Refraction Seismic Method

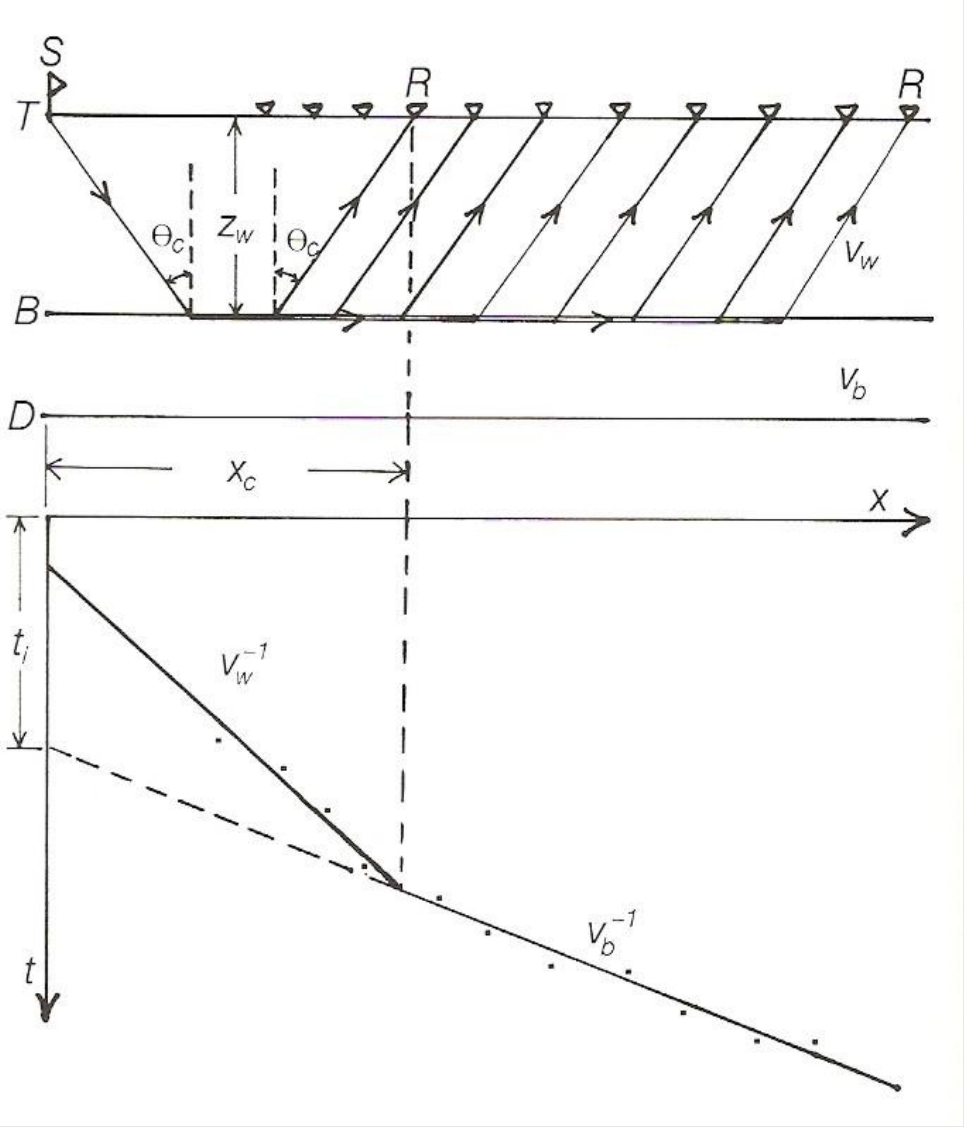
- Intercept times and apparent velocities
- Critical and crossover distances
- Hidden layers
- Determination of the refractor velocity and depth
- The case of dipping refractor
- Inversion methods:
  - "Plus-minus" method
  - Generalized Reciprocal Method
  - Travel-time continuation

- Reading:
  - Reynolds, Chapter 5
  - Shearer, Chapter 4
  - Telford *et al.*, Sections 4.7.9, 4.9

Refraction Seismic Method

- Uses **travel times** of refracted arrivals to derive:

1) Depths to velocity contrasts ("refractors")
2) Shapes of refracting boundaries
3) Seismic velocities



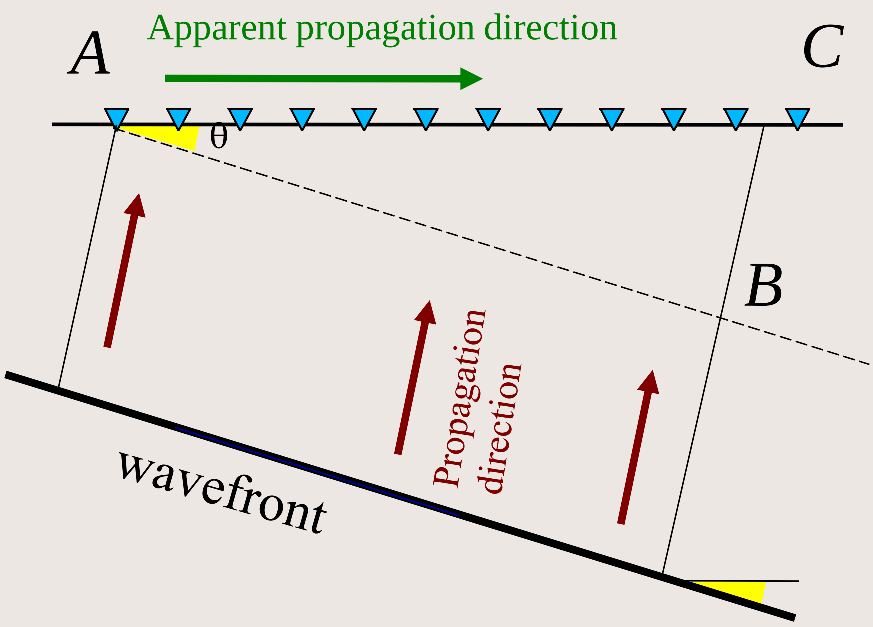
Apparent Velocity

Relation to wavefronts

- *Apparent velocity*, V_{app}, is the velocity at which the wavefront sweeps across the geophone spread
  - Because the wavefront also propagates upward, $V_{app,} \geq V_{true}$:

