Digital Filtering

- Convolution of time series
- Convolution as filtering process
- Cross- and auto-correlation
- Frequency filtering
- Deconvolution

- Reading:
  - Telford et al., Sections A.10,11

Convolution of time series

- *Convolution* for time (or space) series is what commonly is multiplication for numbers.
- Example of a '*convolutional model*': rise in lake level resulting from rainfall
  - Let's assume that the recorded rainfall over 5 months is: 2, 3, 1, 4, 3 cm, respectively.
  - The lake level will respond to a 1 cm of rain fall, say, with a 2-cm rise in the first month and 1-cm in the second. This is called the '*impulse response*'.
  - Lake level rises from the different months of rainfall will accumulate *linearly.*

Lake Level Rise

Numerical example

- The resulting lake levels can be calculated by the following procedure, called *convolution*:

	1	*2*	*3*	*4*	*5*	*Months*
	2	3	1	4	3	*Rainfall time series*
1	2					Lake response (*reversed*)
	4					Rise in the first month
	2	3	1	4	3	
	1	2				
	2+ 6=8					Rise in the second month
	2	3	1	4	3	
		1	2			
		3+ 2=5				Rise in the third month
	2	3	1	4	3	
			1	2		
			1+ 8=9			Rise in the fourth month
	4	8	5	9		Resulting lake level sequence

- The impulse response series is reversed and shifted, and sample-by sample dot product is taken to find the response at any moment

Convolution

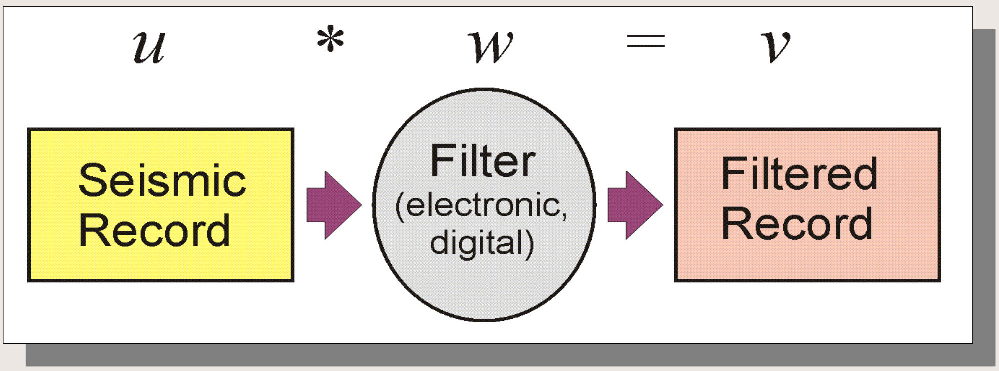
General formulae

- The resulting lake levels can be calculated by the following procedure, called *convolution.* Convolution of two series, u_i, and w_i, denoted $u*w$, is: $$(u*w)_k = \sum_i u_{k-i} w_i$$
- As multiplication, it is symmetric (commutative):
$$u*w = w*u$$
- Note that if we need to multiply two polynomials, with coefficients u_k and w_k, we would use exactly the formula above. Therefore, **in *Z* or *frequency* domains, convolution becomes simple multiplication** of polynomials (show this!):
$$u*w \leftrightarrow Z(u)Z(w) \leftrightarrow F(u)F(w)$$
- This is the key property facilitating efficient digital filtering.

