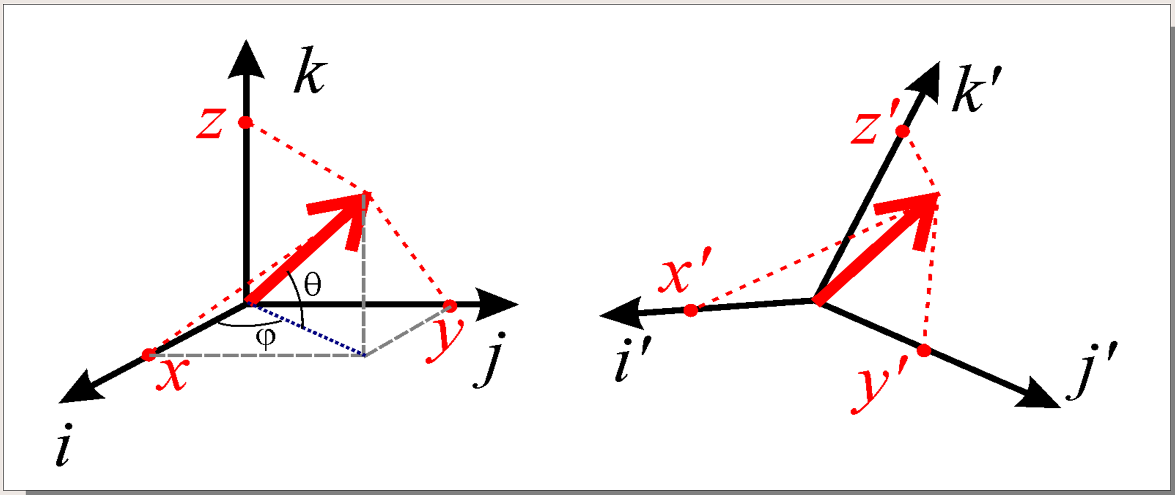
General Concepts

- Scalars, Vectors, Tensors, Matrices
- Fields
- Waves and wave equation
- Signal and Noise
- Reading:
  - Telford *et al*., Sections A.2-3, A.5, A.7
  - Shearer, 2.1-2.2, 11.2, Appendix 2

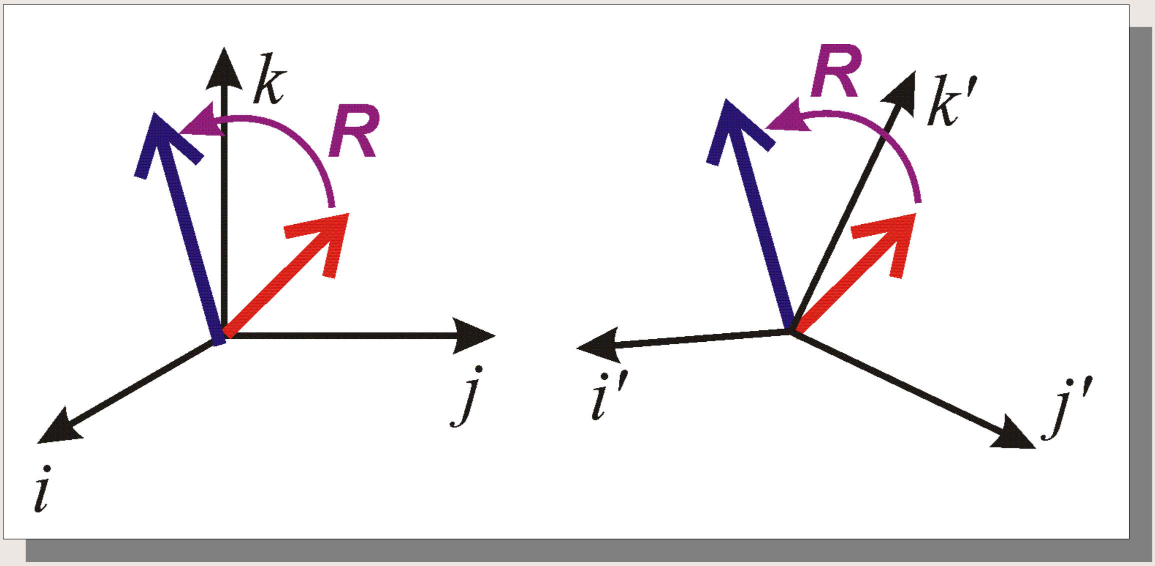
Vector

- Directional quantity
  - Possesses 'amplitude' and 'direction' and nothing else...
    - Thus it can be described by its amplitude and two directional angles (*e.g.*, *azimuth* and *dip*).
  - Characterized by projections on three selected axes: (*x*,*y*,*z*)...
    - ...plus an agreement that the projections are transformed appropriately whenever the frame of reference is rotated.



