Formula reminder sheet for GEOL335.3

You are expected to be able to give and *explain* the following equations. Those marked with asterisks (*) do not need to be memorized in their exact form but still need to be understood and interpreted. What physical quantities do they relate? What processes do they describe and what are their consequences?

General

Relations between wave length, period, velocity, frequency, and wavenumber: $\lambda = VT$, $f = \frac{1}{T}$,

$\omega = 2\pi f = \frac{2\pi}{T}$, $k = \frac{2\pi}{\lambda}$.

Phase (wave) velocity: $V = \frac{\omega}{k}$

Group (energy packet) velocity*: $U = \frac{d\omega}{dk}$

Equations for plane waves:

general: $u(x,t) = f\left(t - \frac{x}{V}\right)$

cosine form: $u(x,t) = A\cos(\omega t - kx)$

complex exp form: $u(x,t) = Ae^{i(\omega t - kx)}$

Amplitude (absolute value, *A*) and phase (argument, ϕ) of a complex number (e.g., spectrum): $Z = Ae^{i\phi}$

Decibel expression of relative signal amplitudes: $\left(\frac{A_1}{A_2}\right)_{dB} = 20\log_{10}\left(\frac{A_1}{A_2}\right)$

The same in terms of signal power: $\left(\frac{P_1}{P_2}\right)_{dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$

Sampling frequency: $f_S = \frac{1}{N\Delta t} = \frac{1}{T}$.

Nyquist frequency: $f_N = \frac{f_S}{2} = \frac{1}{2\mathrm{T}}$.

Z-transform: $U(z) = u_0 + u_1 z^1 + u_2 z^2 + u_3 z^3 + \ldots = \sum_{i=0}^{\infty} u_i z^i$.

Fourier transform*: $U_F(f_j) = U_z(e^{-2i\pi f_j}). = \sum_{k=0}^{N} u_k e^{-2i\pi f_j t_k}$.

Inverse Fourier transform*: $u(t_k)=\frac{1}{N}\sum_{j=0}^{N} U(f_k)e^{2i\pi f_j t_k}.$

Wave equation (general): $\left[\frac{\partial^2}{\partial t^2}-\nabla^2\right]f(t,\boldsymbol{r})=0.$

Geometrical spreading for a seismic wave:

spherical (body wave): $A(R)=\frac{A_0}{R}$

cylindrical (surface wave): $A(R)=\frac{A_0}{\sqrt{R}}$

plane wave $A(R)=A_0=const$

Attenuation*: $A(x)=A_0 e^{-\alpha x}=A_0 e^{-\pi\frac{ft}{Q}}$

Quality factor: $Q=2\pi\frac{E}{\delta E}$

Elasticity

Elementary force acting on a surface within elastic medium*: $dF_i=dS\sum_{j=1,2,3}\sigma_{ij}n_j.$

Elementary force acting on a volume within elastic medium*: $dF_i=dV\sum_{j=1,2,3}\frac{\partial\sigma_{ij}}{\partial x_j}.$

Strain: $\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right).$

Dilatational strain: $\Delta=\frac{\delta V}{V}=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$

Hooke's Law for isotropic medium*: $\sigma_{ij}=\lambda'\Delta+2\mu\varepsilon_{ij}$ for $i=j$, and $\sigma_{ij}=2\mu\varepsilon_{ij}$ for $i\neq j$. [Combined, this can also be written as: $\sigma_{ij}=\lambda'\Delta\,\delta_{ij}+2\mu\varepsilon_{ij}$ (the “Kronecker symbol”, $\delta_{ij}=1$ if $i=j$ and 0 otherwise)].

Elastic energy density*: $E=\frac{1}{2}\sum_{i,\,j=1,2,3}\sigma_{ij}\varepsilon_{ij}.$

Elastic moduli:

- Young's modulus*: $E=\frac{2\mu(3\lambda+2\mu)}{\lambda+\mu}.$

- Poisson's ratio (modulus)*: $\nu = \frac{\lambda}{2(\lambda+\mu)}$.
- Bulk modulus*: $K = \lambda + \frac{2}{3}\mu$
- Rigidity modulus = μ .

Newton's law for elastic medium: $\rho \frac{\partial^2 U_i}{\partial^2 t} = \sum_{j=1,2,3} \frac{\partial \sigma_{ij}}{\partial x_j}$.

P-wave velocity: $V_P = \sqrt{\frac{\lambda+2\mu}{\rho}} = \sqrt{\frac{K-\frac{4}{3}\mu}{\rho}}$.

S-wave velocity: $V_S = \sqrt{\frac{\mu}{\rho}}$

For Poisson's ratio of $\nu = 0.25$, $\frac{V_P}{V_S} = \sqrt{3}$

Seismic Refraction

Snell's law of refraction: $\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = ... = const = p$

Critical angle: $\sin i_c = \frac{V_1}{V_2}$

Critical distance (offset): $x_c = 2h \tan i_c$

Head wave travel time (linear moveout) equation, zero dip: $t(x) = t_{Intercept} + \frac{x}{V}$

Intercept time: $t_{Intercept} = \frac{2\text{h} \cos i_c}{V_{1.}}$

Refraction time*, down dip: $t(x) = \frac{2\text{h}_d \cos i_c}{V_{1.}} + \frac{x}{V_1} \cos(i_c + \alpha)$

up dip: $t(x) = \frac{2\text{h}_u \cos i_c}{V_{1.}} + \frac{x}{V_1} \cos(i_c - \alpha)$

Delay time: $t_{S,R\,Delay} = \frac{h_{s,r} \cos i_c}{V_{1.}}$

Seismic Reflection:

Seismic Impedance: $Z = \rho V$

Reflection coefficient: $R_{PP}=\frac{A_{P_{reflected}}}{A_{P_{incident}}}=\frac{Z_2-Z_1}{Z_2+Z_1}$

Transmission coefficient: $T_{PP}=1-R_{PP}=\frac{2Z_1}{Z_1+Z_2}$

Reflection coefficient <u>for energy</u>: $R_E=R_{PP}^2=\frac{(Z_2-Z_1)^2}{(Z_2+Z_1)^2}$

Transmission coefficient <u>for energy</u>: $T_E=\frac{Z_2}{Z_1}T_{PP}^2=\frac{4Z_1Z_2}{(Z_1+Z_2)^2}$; note that: $T_E=1-R_E$

Energy flux: $E_{flux}=V\,E_{density}=V\frac{\rho}{2}(\omega A)^2=\frac{\omega^2}{2}Z A^2$

Vertical reflection resolution: $\delta z=\frac{\lambda}{4}$.

Horizontal reflection resolution: $\delta x\approx\sqrt{\frac{1}{2}H\lambda}$.

Reflection travel time (normal moveout) equation $t(x)=\sqrt{t_0^2+(\frac{x}{V})^2}\approx t_0+\frac{1}{2t_0}(\frac{x}{V})^2$

RMS velocity*: $V_{RMS}=\sqrt{\frac{\sum_{i=1}^{n}t_iV_i^2}{\sum_{i=1}^{n}t_i}}$.

NMO correction: $t(x)\rightarrow t_0\approx t(x)-\frac{1}{2t(x)}\left(\frac{x}{V}\right)^2$

Instrumentation and Sources

Radius of explosion cavity*: $R[ft] = BW^{\frac{1}{3}}$

Seismic pulse width*: $T[ms] = 2.8 \cdot W^{\frac{1}{3}}$

Current in electromechanical geophone*: $\frac{d^2 i}{dt^2} + 2h\omega_0 \frac{di}{dt} + \omega_0^2 i = \frac{2\pi rnH}{R} \frac{d^3 z}{dt^3}$

Ground-penetrating radar

Velocity of radar waves (low-loss medium): $c = \frac{c_0}{\sqrt{\varepsilon\mu}} \approx \frac{c_0}{\sqrt{\varepsilon}}$.

GPR reflection impedance $Z = \sqrt{\frac{\mu}{\varepsilon}} \approx \sqrt{\frac{1}{\varepsilon}}$, reflection coefficient: $R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$.