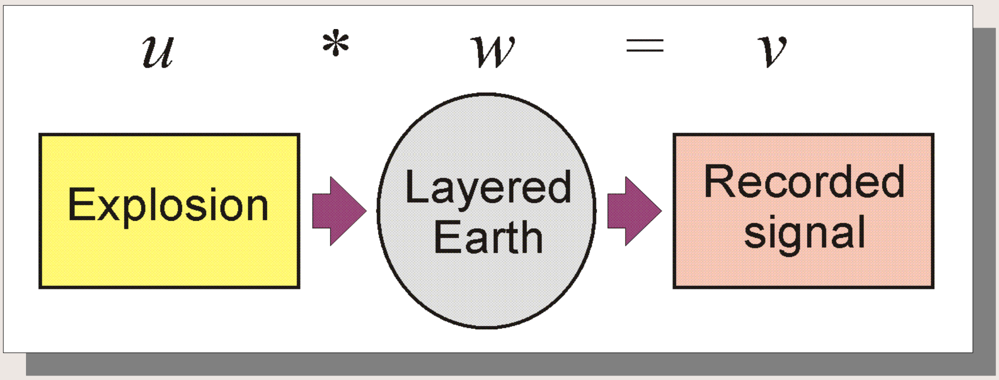
Convolution

Two important cases of interest

- **Digital signal filtering**

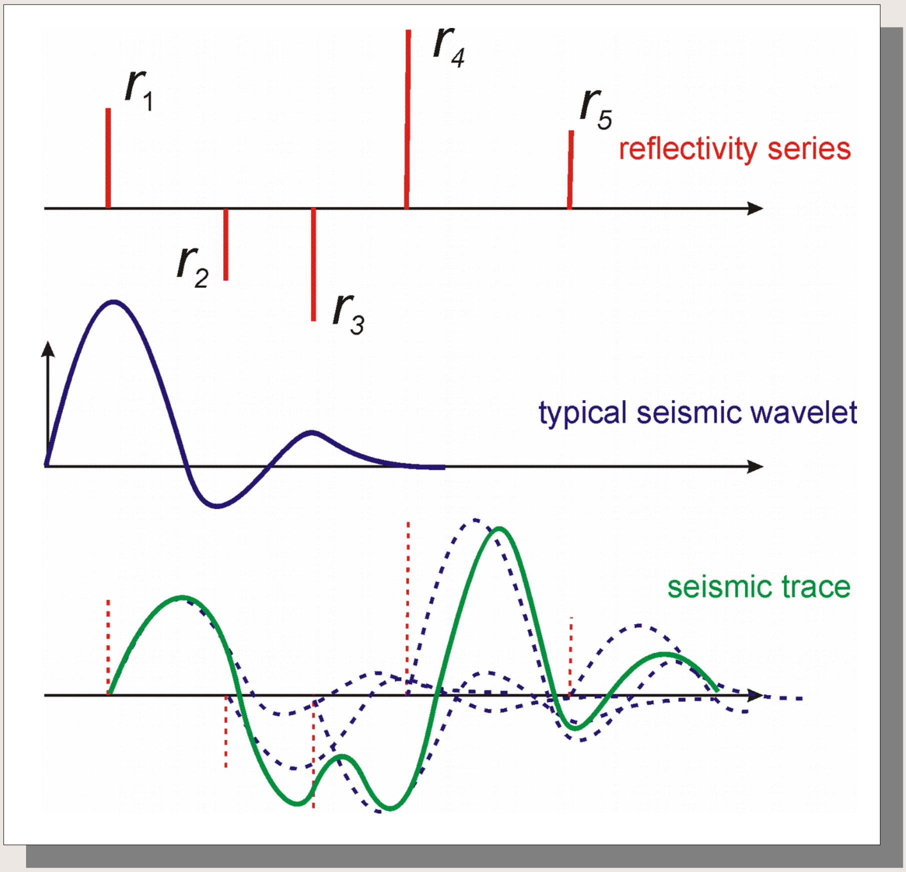


- **Earth's response** is also a filter. Note that in this case, the *impulse response* is unknown and is of primary interest
  - Hence reflection processing deals with *inverse filtering*… (*i.e.*, finding the filter)



