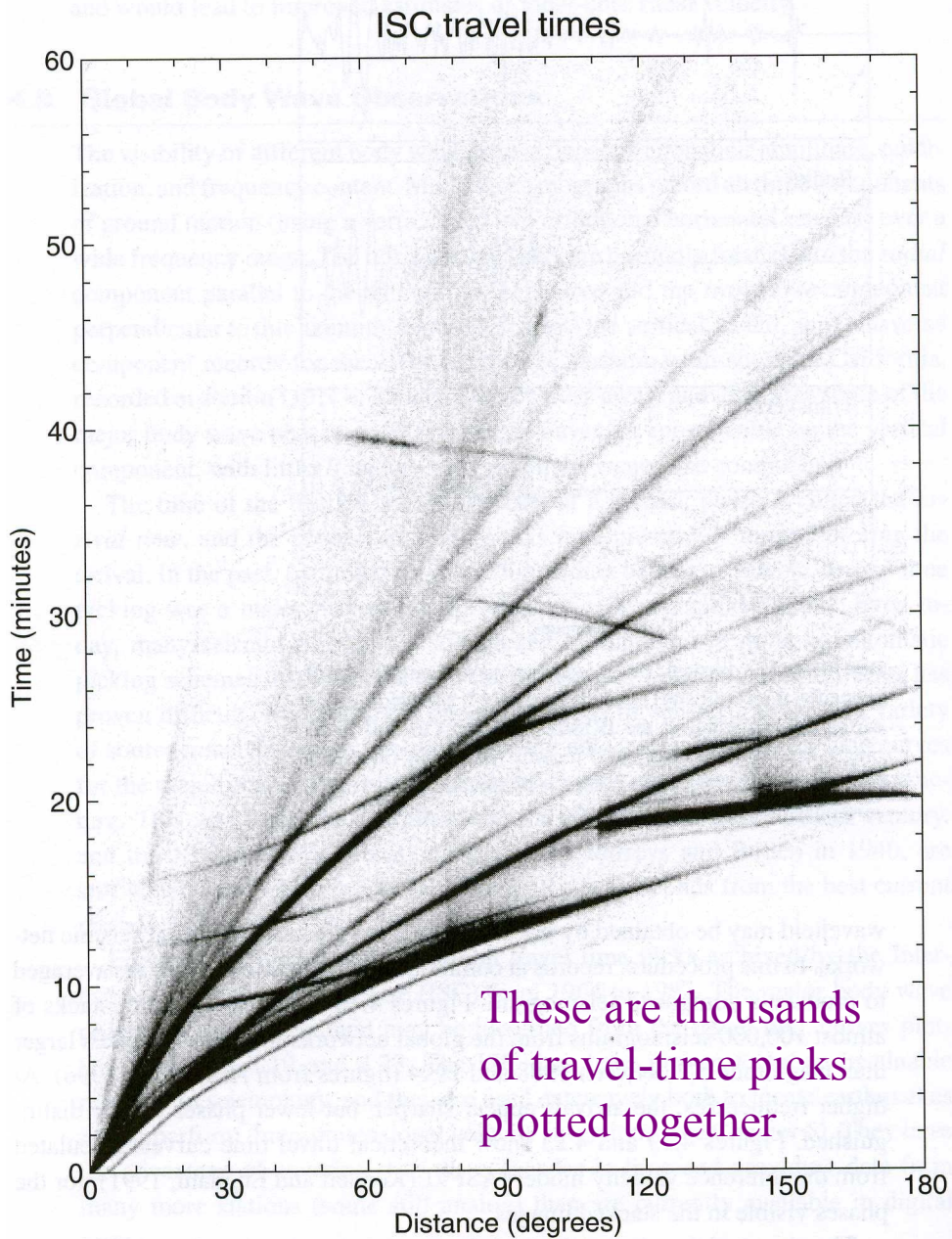V_2, ρ_2

Head waves

S waves