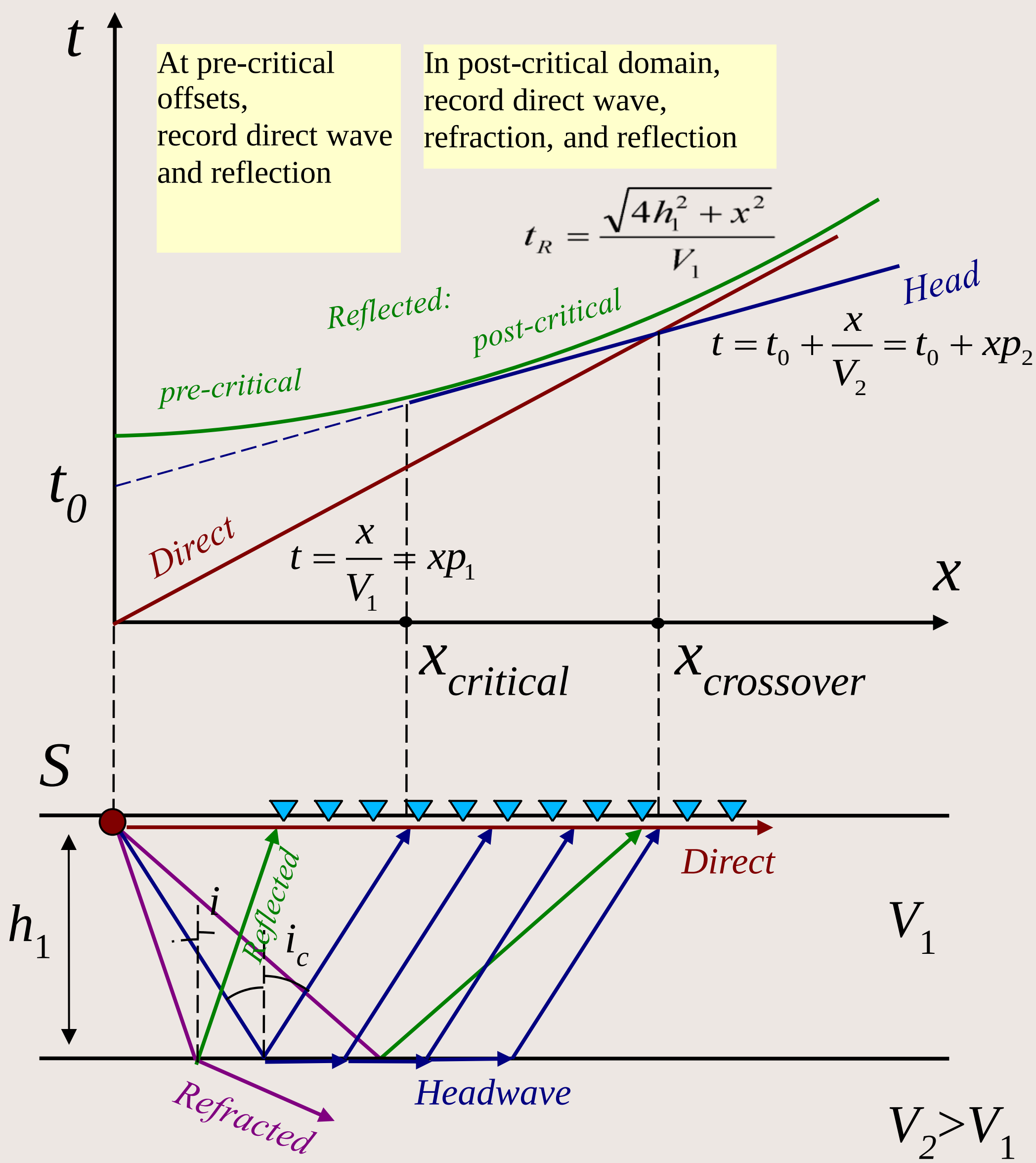
$$AC = \frac{BC}{\sin\theta} \rightarrow \boxed{V_{app} = \frac{V}{\sin\theta}}$$

  - 2 extreme cases:
    - $\theta = 0$: $V_{app} = \infty$;
    - $\theta = 90°$: $V_{app} = V_{true}$.



