Geometrical Seismics
(Seismic phenomenology)

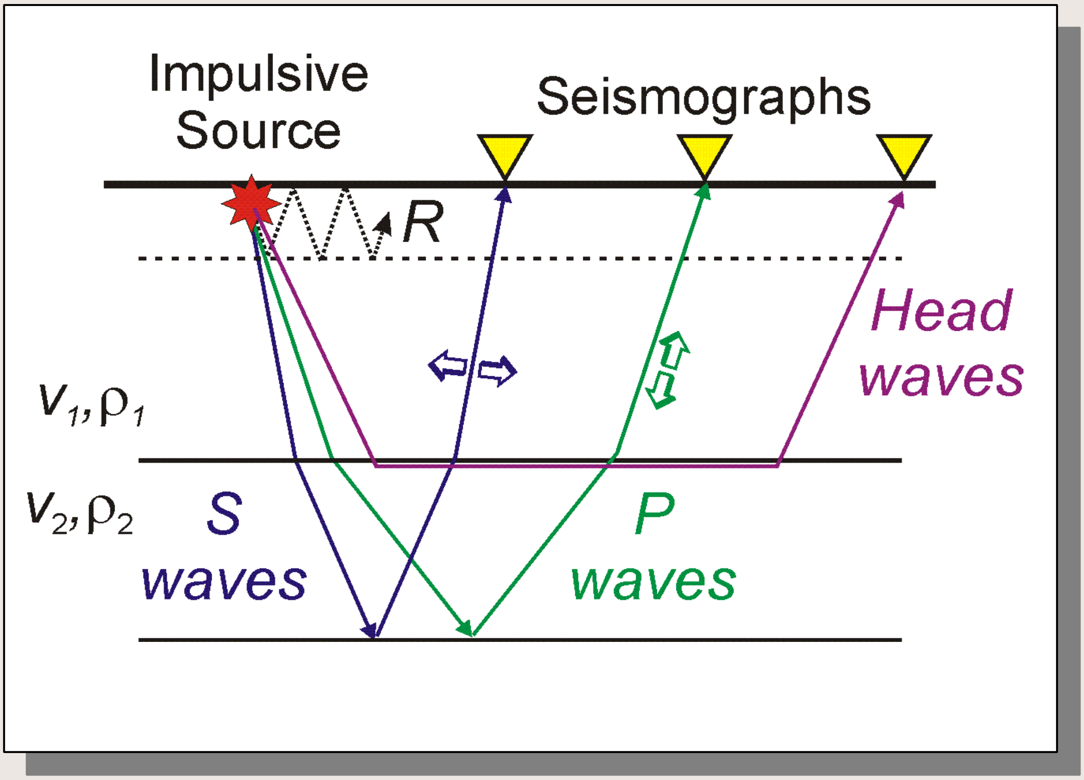
- *P*- and *S*-waves, typical velocities;
- Wavefronts;
- Rays;
- Reflection, Refraction, Conversion;
- Head wave (critical refraction);
- Huygens' principle;
- Fermat principle;
- Snell's law of refraction;
- Seismic wave nomenclature.
- Reading:
  - Reynolds, Sections 5.1-3, 6.1-6.2.2
  - Shearer, 4.1-4.3, 4.9
  - Telford et al., Sections 4.3-4.

Seismic Method
How it works

- Generate a mechanical elastic *signal*
  - "Controlled source" (impact on the ground): Locations and times are known with precision
  - "Passive source": Locations and times need to be determined
- Several types of seismic waves are generated
  - So we have to be able to recognize them!
- Signal travels through the subsurface
- At boundaries between different media, the energy is *reflected*, *transmitted*, or *refracted*
- The transformed signal is recorded by the receivers on the surface (or borehole, *etc*.)
  - Locations of all detectors are known with precision.
- *Arrivals* are identified and their *times* (and amplitudes) are determined
- Travel-times are used to determine the subsurface *velocities* and the *positions of boundaries*

Forward and Inverse Seismic Problems

- Forward problem
  - Layer thicknesses and velocities are known
  - *Calculate arrival times* (easy to do)
- Inverse problem
  - Arrivals are identified (where possible)
  - Arrival times are known
  - Find *velocities* and *depths* (not so easy to do)



Seismic Properties

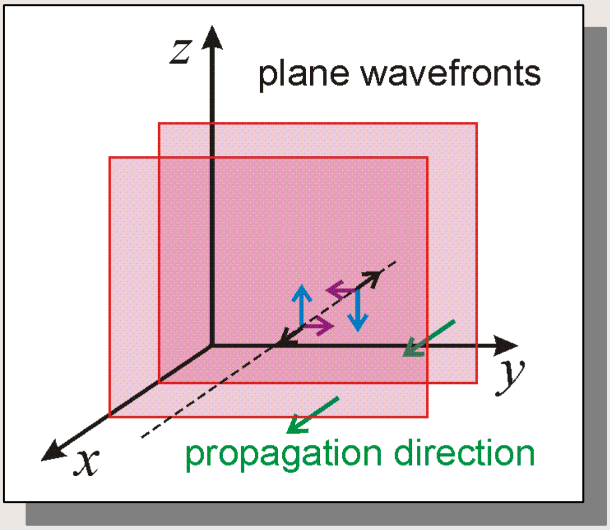
- *P*- (*primary* or "*pressure*", faster) and *S*- (*secondary* or "*shear*", slower) waves are most important
- Their propagation and reflections depend on *elastic velocities* (V_P, V_S) of the medium and its *density*

Material	*Vp (m/s)*	*Vp (ft/s)*	*Vs (m/s)*	*Vs (ft/s)*	*ρ(g!cm^3)*
Air	332	1090			0.0038
Water	1,400 - 1,600	4,600 - 5,250			1.0
Soil	300 - 900	980 - 2,950	120 - 360	390 - 1,180	1.7 - 2.4
Sandstone	2,000 - 4,300	6,560 - 14,100	700 - 2,800	2,300 - 9,190	2.1 - 2.4
Chalk	2,200 - 2,600	7,220 - 8,530	1,100 - 1,300	3,610 - 4,270	1.8 - 3.1
Limestone	3,500 - 6,100	11,490 - 20,000	2,000 - 3,300	6,560 - 10,830	2.4 - 2.7
Dolomite	3,500 - 6,500	11,490 - 21,330	1,900 - 3,600	6,240-11,810	2.5 - 2.9
Salt	4,450 - 5,500	14,600 - 18,050	2,500 - 3,100	8,200 - 10,170	2.1 - 2.3
Granite	4,500 - 6,000	14,770 - 19,690	2,500 - 3,300	8,200 - 10,830	2.5 - 2.7
Basalt	5,000 - 6,400	16,400 - 21,000	2,800 - 3,400	9,190 - 11,160	2.7 - 3.1
Quartz	6,049	19,846	4,089	13,415	2.65
Calite	6,640	21,783	3,436	11,273	2.71

- Velocities are sensitive to multiple factors:
  - Lithology,
  - Pressure, depth of burial (increase)
  - Temperature (decrease)
  - Fractures, porosity, fluid content (decrease)
  - Anisotropy,...