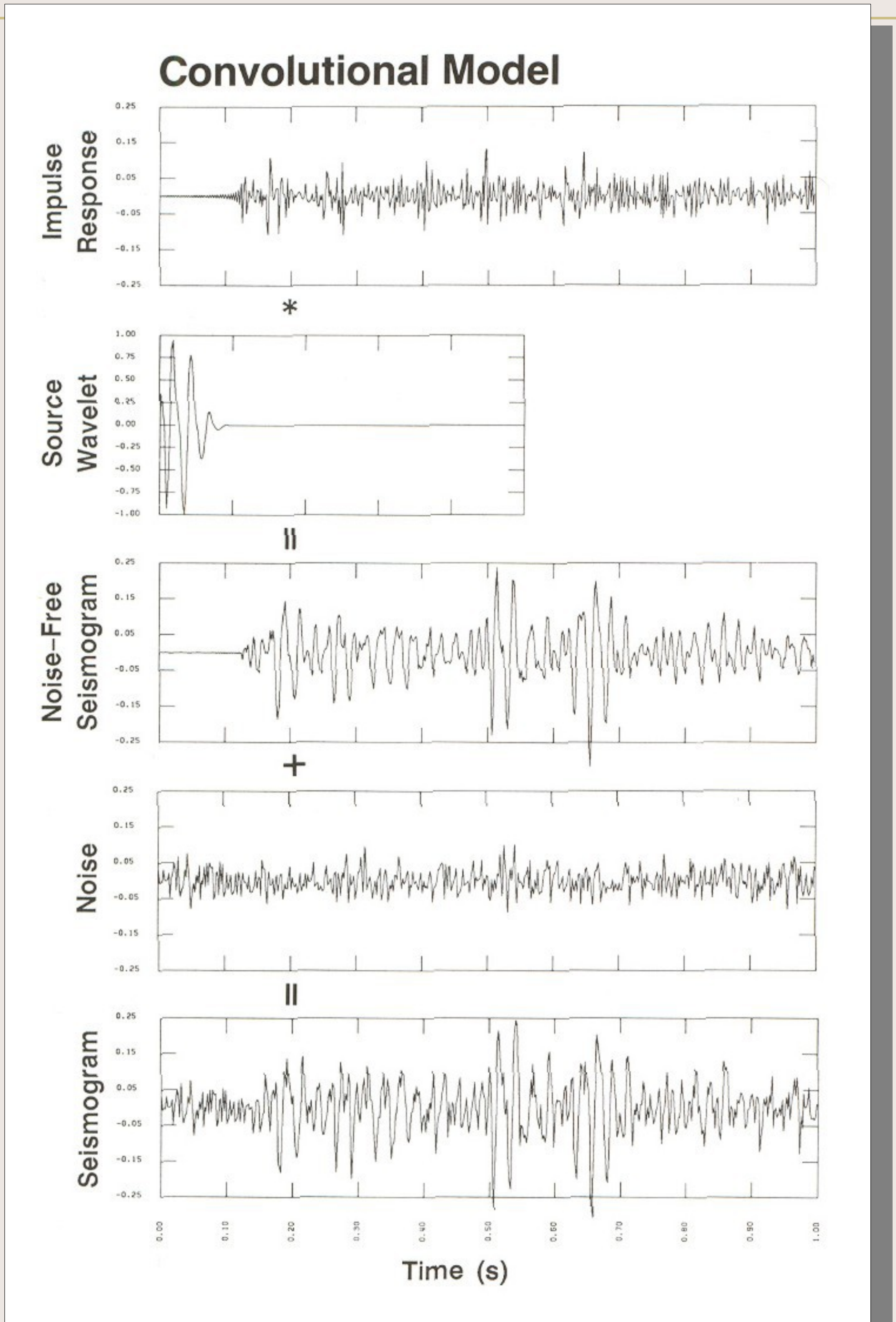
Convolutional model
of a reflection seismic record

- Reflection seismic trace is a convolution of the source wavelet with the Earth's ‘reflectivity series’



Convolutional model

A more realistic example



From Yilmaz, 1987

Cross- and Auto-correlation

- *Cross-correlation* gives the degree of similarity between two signals:
    - For each value of a 'lag' i:
    - Shift the second trace by the lag
    - Calculate dot product:

$$\text{cross}(u, w)_k = \sum_i u_{k+i} w_i$$

- The lag for which the cross-correlation is largest gives the time shift between the two records
    - A most important application – pre-processing of Vibroseis recordings
- *Auto-correlation* of a record is its cross-correlation with itself
    - It is symmetric in terms of positive and negative lags
    - It indicates the degree to which the signal repeats itself.