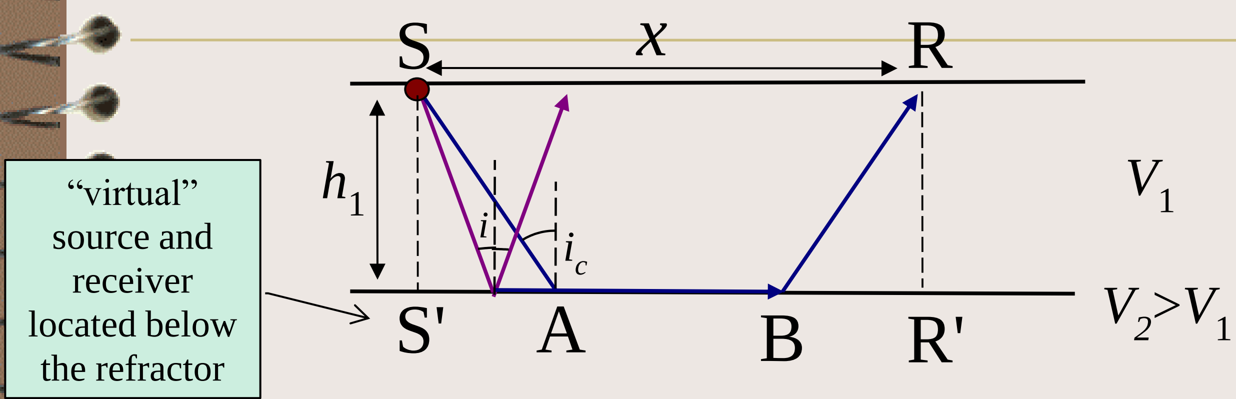
Two-layer problem

(One reflection and one refraction)



Travel-time relations

Two horizontal layers



- For a head wave ("often called refraction"):

$$p = \frac{1}{V_2} \qquad \sin i_c = pV_1 \qquad \cos i_c = \sqrt{1-\left(pV_1\right)^2}$$

$$t = 2\frac{h_1}{V_1 \cos i_c} + \underline{p}\left(x - 2h_1 \tan i_c\right) = t_0 + \underline{p}x$$

"linear moveout" term

intercept time

$$t_0 = 2\frac{h_1}{V_1 \cos i_c}\left(1 - pV_1 \sin i_c\right) = \frac{2h_1}{V_1}\cos i_c$$

this also equals sin i

- For a reflection (we'll use this later):

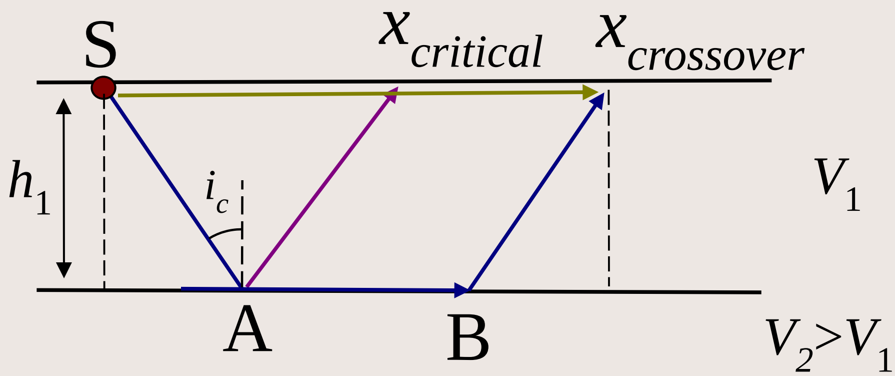
hyperbolic moveout"

$$pV_1 = \sin i \qquad \tan i = \frac{x}{2h_1}$$

$$t = 2\frac{\sqrt{h_1^2 + \left(\frac{x}{2}\right)^2}}{V_1} = \frac{\sqrt{4h_1^2 + x^2}}{V_1}$$

here, p is variable and controlled by arbitrary angle i

Critical and cross-over distances



- Critical distance:

$$x_{critical} = 2h_1 \tan i_c = 2h_1 \frac{V_1/V_2}{\sqrt{1-(V_1/V_2)^2}} = \frac{2h_1V_1}{\sqrt{V_2^2 - V_1^2}}$$

- Cross-over distance:

$$t_{direct}(x_{crossover}) = t_{headwave}(x_{crossover})$$

$$\frac{x_{crossover}}{V_1} = t_0 + \frac{x_{crossover}}{V_2}$$

$$x_{crossover} = \frac{t_0}{(1/V_1 - 1/V_2)}$$

"slownesses"