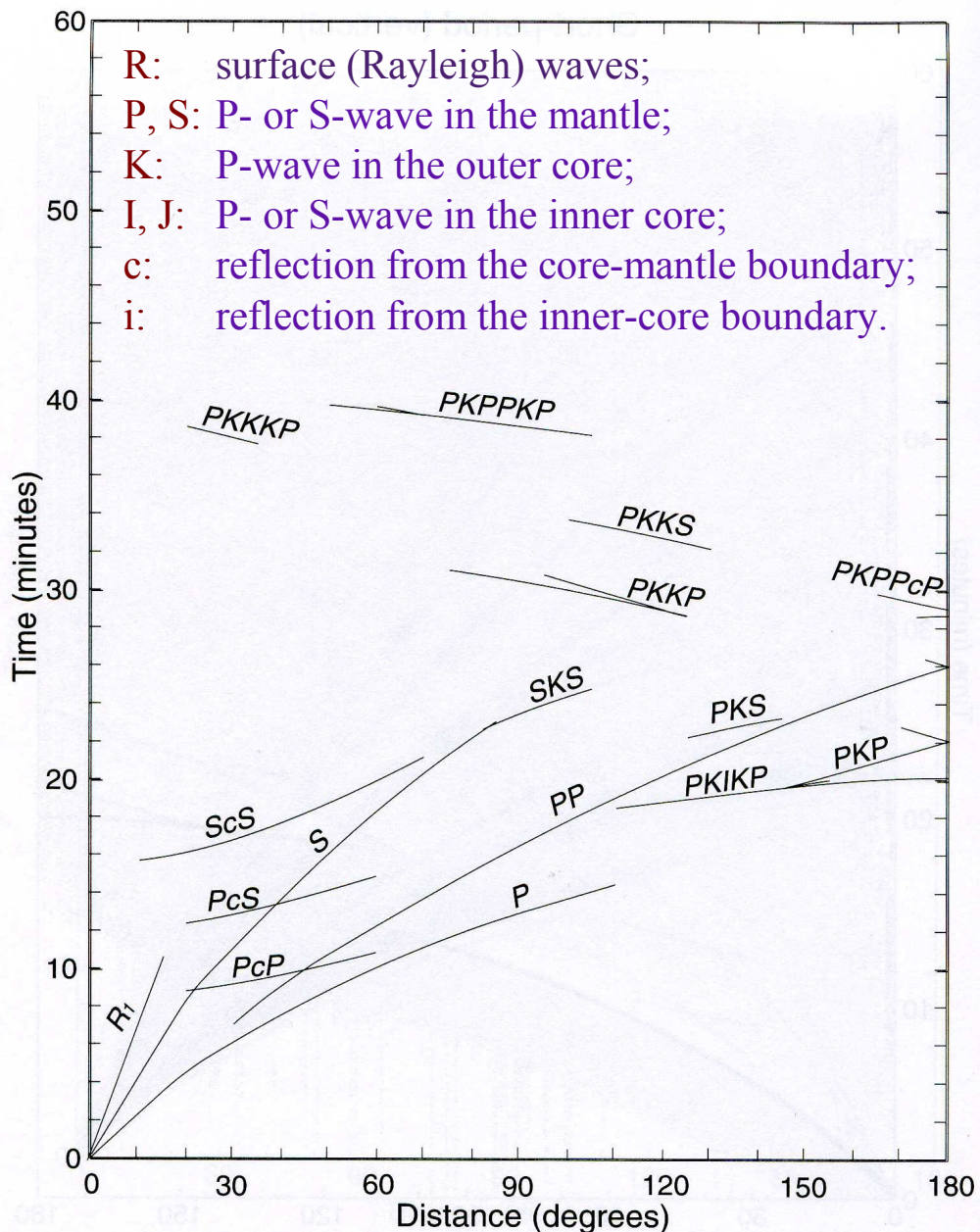P waves