Linear Filtering

- Most operations with seismic signals can be represented by a convolutional operator:

$$v = F\,u$$

  - It is *linear*:

$$F(u_1 + u_2) = F\,u_1 + F\,u_2$$

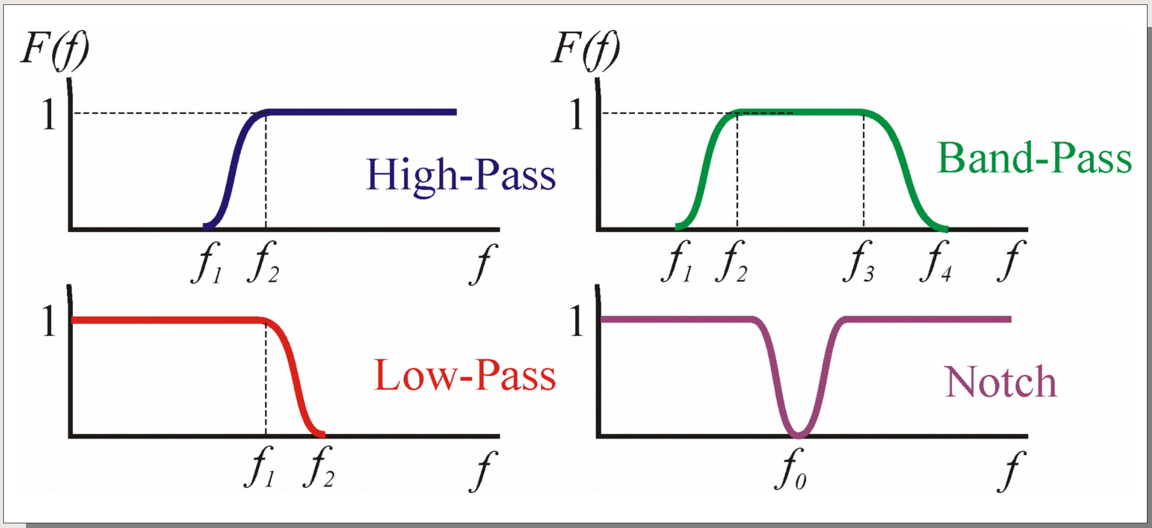
  - It is *translationally (time-) invariant*:

$$F\,time\,shifted_t(u) = time\,shifted_t(F\,u)$$