Multiple-layer case

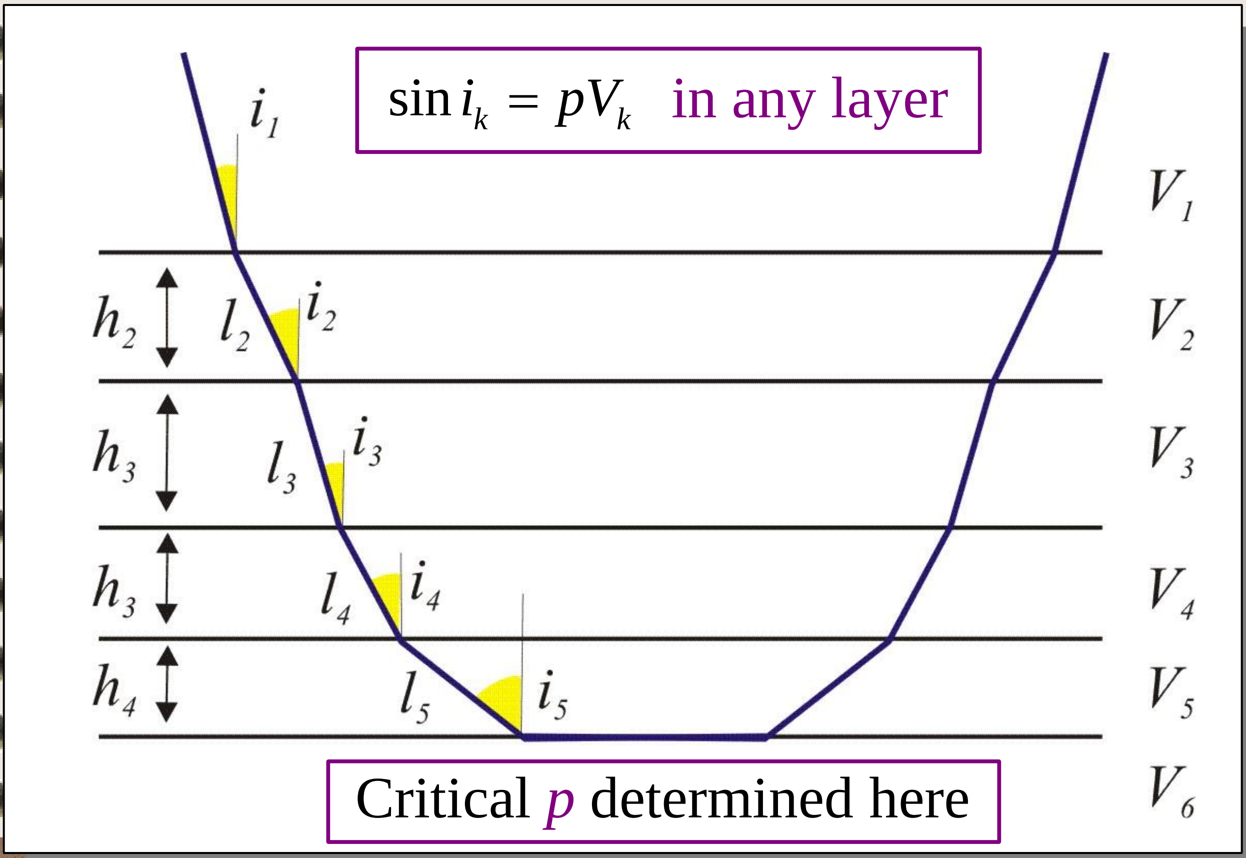
(Horizontal layering)

- p is the same *critical ray parameter*;

$$p = \frac{1}{V_{refractor}}$$

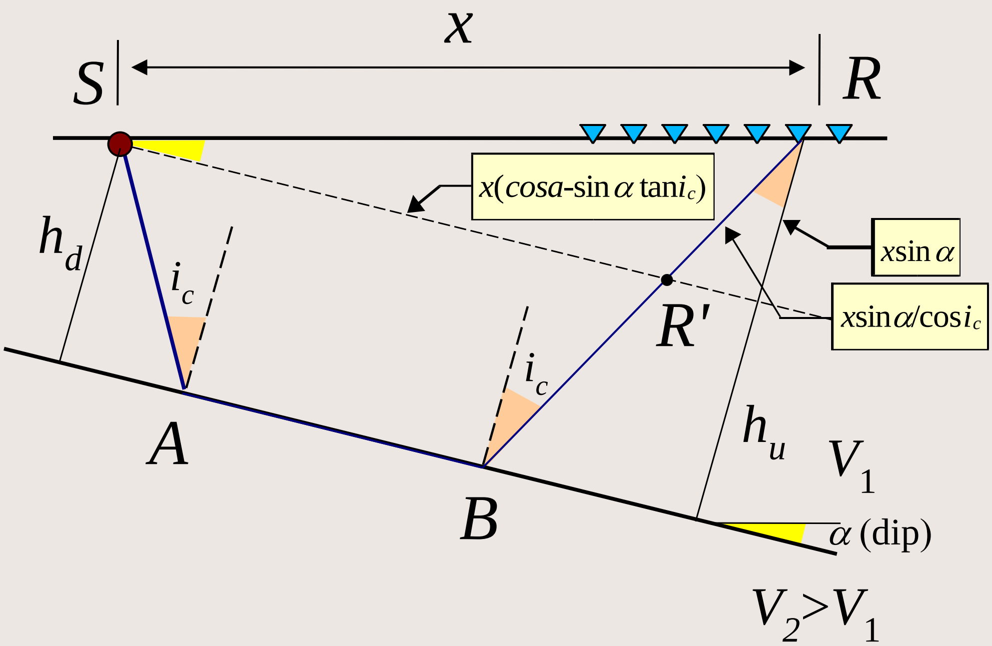
- t_0 is accumulated across the layers:

$$t = \sum_{k=1}^{n} \frac{2h_k}{V_k} \cos i_k + px$$



Dipping Refractor Case

shooting down-dip



$$t = 2\frac{h_d}{V_1}\cos i_c + \frac{1}{V_2}x\left(\cos\alpha - \sin\alpha\tan i_c\right) + \frac{1}{V_1}\frac{x\sin\alpha}{\cos i_c}$$

$\sin i_c$

$$t = \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1\cos i_c}\left[\frac{V_1}{V_2}\left(\cos\alpha\cos i_c - \sin\alpha\sin i_c\right) + \sin\alpha\right]$$

