Lab #4: Seismic Modelling

In this lab, you will study the response of the direct, reflected and refracted waves in a simple layered Earth model, and generate a simple seismic plot using Matlab or Octave. The three-layer and a half-space model is given in the following table:

Layer	Thickness (m)	Velocity (m/s)
1	100	450
2	200	2000
3	200	3000
4	∞	6000

Let us simulate a seismic survey conducted over this layered structure with the following specifications:

- Source located at x = 0;
- Receivers located from x = -1000 m to 1000 m;
- Spacing between receivers 10 m.

Following code lab4.m given on the web page, perform the following modelling steps:

Task (1) (10%) Determine all: a) critical ray parameters; b) critical distances, c) intercept times for the refractions, d) *t*₀ times and RMS velocities for the reflections. In Matlab, assign identifiers to these quantities, such as:

```
p_crit_1 = ...; % critical ray parameter
x_crit_1 = ...; % critical distance
t_int_1 = ...' % intercept time
t_int_2 = ...;
t0_1 = ...; % normal-incidence reflection time;
v_rms_1 = ...;
```

and so on. In the following, use these identifiers rather than constants.

<u>Note:</u> the units for distance should be m, time – ms, velocity – accordingly, m/ms (or km/s).

Task (2) (25%) Using Matlab, compute the arrival times of the direct, reflected and refracted waves from all the acoustic discontinuities for all receivers. Plot these arrival times on a common T-X graph. It would be helpful to write two Matlab *functions* to calculate reflection and refraction travel times, respectively:

```
trefl(t0,v_rms) and trefr(t_intercept,v_refractor)
```

In the subsequent steps, you could simply use those functions, instead of retyping the formulas.

Next, look into the function wavelet_ft(t). This function returns a wavelet (array) of amplitude 1.0, centred at time t=0 and discretized at times specified in array 't' (in milliseconds). The wavelet is obtained by summation of 81 cosine waves of amplitude 1.0/81.0, with frequencies from 20 Hz to 60 Hz with a 0.5 Hz interval. Finally, the waveform is multiplied by a sine "taper" function to ensure that its values at the ends of the time interval equal zero.

This is your *source signature* (aligned at 0 ms time). Such symmetric with respect to time = 0 waveforms are called "zero-phase".

Task (3) (10%) Test function wavelet_ft() and precompute the source waveform by executing something like this:

```
global tref wref

tref = -500:1:500;
wref = wavelet_ft(tref);
plot(tref,wref,'b-')
save wavelet.mat tref wref
```

These commands create an array of times tref centred at t=0, model wavelet wref on it, plot wref, and save it in a file. This precomputed wavelet can be loaded like this:

```
global tref wref
load wavelet.mat
```

and will be used later in this and other labs.

Task (4) (10%) Look into the second Matlab function wavelet (a0, t0, t). This function uses the precomputed tref and wref and returns the same wavelet but with arbitrary amplitude 'a0' and centred at arbitrary time 't0' [ms]. The output is obtained by simple linear interpolation (function interp1 (...)) and is much faster than the summations above.

Test this function like this (you can change parameter values or make multiple tests):

```
t = 0:2:1000;
plot(t, wavelet(5.0,100.0, t));
```

Print out the resulting plot out and indicate whether the time and amplitude of the waveform are as expected.

Task (5) (40%) Using the calculated arrival travel-times and wavelet (...) function, use Matlab to plot a *shot gather* – a set of seismic traces simulated at each of the 200 or so geophone locations. Use 0-1200 ms time range, at 2 ms intervals. Note that for each interface, you will have two arrivals (direct wave and reflection) at pre-critical offsets and three arrivals (direct, reflected, and head wave) beyond that range.

For plotting, you will have to use Matlab's hold on command (to prevent from starting a display window on every plot () call) and arrange a loop over the receivers. For *i*-th receiver (beyond the critical distance), you will first find the time of this arrival in each trace and then plot a wavelet at this time:

```
% determine the source-receiver distance of this trace
x = i*20.0;

% put direct wave into new trace
trace = wavelet(amp, x/v1, t );

% add reflection from the first interface
trace = trace + wavelet(amp, trefl(t0_1,v_rms1), t );

% add refraction from the first interface

if x > x_crit_1 then
    trace = trace + wavelet(amp, trefr(t_int_1,v2), t );
end

% ...and similarly for the other two interfaces

plot(trace+x,t); % plot the resulting trace
```

Here, t_direct, t_head, and t_reflected should be the times of your arrivals. The horizontal axis of the plot will become the distance, and the shifts of \times added to the 'trace' [m] will place your wiggles ("traces") at the positions of geophones. Parameter amp becomes trace excursion (swing amplitude) and should be chosen (I suggest ~15-20) so that the plot looks nice. Time will be plotted along the vertical axis downward, as it is commonly done in refraction/reflection record sections.

Task (6) (5%) Notice the interference between the different waves near critical and crossover distances, and far offsets. Mark these interfering events in your section. Comment on the tuning effects (enhancement or cancellation of arrival amplitudes as they approach each other within a dominant period.

Theory:

The cosine function of amplitude A, frequency f, and centered at time t_0 (used in wavelet ft()) is:

$$u(t) = A\cos(2\pi f(t - t_0))$$

For a stack of layers, travel time of the reflection from the bottom of *n*-th of them is given by the same formula as within a single layer:

$$t(x) = \sqrt{t_0^2 + \left(\frac{x}{V_{RMS}}\right)^2},$$

where t_0 is the vertical-incidence travel time, and V_{RMS} is the effective ("root-mean-square") velocity of the stack of layers:

$$V_{RMS}^2 = \frac{\sum_{i=1}^n V_i^2 \Delta t_i}{\sum_{i=1}^n \Delta t_i},$$

where $\Delta t_i = \Delta z_i / V_i$ is the vertical travel time for each layer.

Hand in:

Code printouts, plots, and comments in a zipped directory.