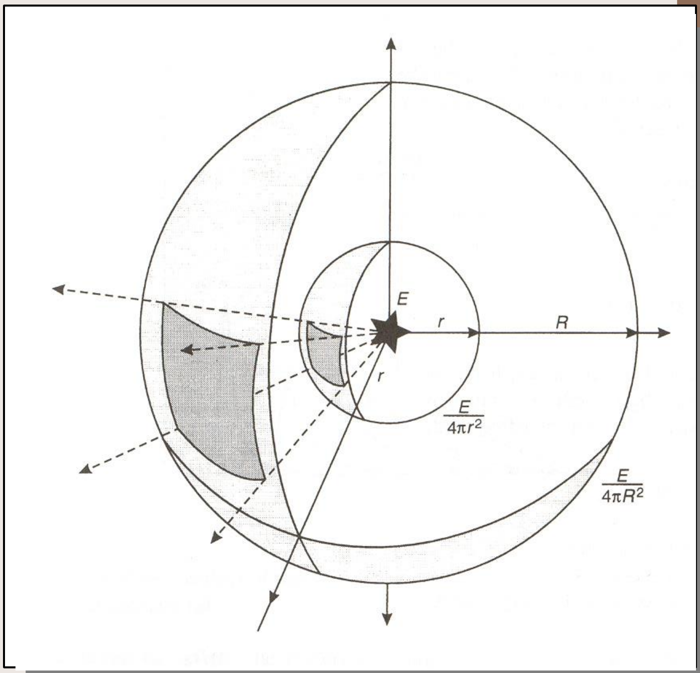
Wavefronts and Rays

- Vibrations originate at the source and propagate away from it
- Wavefronts are defined as surfaces of constant propagation time
- Rays are lines that are orthogonal to the wavefronts at every point
- Wavefronts propagate along the rays at the local wave velocity within the medium
- Rays generally indicate wave propagation direction and energy flux.
  - However, only in relatively simple cases free of 'caustics' and 'diffractions'
- In a homogeneous medium, wavefronts are spheres of progressively increasing radii.
  - At greater distances, spherical wavefronts approach planar shapes:



Spherical divergence

- **Waveform/Ray picture is very commonly used in the seismic method.**
- Ray diagrams also allow estimation of the *wave amplitude decay* due to geometrical (spherical) spreading:



- The amplitude progressively decreases so that the energy *E* passing through the shaded spherical shell remains constant
- Spherical-shell area *S* is proportional to R^2, and therefore the energy density

$$A^2 \propto \frac{E}{S} \propto \frac{1}{R^2}$$

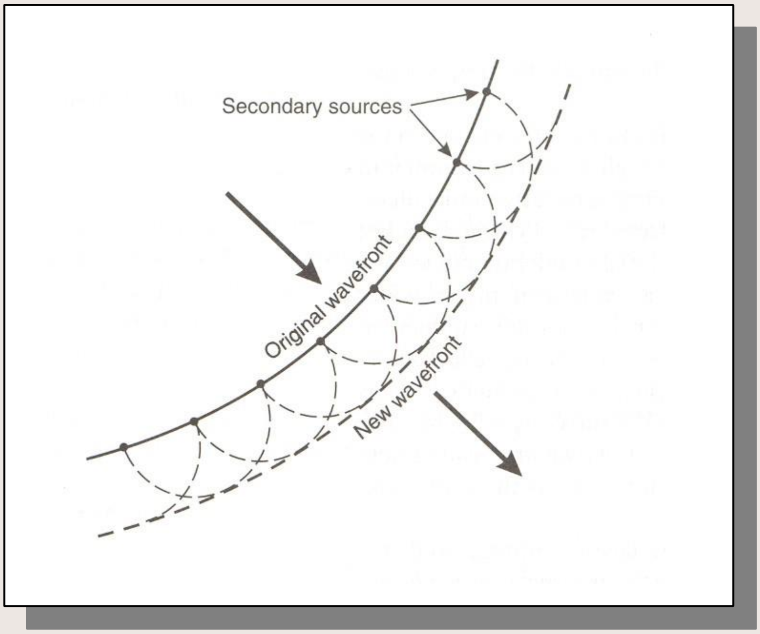
- Therefore, *for spherical wavefronts* (and straight rays), the amplitude *A* decreases with distance, as: $A \propto \frac{1}{R}$