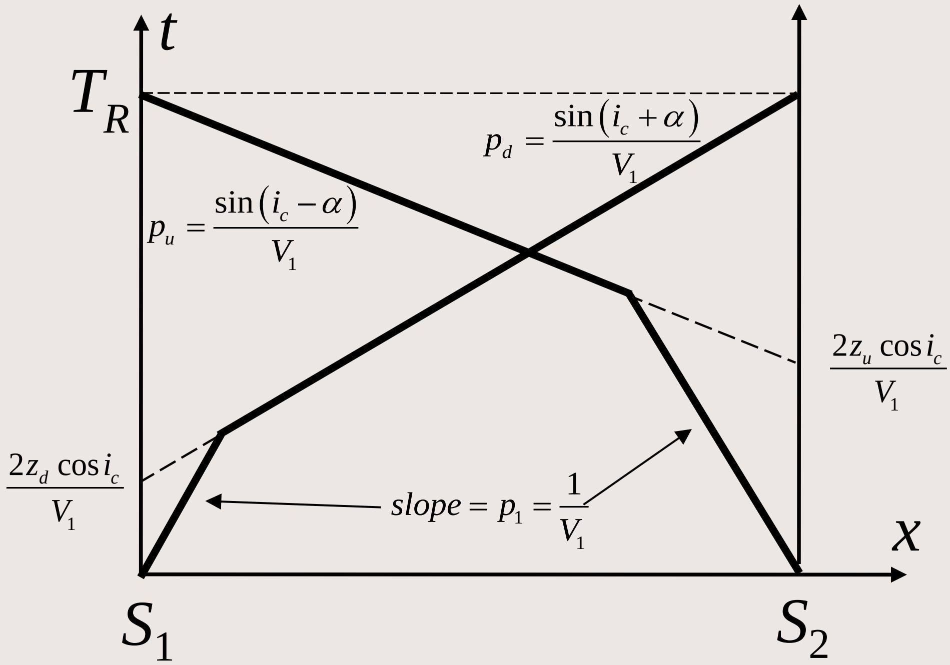
$$t = \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\left(\cos\alpha\sin i_c + \sin\alpha\cos i_c\right)$$

$$\boxed{t = \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\sin\left(i_c + \alpha\right)}$$

would change to '-' for up-dip recording

Refraction Interpretation
Reversed travel times

- One needs *reversed* recording (in opposite directions) for resolution of dips
- The *reciprocal times*, T_R, must be the same for reversed shots
- Dipping refractor is indicated by:
  - Different *apparent velocities* (=1/p, TTC slopes) in the two directions;
    - determine V_2 and α (refractor velocity and dip).
  - Different *intercept times*.
    - determine h_d and h_u (interface depths).



Determination of Refractor Velocity and Dip

- *Apparent velocity* is $V_{app} = 1/p$, where p is the *ray parameter (*i.e., slope of the travel-time curve)
  - Apparent velocities are measured directly from the observed TTCs;
  - $V_{app} = V_{refractor}$ only in the case of a horizontal layering
  - For a dipping refractor:
    - Down dip: $V_d = \dfrac{V_1}{\sin(i_c + \alpha)}$ (*slower than* V_2);
    - Up-dip: $V_u = \dfrac{V_1}{\sin(i_c - \alpha)}$ (*faster*).
- From the two reversed apparent velocities, i_c and α are determined:

$$i_c + \alpha = \sin^{-1}\frac{V_1}{V_d} \qquad i_c - \alpha = \sin^{-1}\frac{V_1}{V_u}$$

$$\boxed{\begin{aligned} i_c &= \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} + \sin^{-1}\frac{V_1}{V_u}\right) \\ \alpha &= \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} - \sin^{-1}\frac{V_1}{V_u}\right) \end{aligned}}$$

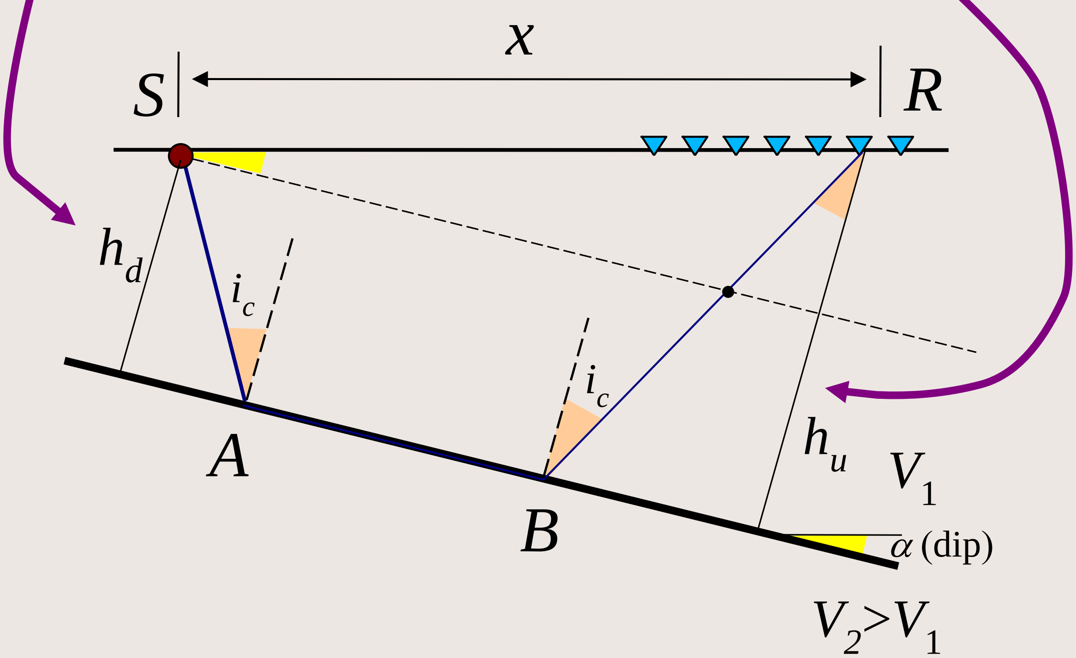
- From i_c, the refractor velocity is: $\boxed{V_2 = \dfrac{V_1}{\sin i_c}}$