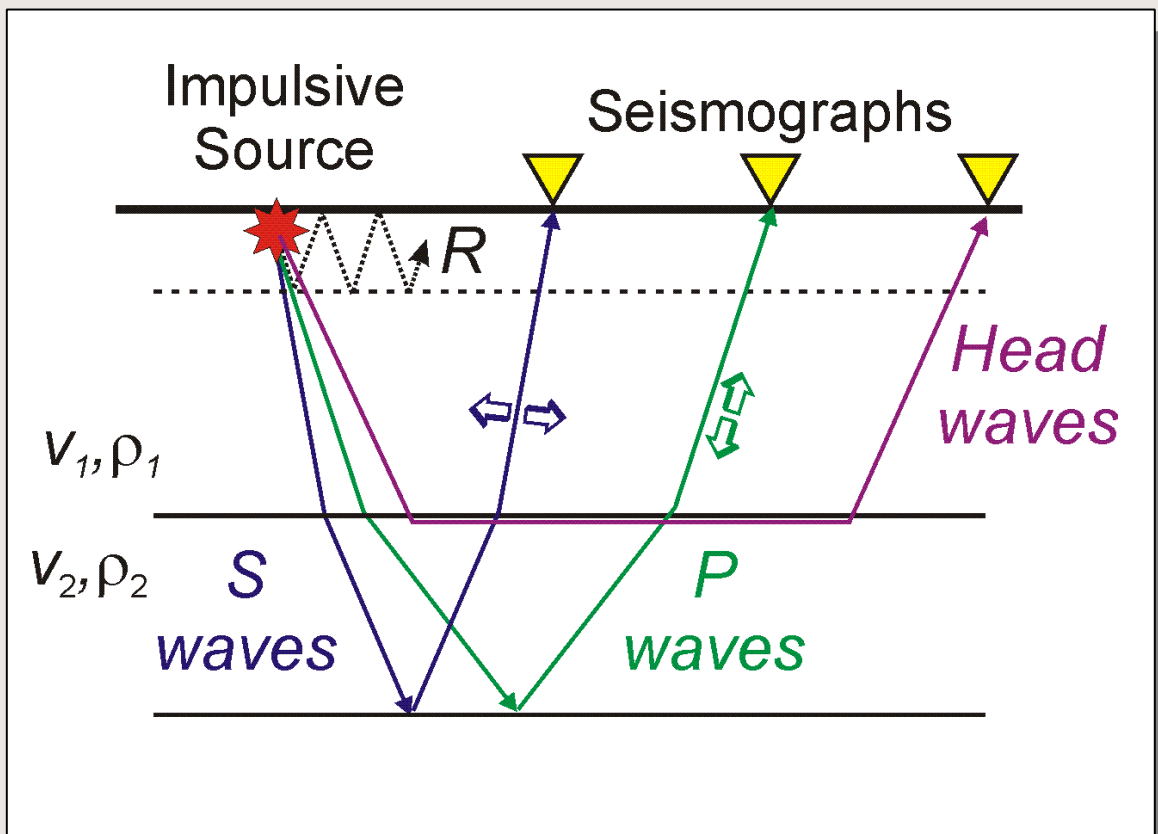
Travel-Time Dependencies
("Travel-Time Curves")

- When an arrival is identified in a dense geophone grid within a range of source-receiver *offsets* (distances), its travel times form a a set of *t*(*offset*) points, called the *travel-time curve* (TTC).

- Convex, piecewise-linear segments in the *first arrivals* are characteristic of *refractions*, strong, concave, hyperbolic secondary arrivals typically are reflections.

- The goal is to derive the velocity structure that could explain both:

  - Shapes of all the observed TTCs.

 - The offset ranges within which the arrivals are observed.

Refraction TTC   Reflection TTC

From Sheriff and Geldart, 1995

Global travel times

These are thousands of travel-time picks plotted together

Global seismic phase nomenclature

R: surface (Rayleigh) waves;
P, S: P- or S-wave in the mantle;
K: P-wave in the outer core;
I, J: P- or S-wave in the inner core;
c: reflection from the core-mantle boundary;
i: reflection from the inner-core boundary.

Global seismic ray paths

P, S: P- or S-wave in the mantle;
K: P-wave in the outer core;
I, J: P- or S-wave in the inner core;
c: reflection from the core-mantle boundary;
i: reflection from the inner-core boundary.
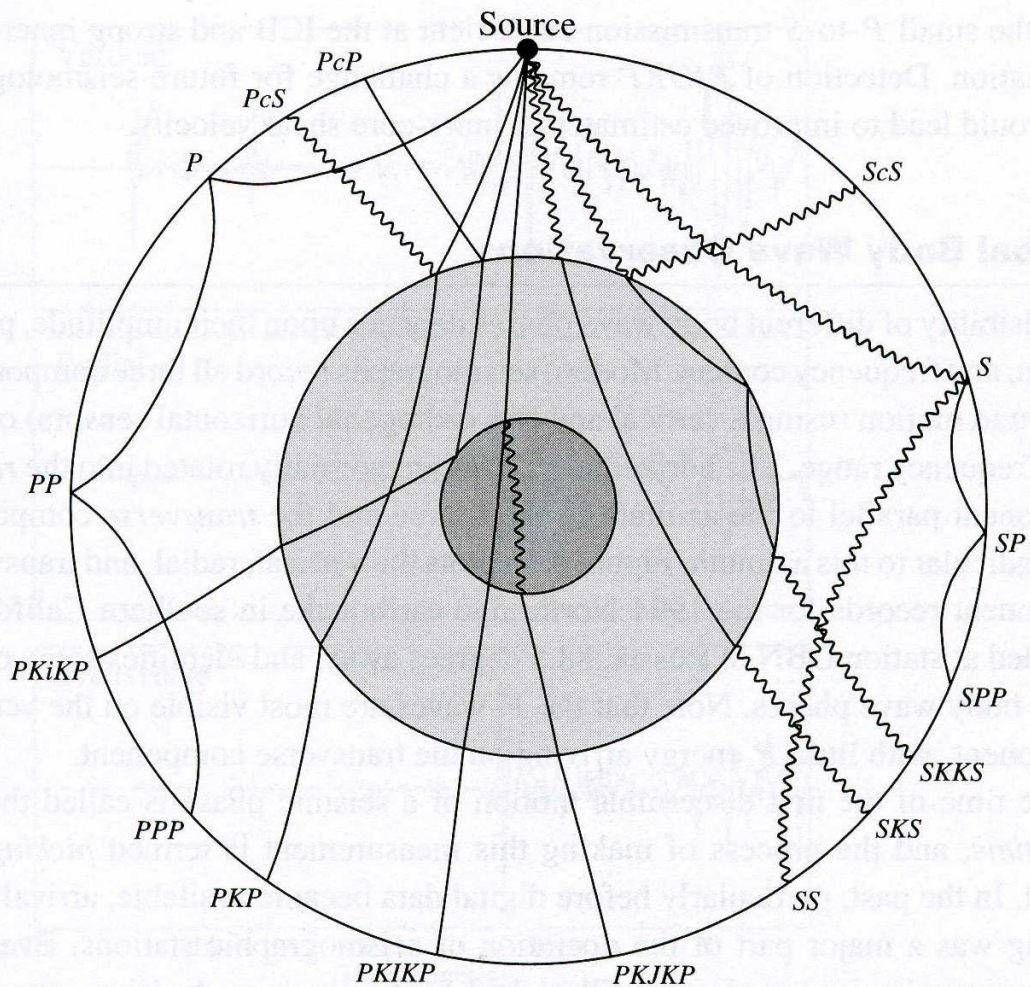
Refraction in a *laterally homogeneous* structure: Snell's law

- When waves (rays) penetrate a medium with a different velocity, they *refract*, *i.e.* bend toward or away from the normal to the velocity boundary.

- The *Snell's Law of refraction* relates the angles of incidence and emergence of waves refracted on a velocity contrast:

$$\frac{\sin\theta_1}{V_{P1}} = \frac{\sin\theta_2}{V_{P2}} = \frac{\sin\varphi_1}{V_{S1}} = \frac{\sin\varphi_2}{V_{S2}} = p$$
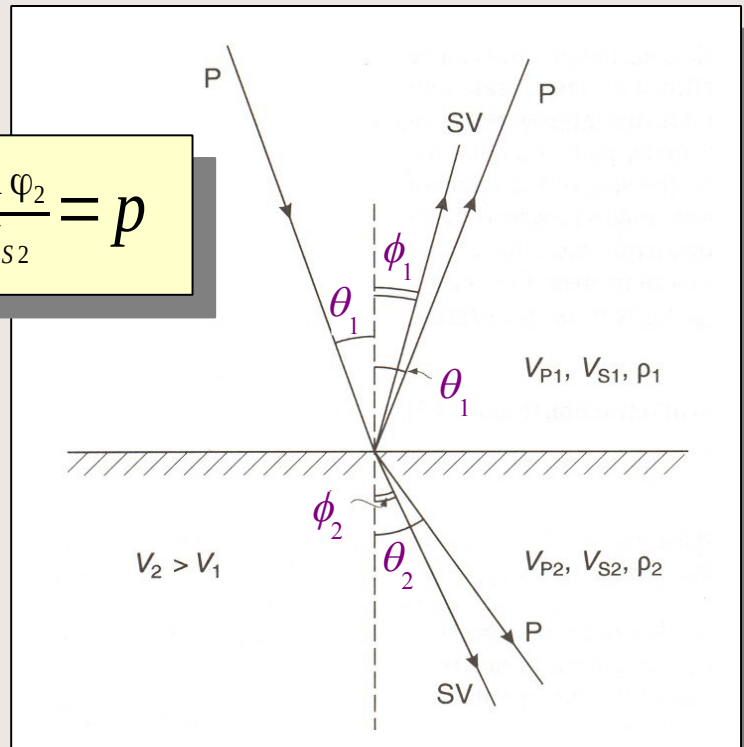
- The constant p is called *ray parameter*

- Note that refraction angles depend on the velocities alone!
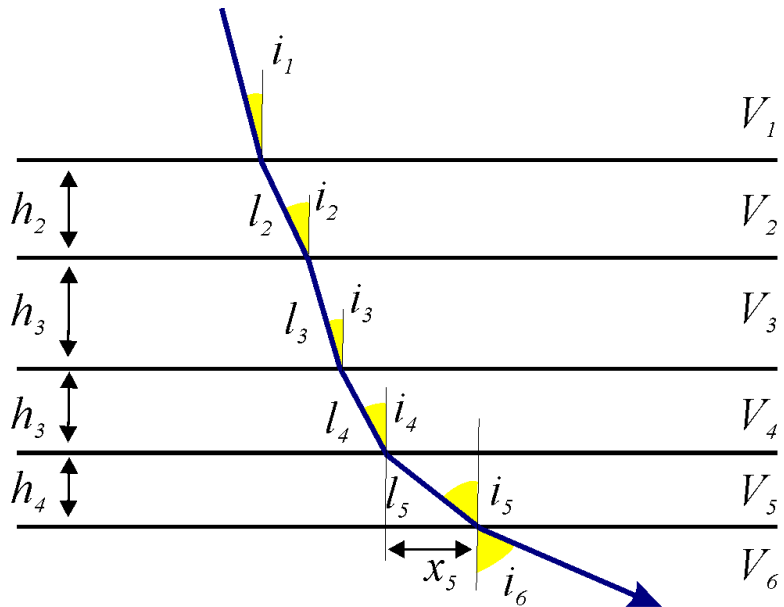
Refraction in a stack of horizontal layers

Ray parameter, *p*, *uniquely specifies* the entire ray.

It does not depend on layer thicknesses or velocities.

Travel times and distances accumulate along the ray to yield the total *T*(*X*)

For any layer:

$$\sin i_k = pV_k$$

$$l_k = \frac{h_k}{\cos i_k} = \frac{h_k}{\sqrt{1 - (pV_k)^2}}$$

$$t_k = \frac{l_k}{V_k} = \frac{h_k}{V_k \sqrt{1 - (pV_k)^2}}$$

$$x_k = l_k \sin i_k = \frac{h_k (pV_k)}{\sqrt{1 - (pV_k)^2}}$$

$$T_n = \sum_{k=1}^{n} t_k \qquad X_n = \sum_{k=1}^{n} x_k$$

Critical angle of refraction

- Consider a faster medium overlain with lower-velocity layer (this is the typical case).