Exercise: show that for cylindrical waves, spreading is

$$A \propto \frac{1}{\sqrt{R}}$$

Huygens' Principle

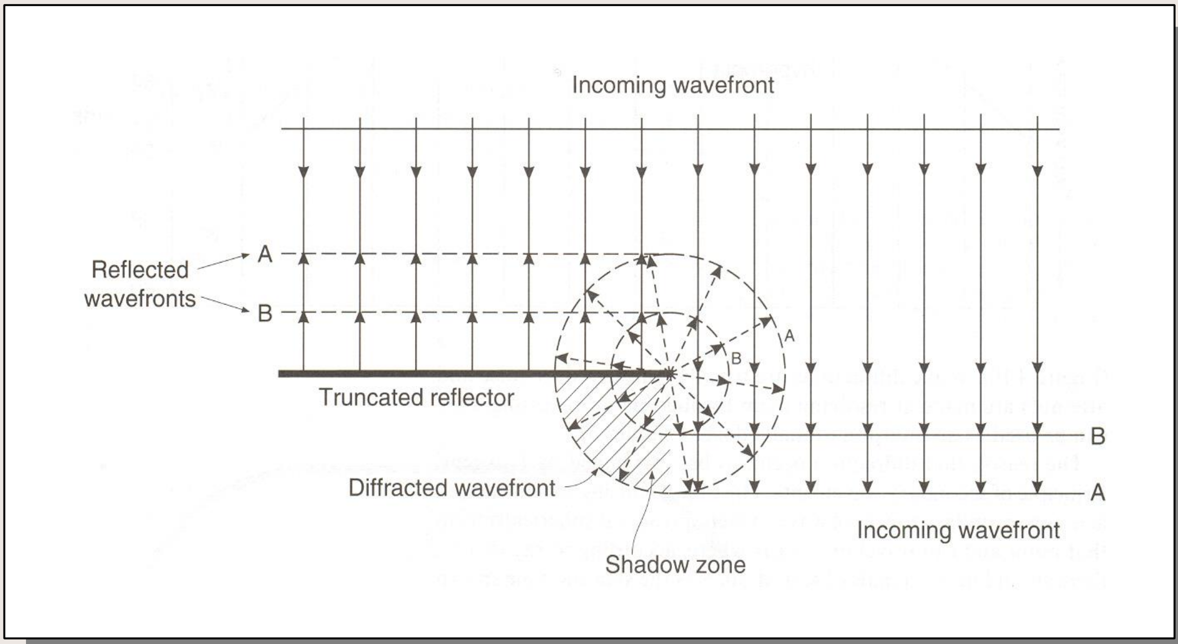
- Energy spreads from a point source in a spherical manner. So, near the source, the wavefronts are spherical (circular in 2-D)
- Every point on a wavefront can be viewed as a source of secondary waves that spread put in spheres (circles). Envelope of these spheres is the new wavefront



- In Lab 1, you will use this principle to work out the head wave propagation problem.
- A more rigorous treatment of this principle is known as the Kirchhoff theory.

Diffraction

- Secondary wavefronts can penetrate into '*shadow zones*' into which the normal, '*specular*' rays from the source cannot enter.
- This is a fundamental effect of wave propagation called *diffraction*.

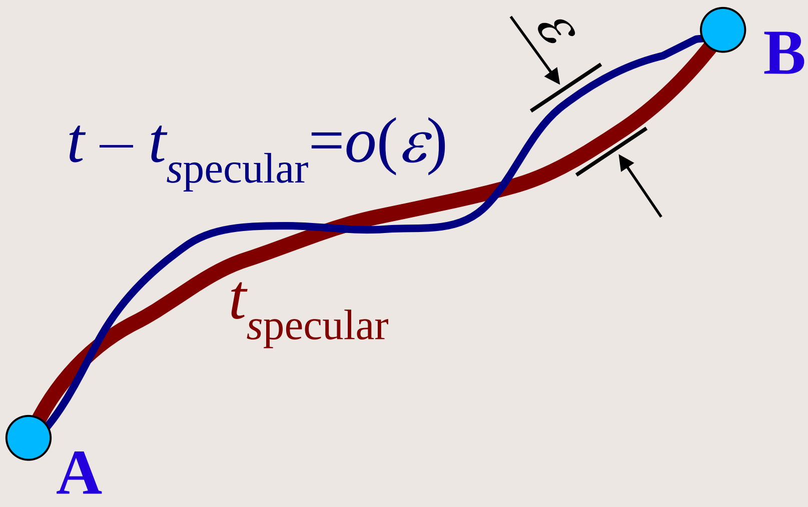


From Reynolds, 1997

Fermat Principle

(Least-time path, *brachistochrone*)

- A wave will take the path for which the travel time is *stationary* with respect to minor variations of the ray path.
  - Stationary means when the ray path is slightly perturbed, variation of its travel time is zero (to the first order of perturbation).
- Usually, the ray path has the smallest travel time among its small perturbations.

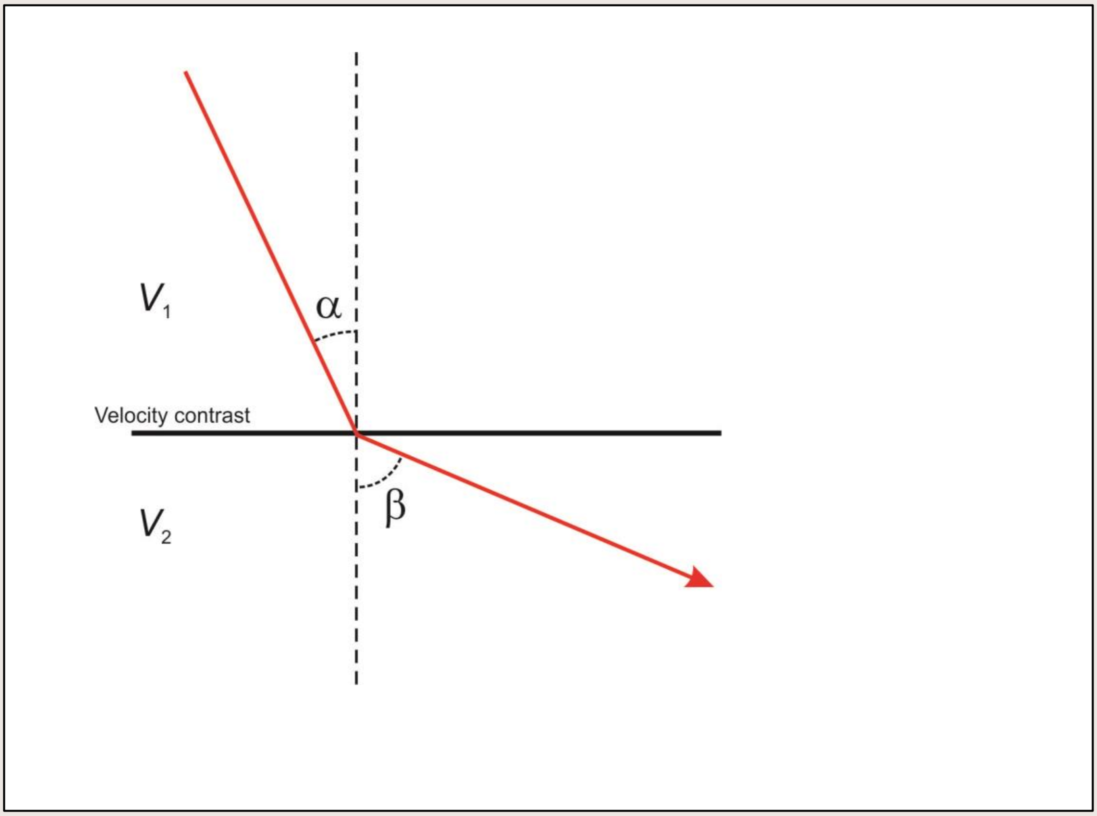


Example of using Fermat principle: Snell's law of refraction

- Show that the travel time is stationary ($\delta t = 0$) for a ray bending at the velocity interface; so that:

$$\frac{\sin\alpha}{V_1} = \frac{\sin\beta}{V_2}$$

- This relation is called the Snell's law or refraction

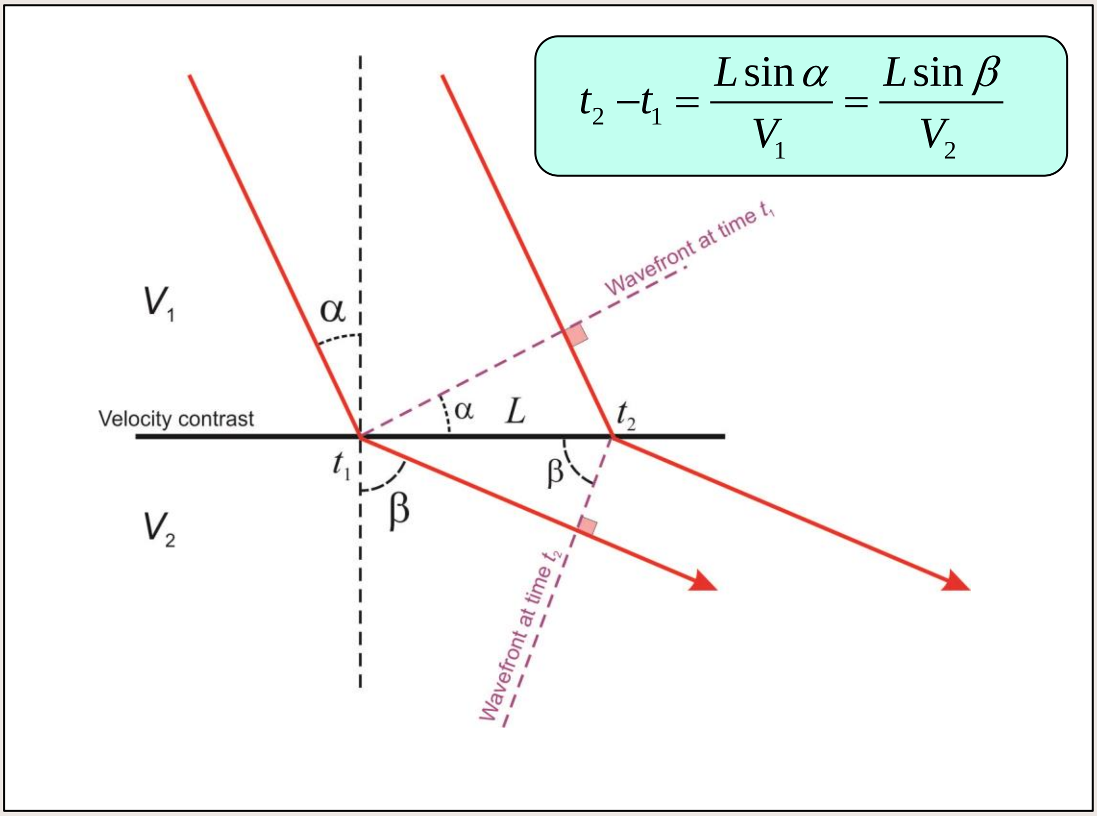


Example of using Fermat principle: Snell's law of refraction

- Show that the travel time is stationary ($\delta t = 0$) for a ray bending at the velocity interface; so that:

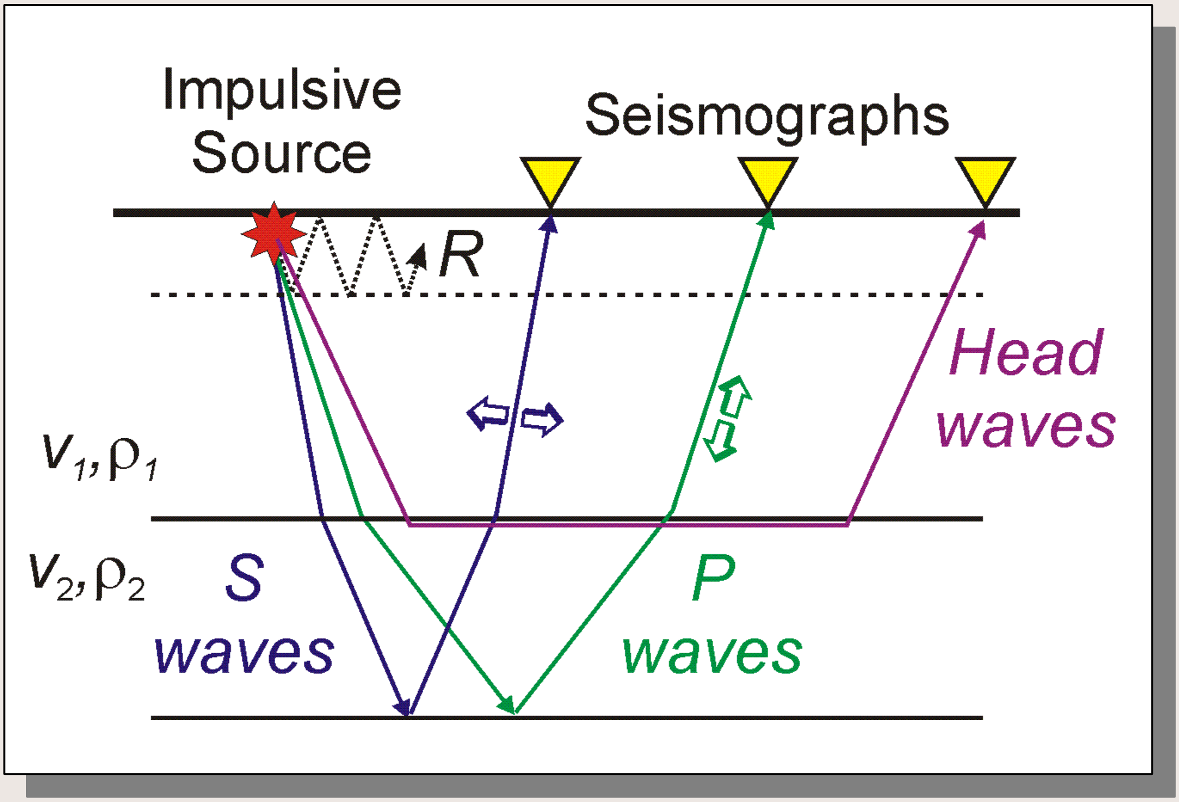
$$\frac{\sin\alpha}{V_1} = \frac{\sin\beta}{V_2}$$

- Solution: draw in the wavefronts as well:



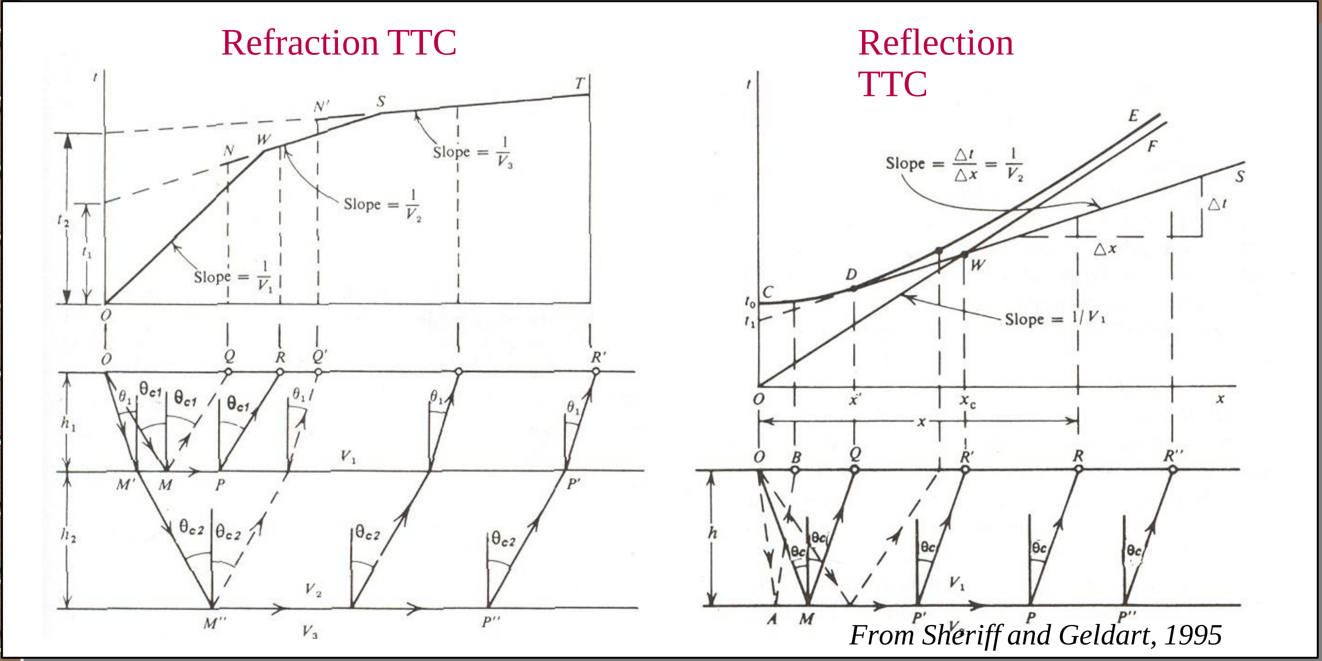
Seismic Phases
used in refraction/reflection seismics

- *P*- and *S*- body waves:
  - Refracted (bending) across velocity interfaces.
  - 'Head waves' traveling along velocity discontinuities.
  - Reflected from velocity and/or density contrasts.
- Sometimes surface waves (called 'Rayleigh' and 'Love')



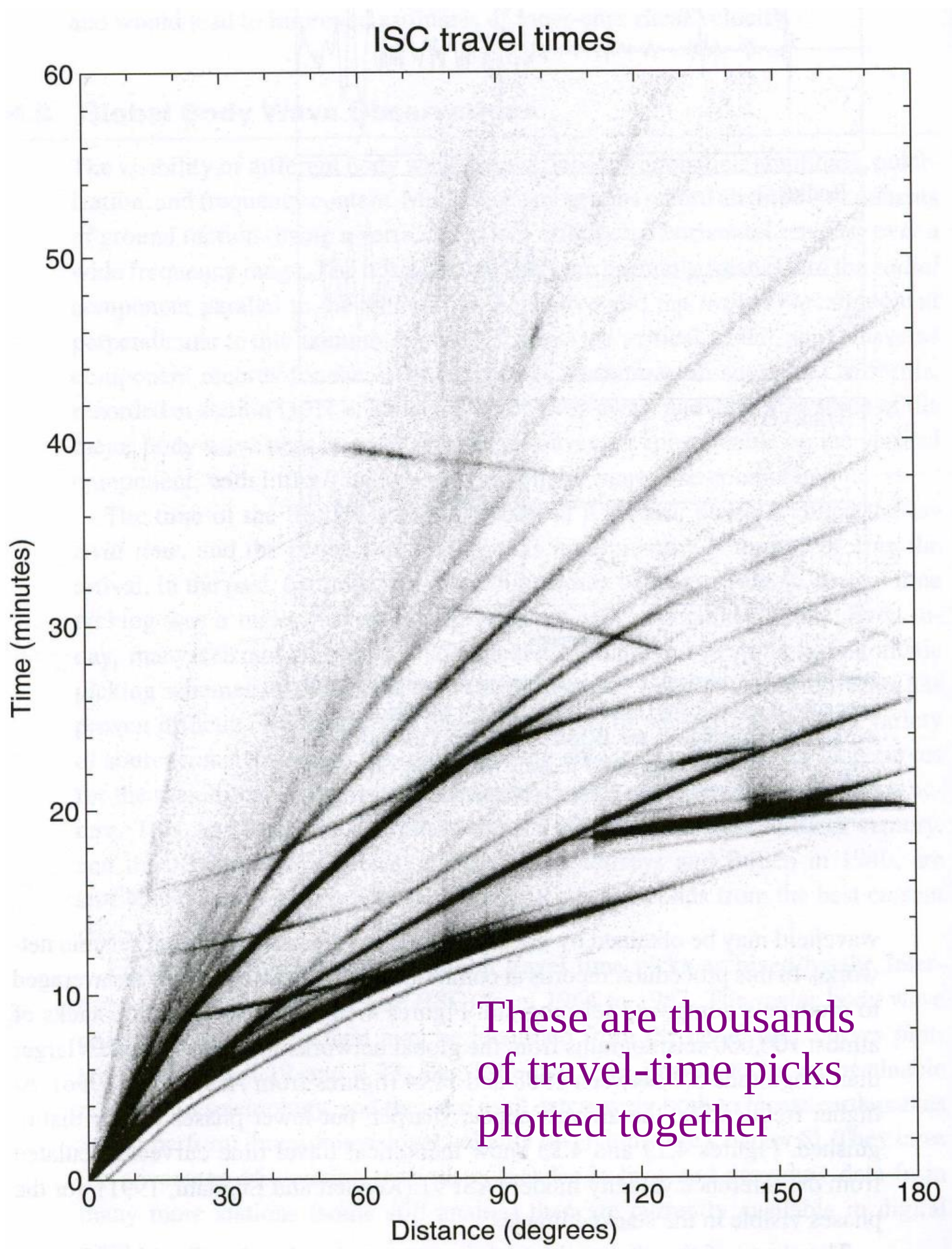
Travel-Time Dependencies
("Travel-Time Curves")