Determination of Refractor Depth

- From the *intercept times*, t_d and t_u, *refractor depth* is determined:

$$h_d = \frac{V_1 t_d}{2\cos i_c}$$

$$h_u = \frac{V_1 t_u}{2\cos i_c}$$



Delay time

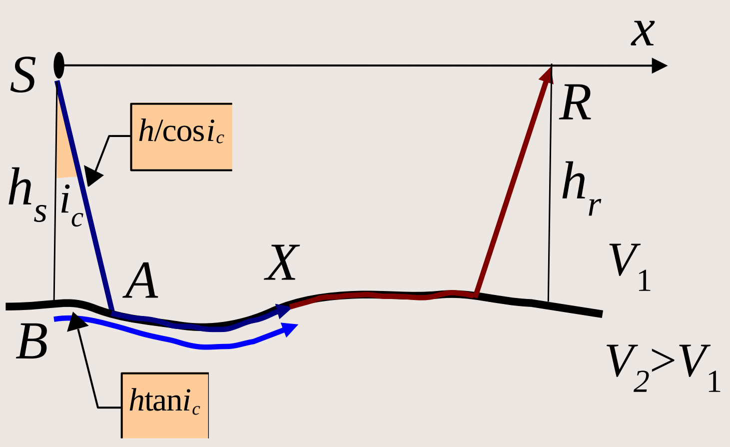
- Consider a nearly horizontal, shallow interface with strong velocity contrast (a typical case for weathering layer).
    - In this case, we can separate the times associated with the source and receiver vicinities: $t_{SR} = t_{SX} + t_{XR}$.
- Relate the time t_{SX} to a time along the refractor, t_{BX}:
$t_{SX} = t_{SA} - t_{BA} + t_{BX} = t_{S\,Delay} + x/V_2$.

$$t_{SDelay} = \frac{SA}{V_1} - \frac{BA}{V_2} = \frac{h_s}{V_1 \cos i_c} - \frac{h_s \tan i_c}{V_2} = \frac{h_s}{V_1 \cos i_c}\left(1 - \sin^2 i_c\right) = \frac{h_s \cos i_c}{V_{1.}}$$

Note that $V_2 = V_1/\sin i_c$

- Thus, source and receiver *delay times* are:

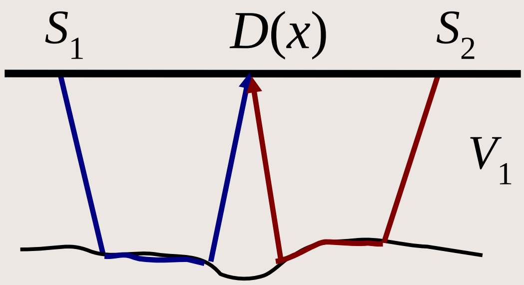
$$\boxed{t_{S,RDelay} = \frac{h_{s,r} \cos i_c}{V_{1.}}}$$ and $$\boxed{t_{SR} = t_{S\ \mathrm{Delay}} + t_{R\ \mathrm{Delay}} + \frac{SR}{V_2}}$$

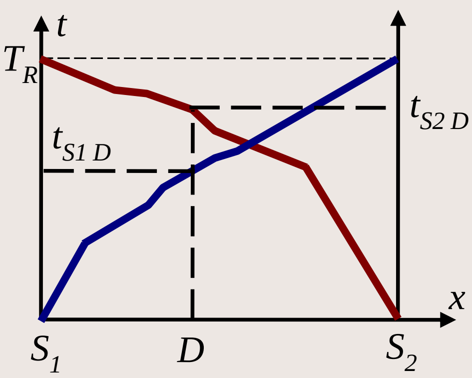


Plus-Minus Method

(Weathering correction; Hagedoorn)

- Assume that we have recorded two headwaves in opposite directions, and have estimated the velocity of overburden, V_1
  - How can we map the refracting boundary?





- Solution:
  - Profile $S_1 \rightarrow S_2$: $t_{S_1D} = \frac{x}{V_2} + t_{S_1} + t_D$
  - Profile $S_2 \rightarrow S_1$: $t_{S_2D} = \frac{(S_1S_2 - x)}{V_2} + t_{S_2} + t_D$
  - Form PLUS travel-time:

$$t_{PLUS} = t_{S_1D} + t_{S_2D} = \frac{S_1S_2}{V_2} + t_{S_1} + t_{S_2} + 2t_D = t_{S_1S_2} + 2t_D$$

Hence: $$\boxed{t_D = \frac{1}{2}\left(t_{PLUS} - t_{S_1S_2}\right)}$$

  - To determine i_c (and depth), we still need to find V_2

Plus-Minus Method
(Continued)

- To determine V_2:
  - Form MINUS travel-time:

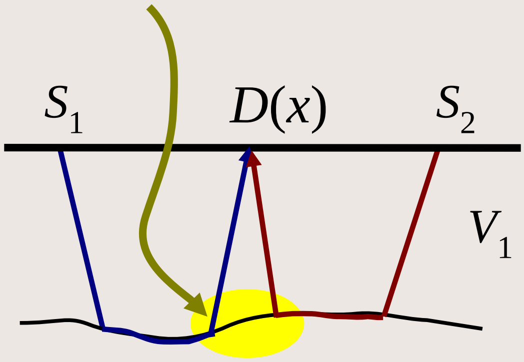
this is a constant!