Tensor
(informal)

- Bi-Directional quantity
  - 'Relationship' between two vectors;
  - Represented by a *matrix:*
    - 3×3 in three-dimensional space, 2×2 in two dimensions, etc.
    - ...this matrix, however, is transformed whenever the frame of reference is rotated.
- Examples:
  - Rotation operator, ***R*** in the plot below;
  - Stress and strain in an elastic body.



Vector operations

- Summation: $\boldsymbol{c}=\boldsymbol{a}+\boldsymbol{b}$

  $$c_x=a_x+b_x, c_y=a_y+b_y, c_z=a_z+b_z$$

  or simply: $c_i=a_i+b_i$

- Scaling: $\boldsymbol{c}=\lambda\boldsymbol{b}$

  $$c_x=\lambda b_x, c_y=\lambda b_y, c_z=\lambda b_z$$ $$c_i=\lambda b_i$$

- Scalar (dot) product:

  $$c=\boldsymbol{a}\cdot\boldsymbol{b}=a_x b_x+a_y b_y+a_z b_z$$

  "Einstein's" notation: $c=a_i b_i$

- Vector (cross) product:

  $$\boldsymbol{c}=\boldsymbol{a}\times\boldsymbol{b}=\begin{Vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{Vmatrix}.$$

  $$c_i=\epsilon_{ijk} a_j b_k$$

Two key matrices

- Unit (identity):

$$\boldsymbol{I} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}$$

$$I_{ij} = \delta_{ij}$$

- δ_{ij} is called the "Kronecker symbol":

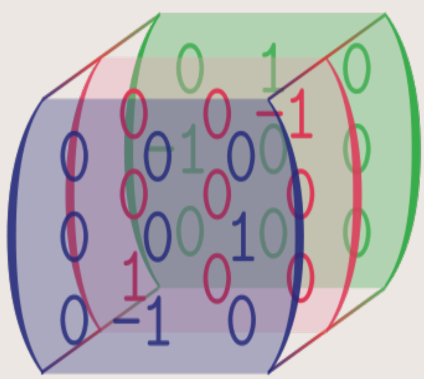
$$\delta_{ij} = \begin{matrix} 1 & for\ i = j \\ 0 & for\ i \neq j \end{matrix}$$

Exercise: evaluate $\delta_{ii} = ?$

Two key matrices

- Antisymmetric (or permutation, “Levi-Civita”) symbol:

$$\epsilon_{ijk} = \begin{cases} 1 & for\ (i,j,k) = even\ permutations\ of\ (1,2,3) \\ -1 & for\ odd\ permutations\ of\ (1,2,3) \\ 0 & otherwise \end{cases}$$

$\epsilon_{ijk} =$ 

key identities: $\epsilon_{ijj} \equiv \epsilon_{kik} \equiv \epsilon_{lli} \equiv 0$

vector cross-product: $c_i = \epsilon_{ijk} a_j b_k$

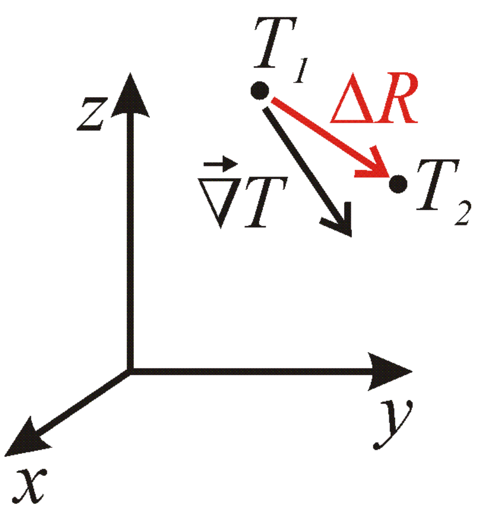
Exercise: evaluate $c_k = \epsilon_{ijk} \delta_{ij}$