- When an arrival is identified in a dense geophone grid within a range of source-receiver *offsets* (distances), its travel times form a a set of *t*(*offset*) points, called the *travel-time curve* (TTC).
  - Convex, piecewise-linear segments in the *first arrivals* are characteristic of *refractions*, strong,
  - Concave, hyperbolic secondary arrivals are typically reflections.
- The goal of interpretation is to derive the velocity structure that could explain both:
  - Shapes of all the observed TTCs.
  - Offset ranges within which the arrivals are observed.



From Sheriff and Geldart, 1995

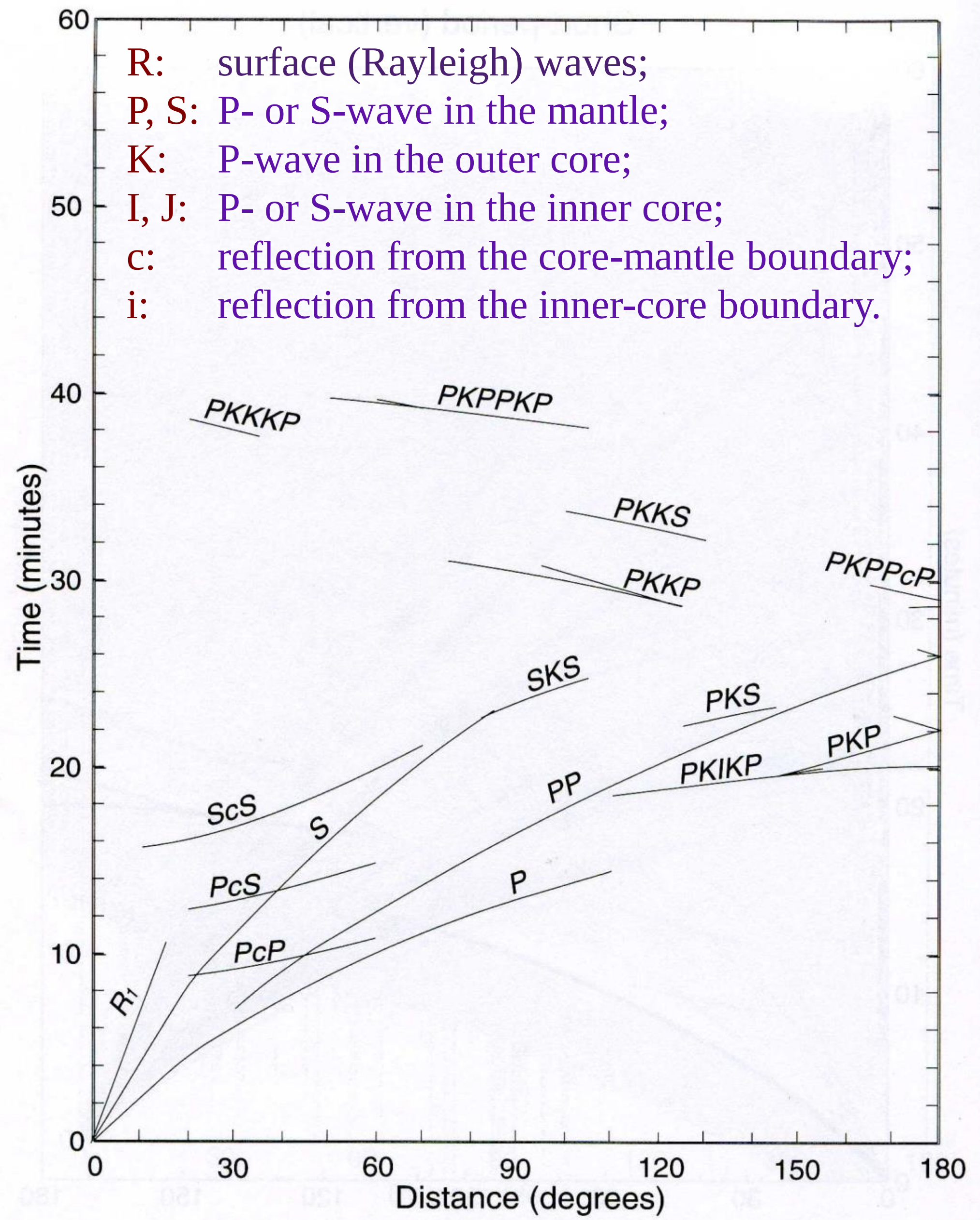
Global travel times



These are thousands of travel-time picks plotted together

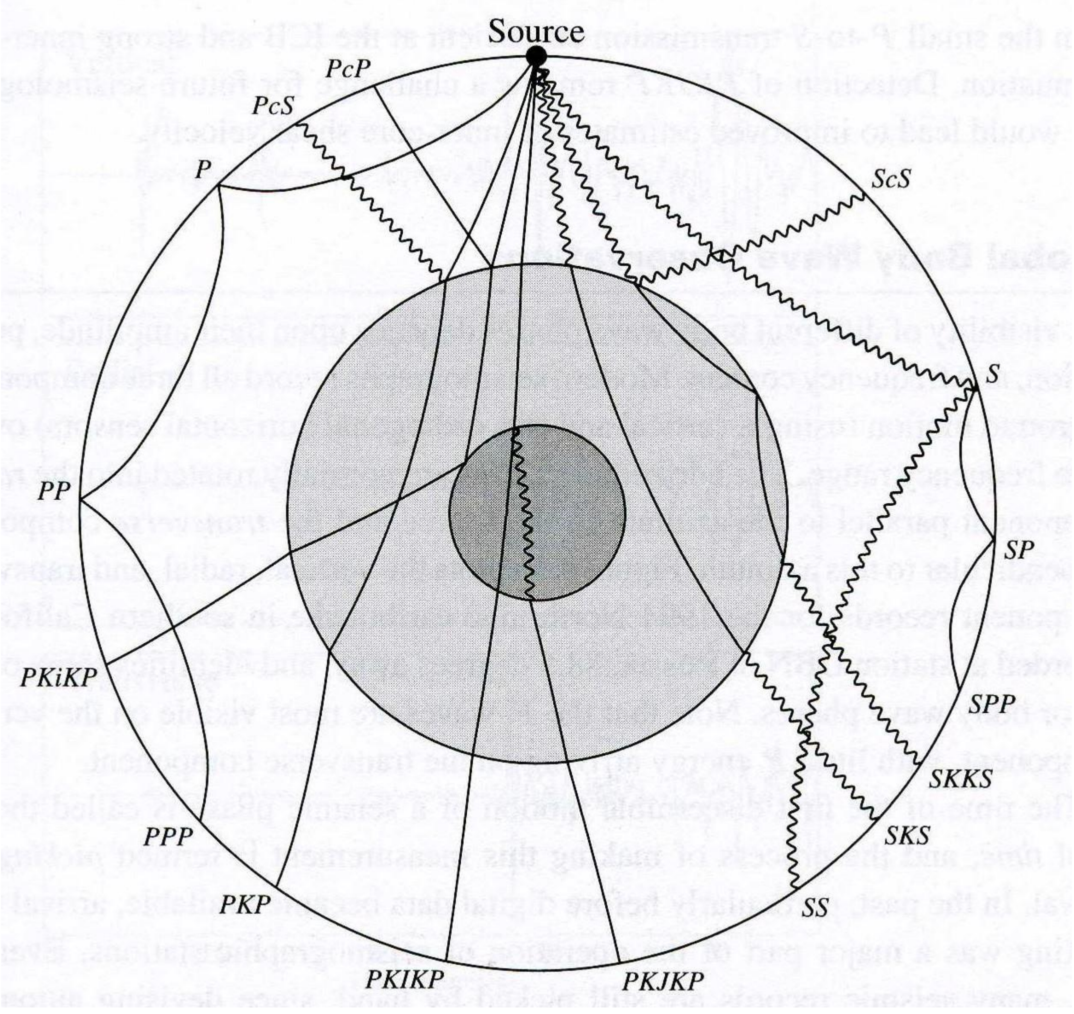
Global seismic phase nomenclature

R: surface (Rayleigh) waves;
P, S: P- or S-wave in the mantle;
K: P-wave in the outer core;
I, J: P- or S-wave in the inner core;
c: reflection from the core-mantle boundary;
i: reflection from the inner-core boundary.



Global seismic ray paths

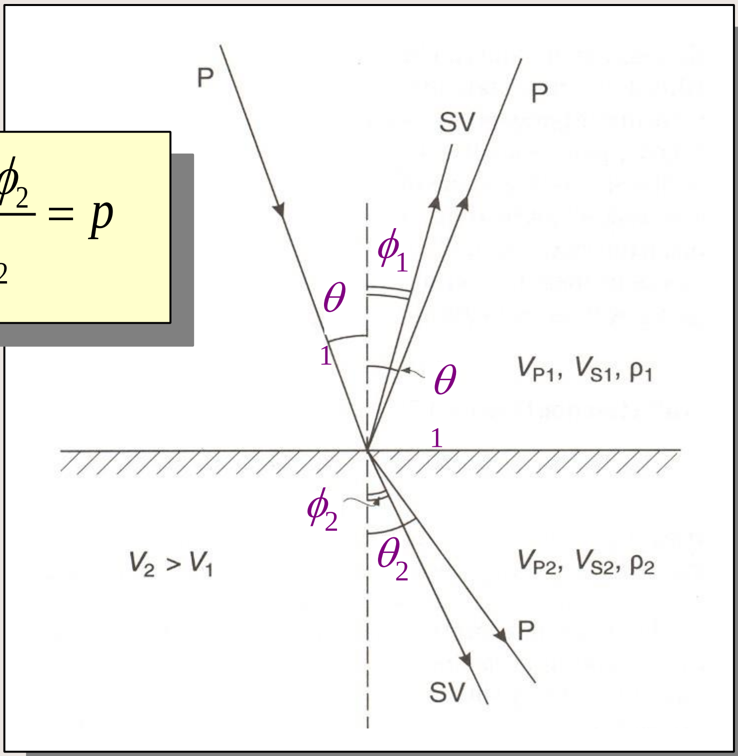
P, S: P- or S-wave in the mantle;
K: P-wave in the outer core;
I, J: P- or S-wave in the inner core;
c: reflection from the core-mantle boundary;
i: reflection from the inner-core boundary.



Refraction in a *laterally homogeneous* structure: Snell's law

- When waves (rays) penetrate a medium with a different velocity, they *refract*, *i.e*. bend toward or away from the normal to the velocity boundary.
- The *Snell's Law of refraction* relates the angles of incidence and emergence of waves refracted on a velocity contrast:

$$\frac{\sin\theta_1}{V_{P1}} = \frac{\sin\theta_2}{V_{P2}} = \frac{\sin\phi_1}{V_{S1}} = \frac{\sin\phi_2}{V_{S2}} = p$$



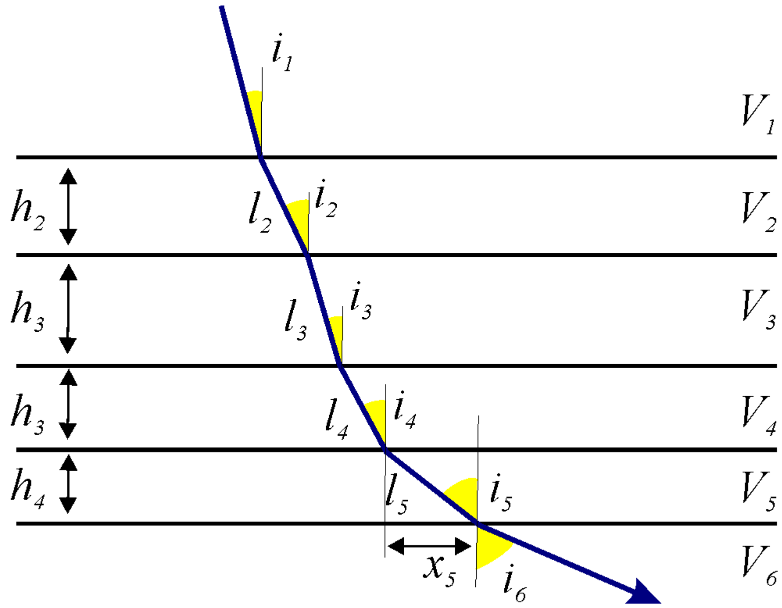
- The constant p is called the *ray parameter*
- Note that refraction angles depend on the velocities alone!

Refraction in a stack of horizontal layers

Ray parameter, *p*, *uniquely specifies* the entire ray.

It does not depend on layer thicknesses or velocities.

Travel times and distances accumulate along the ray to yield the total *T*(*X*)



For any layer: $\sin i_k = pV_k$

$$l_k = \frac{h_k}{\cos i_k} = \frac{h_k}{\sqrt{1-(pV_k)^2}}$$

$$t_k = \frac{l_k}{V_k} = \frac{h_k}{V_k\sqrt{1-(pV_k)^2}}$$

$$x_k = l_k \sin i_k = \frac{h_k(pV_k)}{\sqrt{1-(pV_k)^2}}$$

$$T_n = \sum_{k=1}^{n} t_k \qquad X_n = \sum_{k=1}^{n} x_k$$

Critical angle of refraction

- Consider a faster medium overlain with lower-velocity layer (this is the typical case).
- *Critical angle* of incidence in the slower layer is such that the refracted waves (rays) travel horizontally in the faster layer (sin *r* = 1)
- The critical angles thus are:

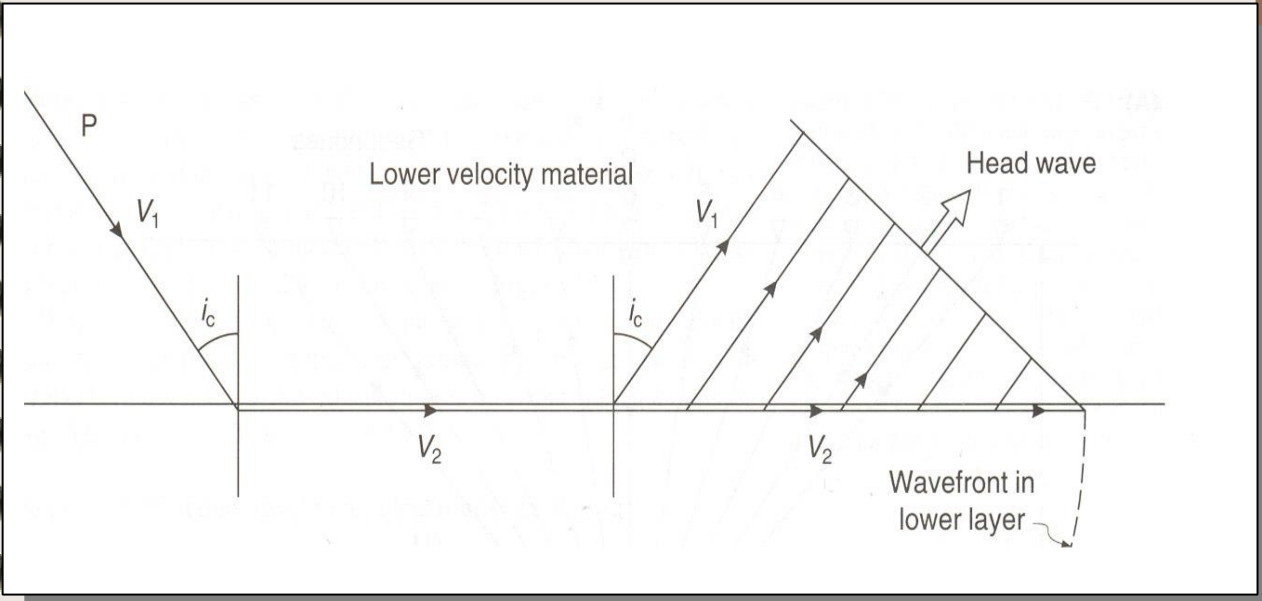
$$i_c = \sin^{-1}\frac{V_{P_1}}{V_{P_2}} \quad \text{for } P \text{ waves}$$

$$i_c = \sin^{-1}\frac{V_{S_1}}{V_{S_2}} \quad \text{for } S \text{ waves}$$

- Critical *ray parameter*: $p^{\text{critical}} = \dfrac{1}{V_{\text{refractor}}}$
- If the incident wave strikes the interface at an angle exceeding the critical one, *no refracted or head wave is generated*

Critical refraction: Head Waves

- At critical incidence from the "slower" medium, a *head wave* is generated in the "faster" one.
- Although in reality head waves carry little energy, they are useful approximation for interpreting seismic wave propagation in the presence of strong velocity contrasts.
- Head waves are characterized by planar wavefronts inclined at the critical angle in respect to the velocity boundary:



Head-wave travel times

- Head-wave travel-time curves are straight lines:

$$t(x) = t_0 + \frac{x}{V_{\text{app}}}$$

Here, t_0 is the *intercept time*, and V_{app} is the *apparent velocity*

Note that "apparent" often means "as observed" (but not necessarily "true")