- *Critical angle* of incidence in the slower layer is such that the refracted waves (rays) travel horizontally in the faster layer (sin $r = 1$)

- The critical angles thus are:

$$i_c = \sin^{-1} \frac{V_{P_1}}{V_{P_2}} \qquad \text{for } P \text{ waves}$$

$$i_c = \sin^{-1} \frac{V_{S_1}}{V_{S_2}} \qquad \text{for } S \text{ waves}$$

- Critical *ray parameter*: $p^{critical} = \dfrac{1}{V_{refractor}}$

- If the incident wave strikes the interface at an angle exceeding the critical one, *no refracted or head wave is generated*.

Critical refraction: Head Waves

- At critical incidence from the "slower" medium, a *head wave* is generated in the "faster" one.

- Although in reality head waves carry little energy, they are useful approximation for interpreting seismic wave propagation in the presence of strong velocity contrasts.

- Head waves are characterized by planar wavefronts inclined at the critical angle in respect to the velocity boundary:
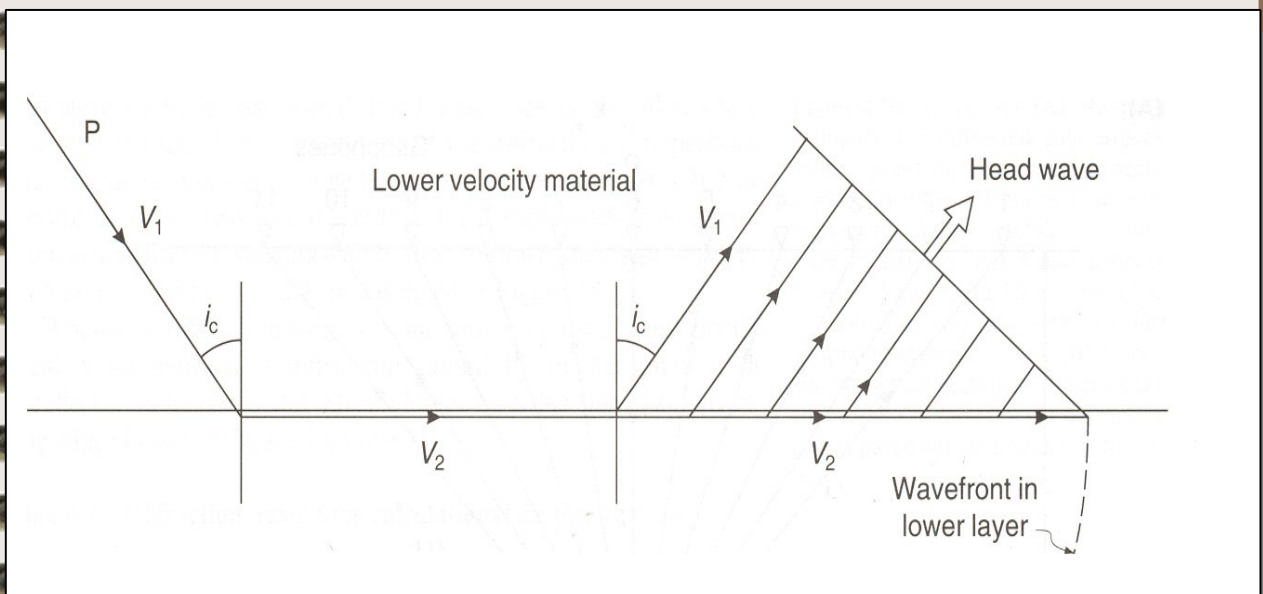
Head-wave travel times

- Head-wave travel-time curves are straight lines:

$$t = t_0 + \frac{x}{V_{app}}$$

Here, t_0 is the *intercept time*, and V_{app} is the *apparent velocity*

Note that "apparent" often means "as observed" (but may not be "true")