Field

- Physical quantity which takes on values at a continuum of points in space and/or time
  - Represented by a function of coordinates and/or time:
    - Scalar: $f(x, y, z, t)$ or $f(\mathbf{r}, t)$
      - Examples: temperature, density, seismic velocity, pressure, gravity, electric potential
    - Vector: $\boldsymbol{F}(\mathbf{r}, t)$
      - Examples: particle displacement, velocity, or acceleration, force, electric or magnetic field, current
    - Tensor
      - 'relation' between two vectors
      - Examples: strain and stress, electromagnetic field in electrodynamics
      - The only way to describe *anisotropy*
  - Always associated with some *source*, carries some kind of *energy*, and often able to propagate *waves*
- *Everything in physics is fields!*

Scalar Fields

- Gradient
  - Spatial derivative of a scalar field (say, temperature, T(x,y,z,t))
  - It is a Vector field, denoted ∇T ('nabla' T):



$$\Delta T = T_2 - T_1$$

$$= \vec{\nabla} T \cdot \Delta \vec{R}$$

$$= \frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y + \frac{\partial T}{\partial z} \Delta z$$

$$\Delta \vec{R} = \vec{i}\, \Delta x + \vec{j} \Delta y + \vec{k} \Delta z$$

$$\vec{\nabla} T = \vec{i}\, \frac{\partial T}{\partial x} + \vec{j}\, \frac{\partial T}{\partial y} + \vec{k}\, \frac{\partial T}{\partial z}$$

Vector Fields

Differential operations

- Gradient of a vector field is a tensor:

$$\left(grad\,U_j\right)_i = \partial_i U_j$$

- Curl operation produces a new vector field:

$$\left(curl\,\boldsymbol{F}\right)_i = \epsilon_{ijk}\partial_j F_k$$

Two Important Relations

- Divergence of a curl is always zero:

  $\mathrm{div}(\mathbf{curl}(\boldsymbol{\psi})) \equiv 0.$

  This will be the S wave

- Curl of a gradient is zero:

  $\mathbf{curl}(\mathbf{grad}(\phi)) \equiv \mathbf{0}.$

  This will be the P wave

- These properties are easily verified using Einstein's notation (try this!):

$$(grad\, U)_i = \partial_i U$$

$$(curl\, \boldsymbol{F})_i = \epsilon_{ijk} \partial_j F_k$$

Static Fields and Waves

- Fields in geophysics typically exhibit either *static* or *wave* behaviours:
  - Static – independent on time:

$$\frac{\partial T}{\partial t}=0.$$

Stationary temperature distribution (geotherm).

  - Wave – stable spatial pattern propagating with time:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}-\nabla^2 p=0$$

Acoustic (pressure) wave.

This is the typical form of wave equation; c is the velocity of propagation.

$$p=f(x-ct)$$

Plane wave propagating along the X-axis.

$f(...)$ is the waveform at time t, it has its "zero" at $x = ct$ and propagates to $x > 0$

Exercise: show this.

Signal and Noise

- Geophysical data always contain some SIGNAL and some NOISE
  - Signal - 'deterministic' part that we want to know
    - Consistent with the method employed
  - Noise - anything else mixed into the measurement
- Sources of noise:
  - Instrument
  - Geologic sources
  - Too simple theory (*e.g.*, 2-D sounding in a 3-D Earth)
- Types of noise
  - Coherent (caused by the signal itself, worst of all)
  - Incoherent (random, coming from unrelated sources)
    - Such noise can be reduced by filtering
- Main task of data processing is to increase the signal/noise (S/N) ratio

S/N improvement by stacking

- "Stacking" (summation) is the most common approach to increasing the Signal/Noise ratio
- To derive the *S/N improvement factor,* consider stacking of *N* records with identical signals and random noise:

$$u_i(t) = s(t) + n_i(t)$$

- Stacked signal amplitude is proportional to *N*:

$$\sum_{i=1}^{N} u_i(t) = Ns(t) + \sum_{i=1}^{N} n_i(t)$$

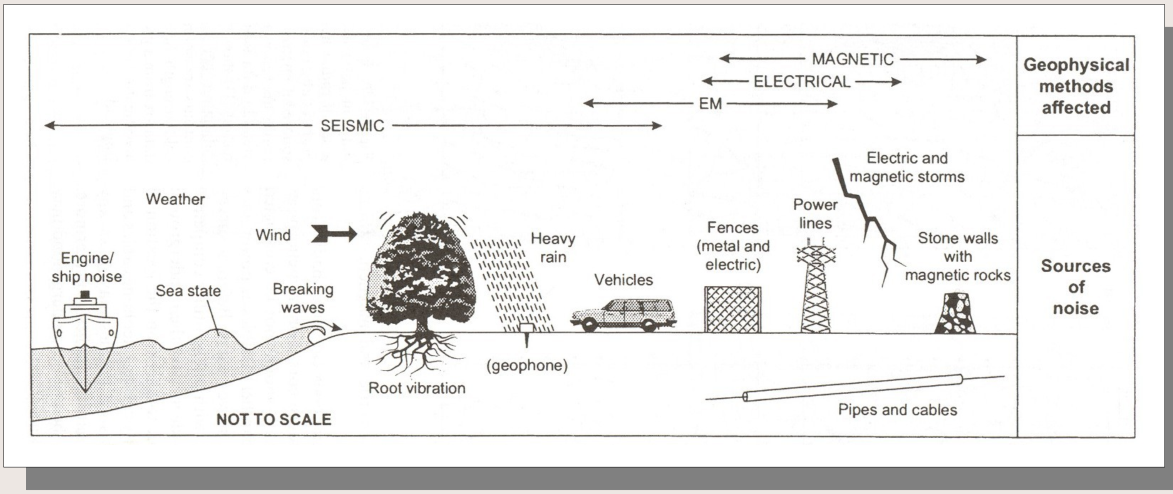
- Noise power increases $\propto N$ (despite what it is commonly said, noise is not "attenuated" by stacking!):

$$\langle (\sum_{i=1}^{N} n_i(t))^2 \rangle = \sum_{i=1}^{N} \sum_{j=1}^{N} \langle n_i(t) n_j(t) \rangle = N \langle n^2(t) \rangle$$

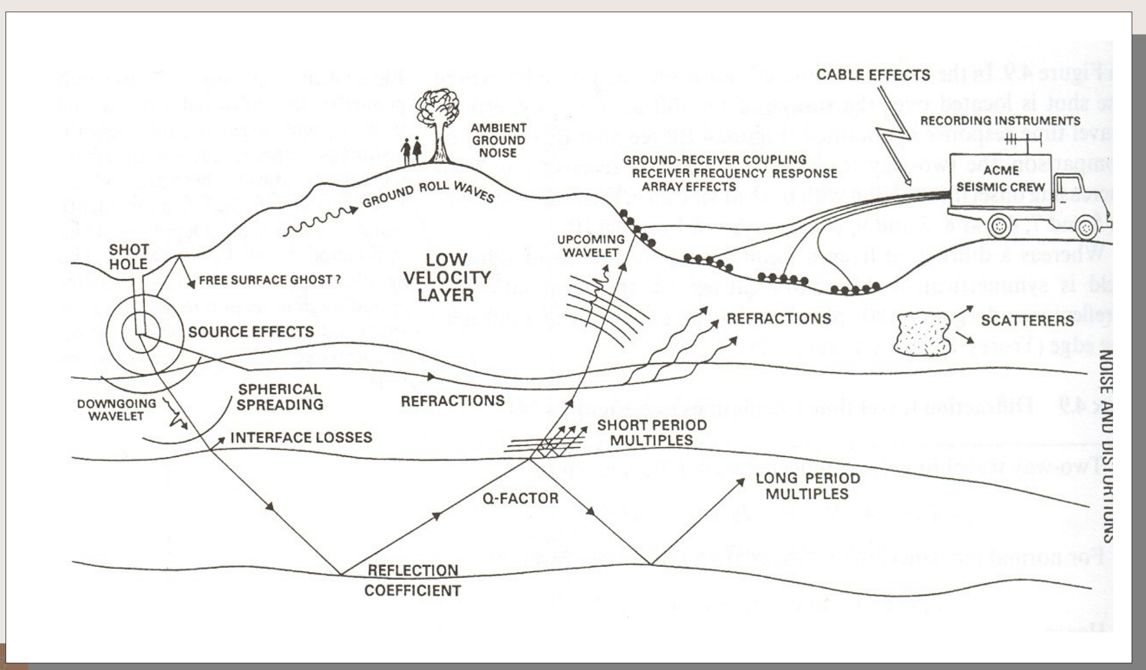
- Therefore: $\frac{S}{N} = \sqrt{N} \frac{s}{n}$

S/N ratio increases as $\sqrt{N}$

Noise in Geophysical Measurements



- For seismics, the *signal* is represented by reflections and refractions
  - For 2D, also only those coming in-plane.
- Several factors cause degradation of the seismic signal:



From Reynolds, 1997