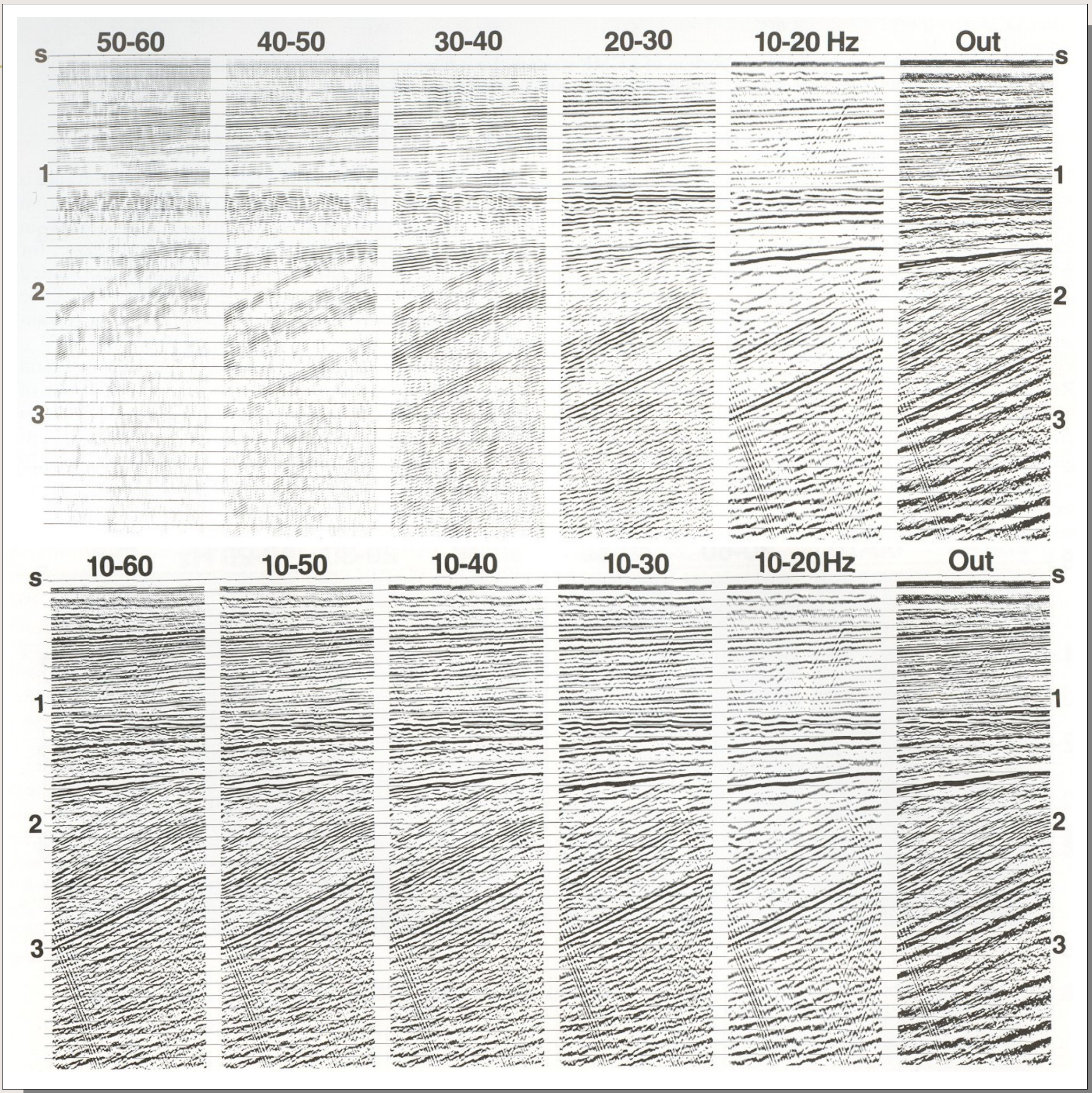
- The filtering operator is represented differently in different domains:
  - Convolution in time domain
  - Complex-value multiplication in *Z*- and frequency domains
    - This allows easy frequency filtering (selective enhancement or suppression of harmonic components in the signal)

Frequency Filtering

- **Key element** of seismic and GPR processing.
- Low-pass (high-cut), Band-pass, High-pass (low-cut), Notch
    - Suppressing the unwanted (noisy) parts of the frequency spectrum.
    - Usually *zero-phase*, to avoid phase (travel-time) distortion. This means that the filter does not change the phase spectrum.



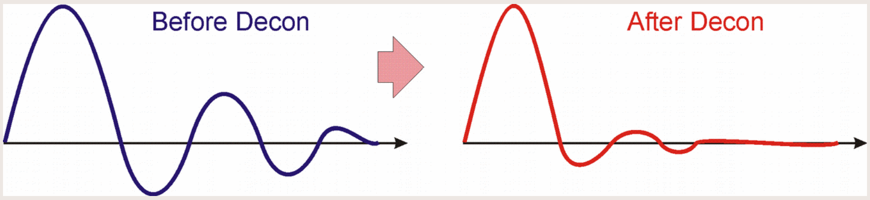
Filter panels



- Trial band-pass filter panels are designed in order to determine the best frequency range for data display and analysis. For a final display, *time-variant* filters are used.

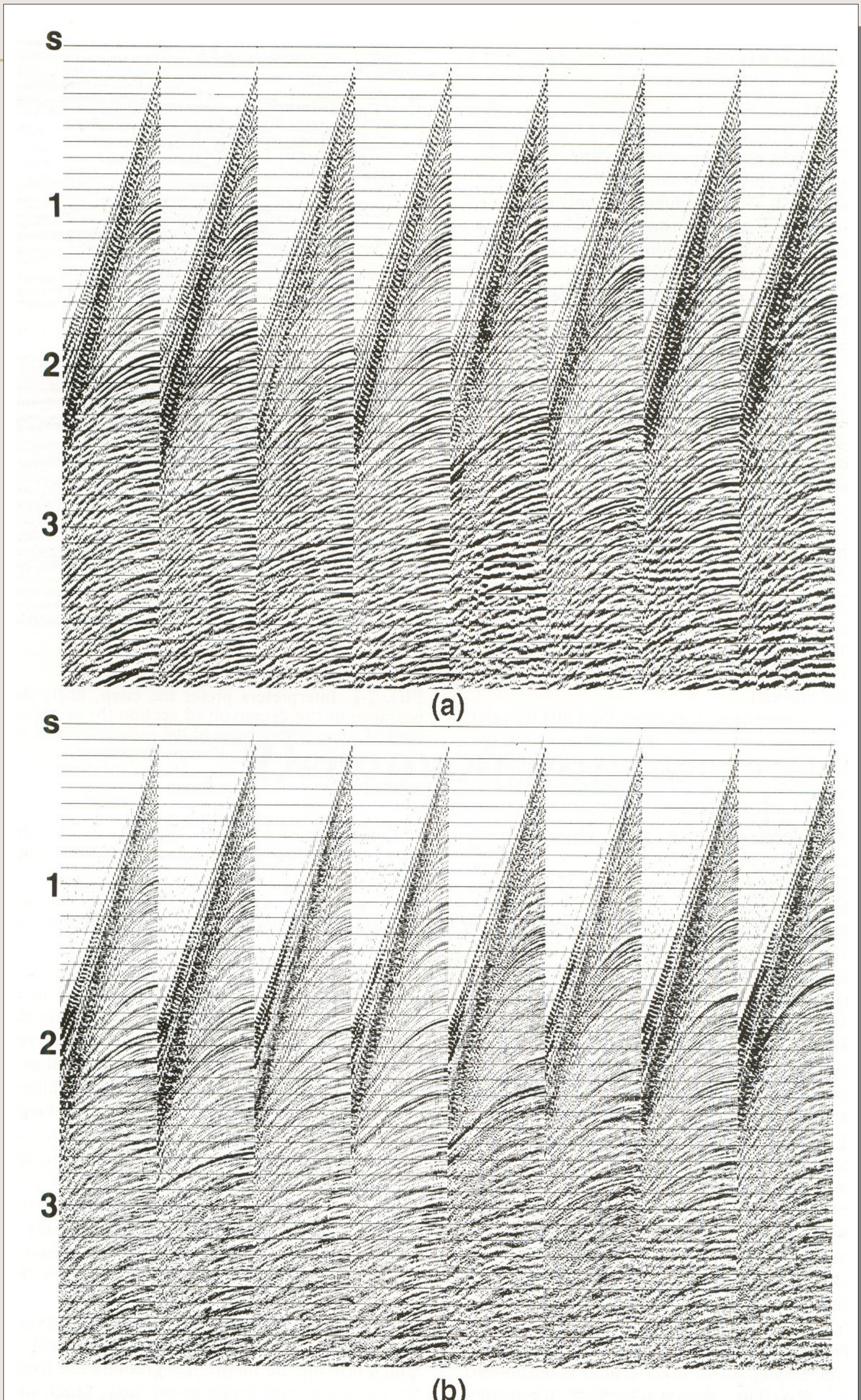
Deconvolution

- Deconvolution (inverse) filters
  - To remove (reduce) the effects of wavelet's complexity on the resulting image.
  - Based on the known (or estimated) wavelet shape, an *inverse* filter is designed with the objective to compress this wavelet in time:



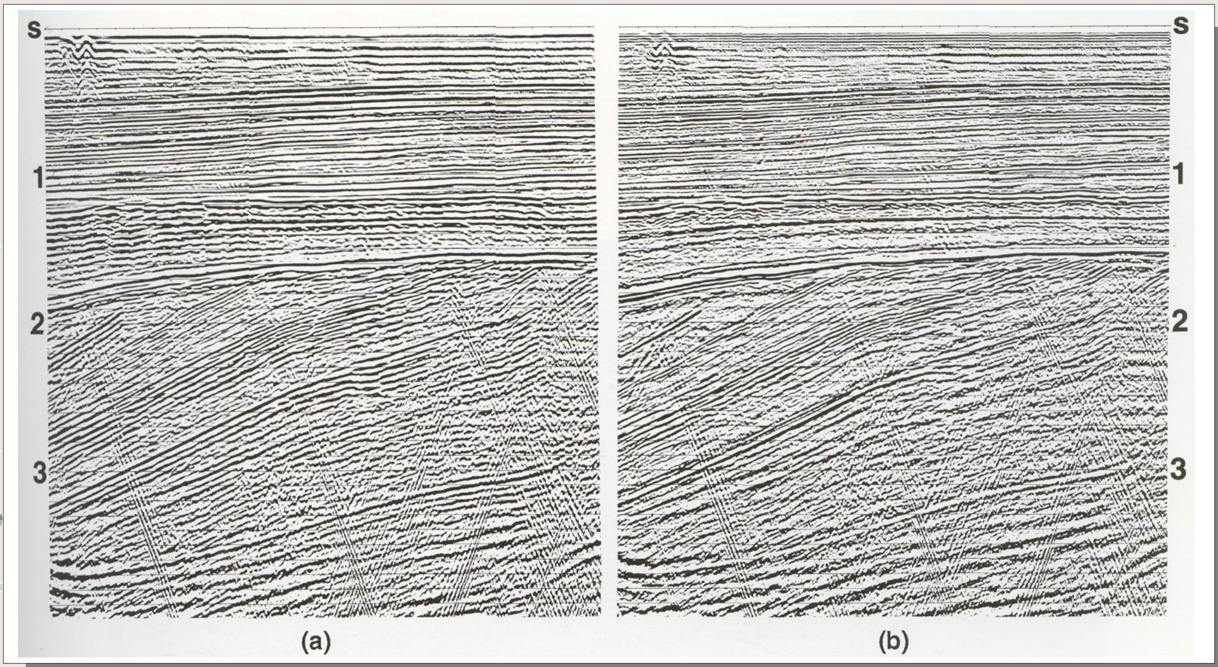
- Numerous deconvolution techniques are available
  - Performed in time or frequency domains.

Deconvolution
(shot gathers)



From Yilmaz, 1987

Deconvolution
(stack)



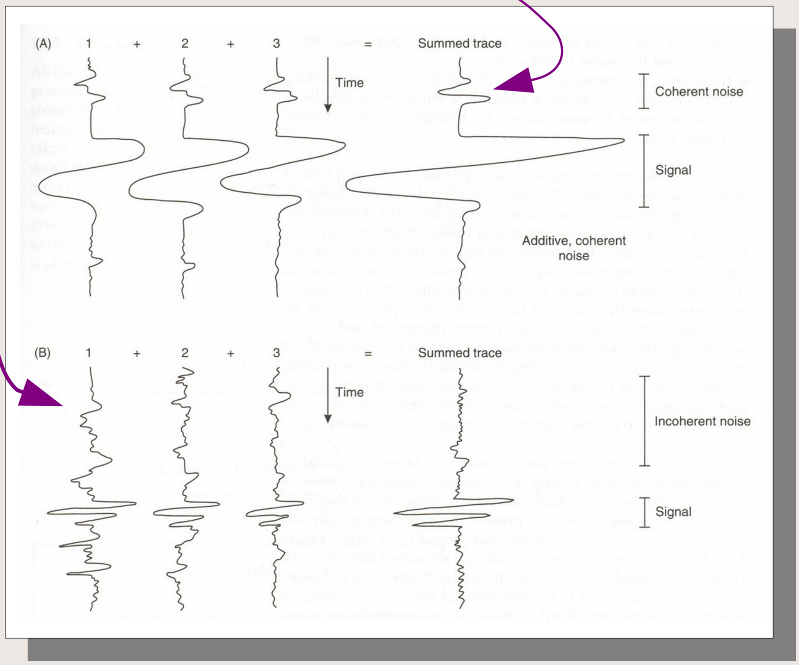
From Yilmaz, 1987

- Interpreters certainly prefer working with the crisper, high-resolution image (b) that uses deconvolution.

Example of Multichannel filter:

Stacking

- Records are summed ('*stacked*') in order to increase the S/N ratio:
  - Signal is assumed the same in all channels, therefore, its **amplitude** is increased $\propto N$ (the number of records);
  - For *incoherent noise*, the **energy** becomes proportional to N, and so the amplitude increases as $\sqrt{N}$;
  - Therefore, the S/N ratio $\propto \sqrt{N}$.
  - Note: *coherent noise* cannot be suppressed by stacking!



From Reynolds, 1997