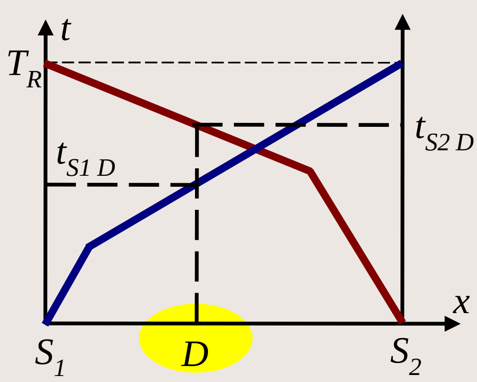
$$t_{MINUS} = t_{S_1D} - t_{S_2D} = \frac{2x}{V_2} \boxed{- \frac{S_1S_2}{V_2} + t_{S_1} - t_{S_2}}$$

Hence: $$\boxed{slope\left[t_{MINUS}(x)\right] = \frac{2}{V_2}}$$

  - The slope is usually estimated by using the *Least Squares method*

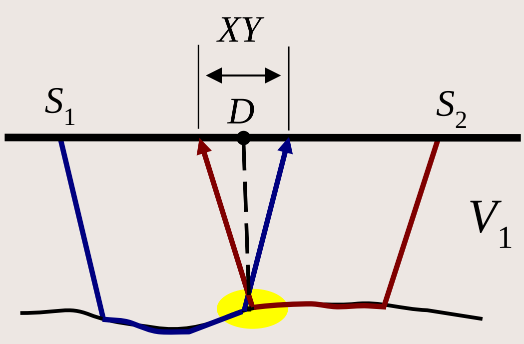
- Drawback of this method – averaging over the pre-critical region

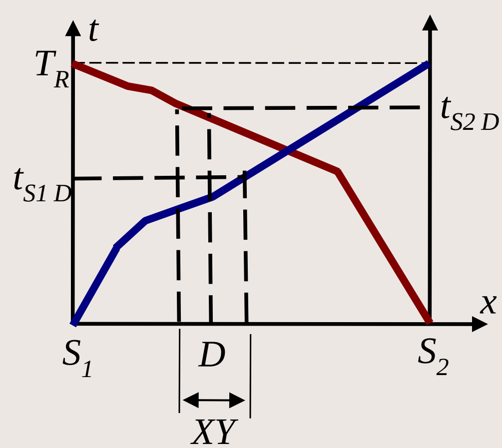




Generalized Reciprocal Method (GRM)

- Introduces offsets ('*XY*') in travel-time readings in the forward and reverse shots;
  - so that the imaging is focused on a compact interface area
- Proceeds as the plus-minus method;
- Determines the '*optimal*' *XY*:
  1) Corresponding to the most linear *velocity analysis function*;
  2) Corresponding to the *most detail* of the refractor.





- The *velocity analysis function*:

$$t_V = \frac{1}{2}\left(t_{S_1D} - t_{S_2D} + t_{S_1S_2}\right)$$

should be linear, slope $= 1/V_2$;

- The *time-depth function*:

$$t_D = \frac{1}{2}\left(t_{S_1D} + t_{S_2D} - t_{S_1S_2} - \frac{XY}{V_2}\right)$$

this is related to the desired depth:

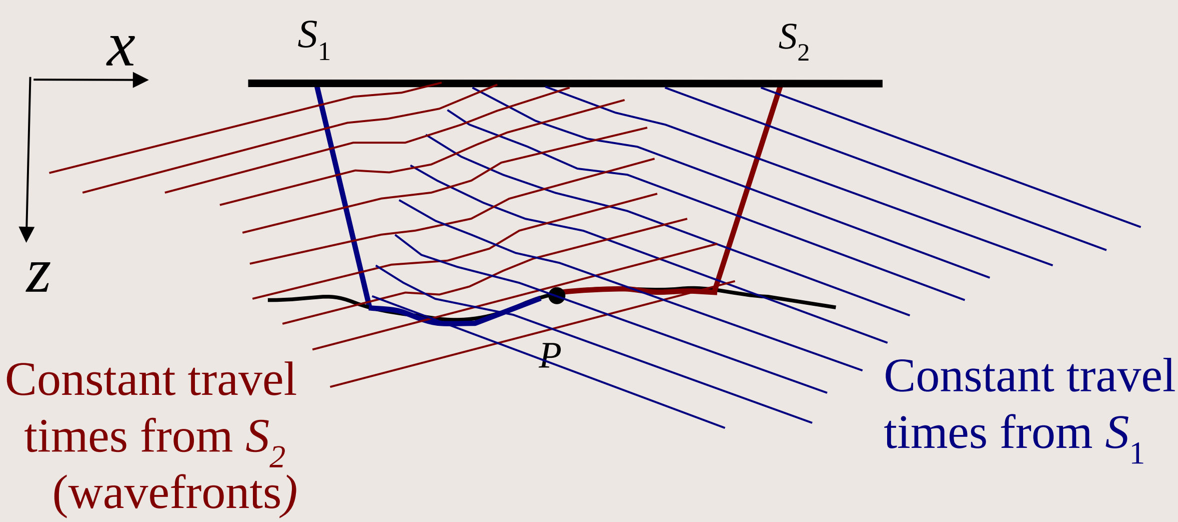
$$h_D = \frac{t_D V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

Head-wave "migration" (travel-time continuation) method

- "Migration" refers to transforming the space-time picture (travel-time curves here) into a depth image (position of refractor)
- We already used this method in Lab 1b
- Refraction (head-wave) migration:
  - Using the observed travel times, draw the head-wave wavefronts in depth;
  - Identify the surface on which:

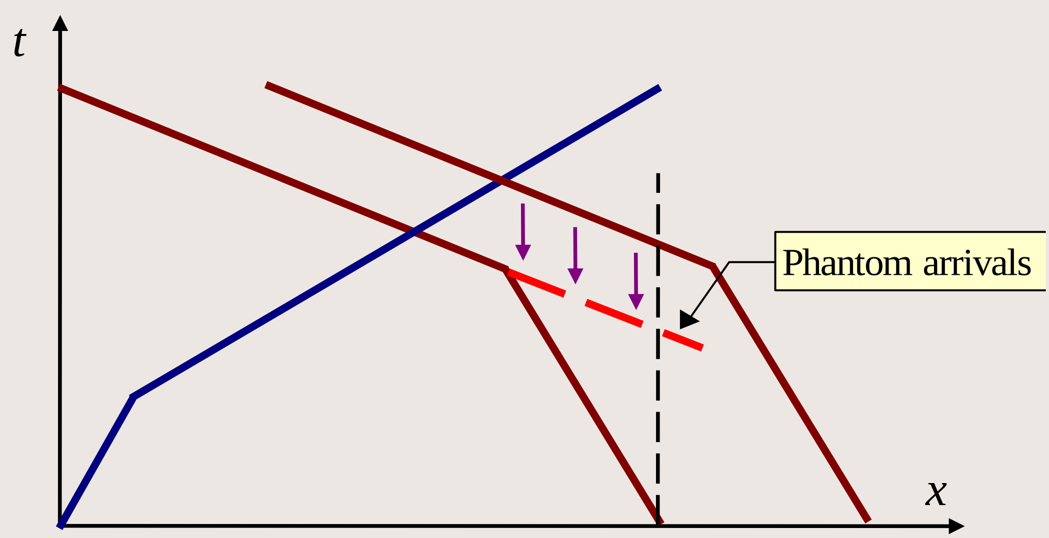
$$t_{forward}\left(x,z\right)+t_{reversed}\left(x,z\right)=T_R$$

  - This surface is the position of the refractor



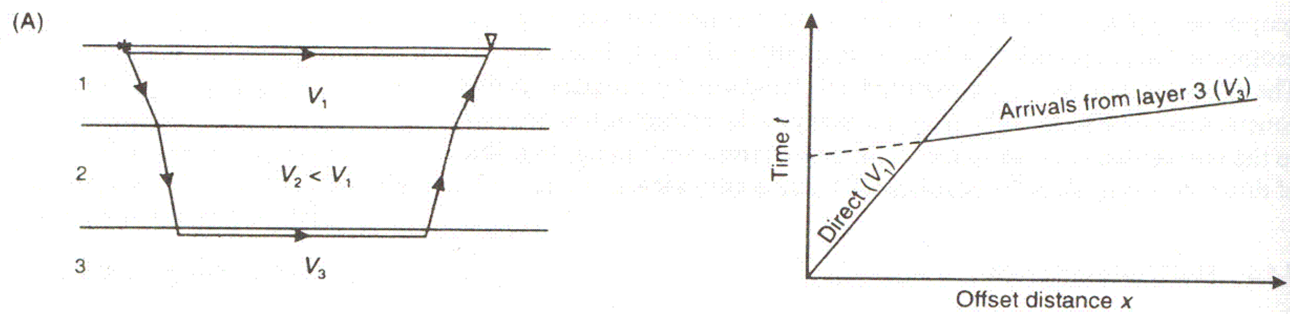
Phantoming

- Refraction imaging methods work within the region sampled by head waves, that is, beyond critical distances from the shots;
- In order to extend this coverage to the shot points, *phantoming* can be used:
  - Head wave arrivals are extended using time-shifted picks from other shots;
  - *However*, this can be done only when horizontal structural variations are small.

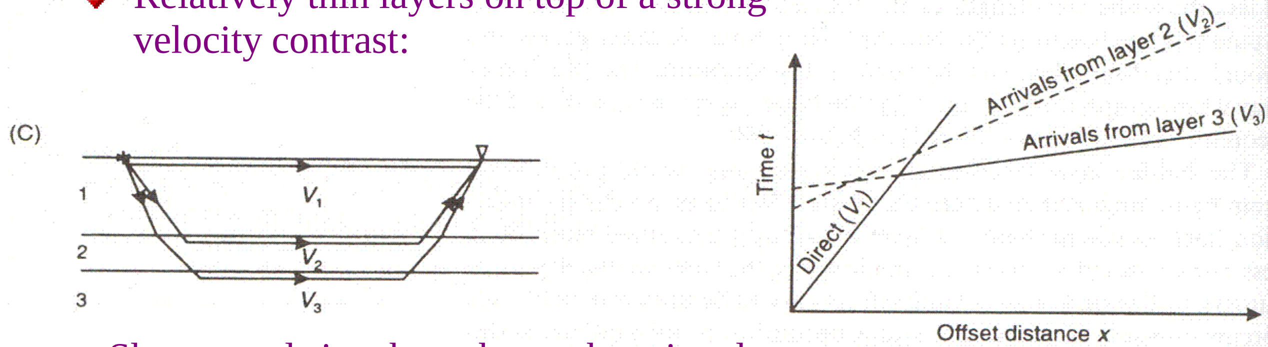


The "Hidden-Layer" Problem

- Velocity contrasts *may not be visible* in refraction (first-arrival) travel times. Three typical cases:
  - Low-velocity layers do not appear in first arrivals in principle:



  - Relatively thin layers on top of a strong velocity contrast:



  - Short travel-time branch may be missed when using sparse geophone coverage: