General Concepts

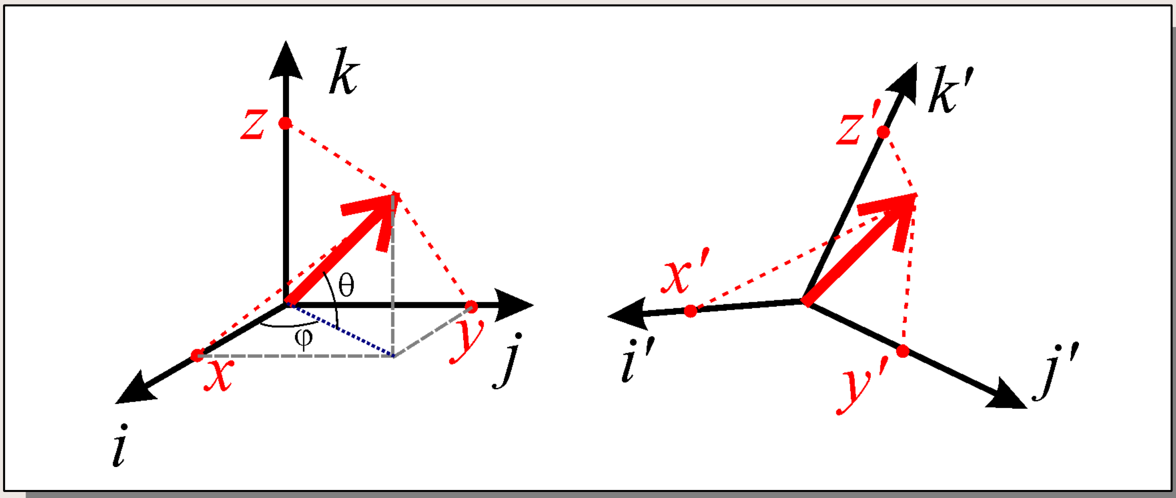
- Scalars, Vectors, Tensors, Matrices
- Fields
- Waves and wave equation
- Signal and Noise
- Reading:
  - Telford *et al*., Sections A.2-3, A.5, A.7
  - Shearer, 2.1-2.2, 11.2, Appendix 2

Scalar

- Non-directional quantity
  - Independent of the frame of reference used for measurement of spatial coordinates and orientation
  - Examples in geophysics:
    - Material density, fluid content
    - Temperature, pressure
    - Resistivity, electric potential
    - Gravity

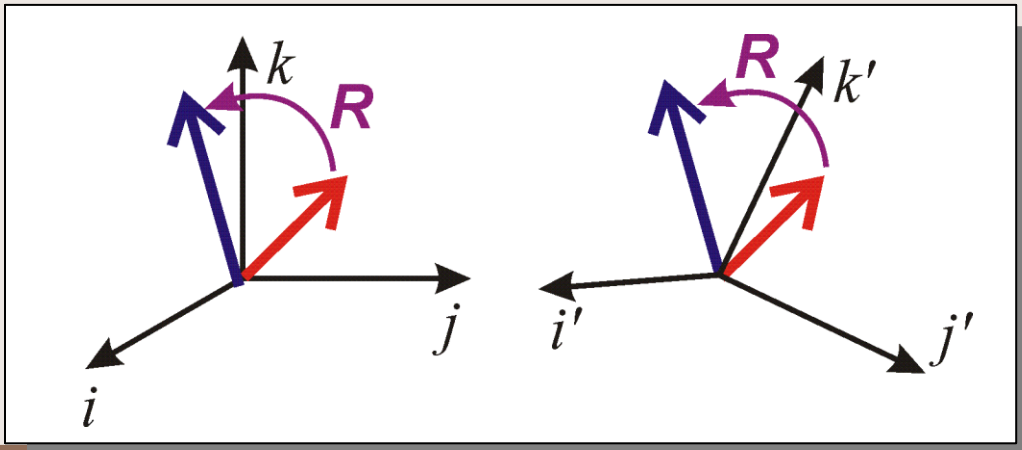
Vector

- Directional quantity
  - Possesses magnitude' and 'direction' and nothing else...
    - Thus it can be described by its amplitude and two directional angles (*e.g.*, *azimuth* and *dip*)
  - Examples:
    - Particle displacement, velocity, acceleration
    - Electric, magnetic fields, current density
  - Characterized by projections on three selected axes: (*x*,*y*,*z*)...
    - ...plus an agreement that the projections are transformed appropriately whenever the frame of reference is rotated



Tensor
(informal)

- Bi-Directional quantity
  - 'Relation' between two vectors;
  - Represented by a *matrix:*
    - 3×3 in three-dimensional space, 2×2 in two dimensions, etc.
    - ...however, this matrix is transformed whenever the frame of reference is rotated
- Examples:
  - Rotation operator, ***R*** in the plot below;
  - Stress and strain in an elastic body
  - Tensor quantities often arise when describing *anisotropy* of the medium (dependence on the direction of applied force or wave propagation)



Vector operations

- Summation: $c = a + b$

$$c_x = a_x + b_x, \quad c_y = a_y + b_y, \quad c_z = a_z + b_z$$

or simply: $c_i = a_i + b_i$

- Scaling: $\mathbf{c} = \lambda \mathbf{b}$

$$c_x = \lambda b_x, \quad c_y = \lambda b_y, \quad c_z = \lambda b_z$$

$$c_i = \lambda b_i$$

- Scalar (dot) product:

$$c = a \cdot b = a_x b_x + a_y b_y + a_z b_z$$

"Einstein's" notation: $c = a_i b_i$

- Vector (cross) product:

$$\mathbf{c} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$c_i = \varepsilon_{ijk} a_j b_k$$

Two key matrices (1)

- Unit (identity):

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{I_{ij} = \delta_{ij}}$$

  - δ_{ij} is called the "Kronecker symbol":

$$\delta_{ij} = \begin{cases} 1 & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases}$$

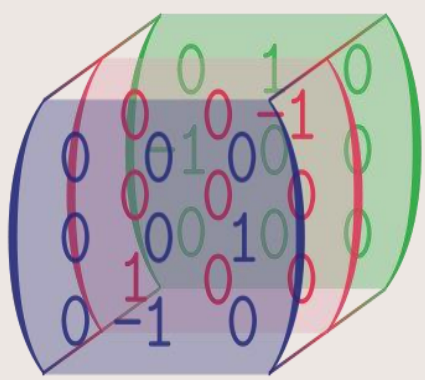
Exercise: evaluate δ_{ii} = ?

Two key matrices (2)

- Antisymmetric (or permutation, “Levi-Civita”) symbol:

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{for } (i, j, k) = \text{even permutations of } (1,2,3) \\ -1 & \text{for odd permutations of } (1,2,3) \\ 0 & \text{otherwise} \end{cases}$$

$\epsilon_{ijk} =$



key identities: $\varepsilon_{ijj} \equiv \varepsilon_{kik} \equiv \varepsilon_{lli} \equiv 0$

vector cross-product: $c_i = \varepsilon_{ijk} a_j b_k$

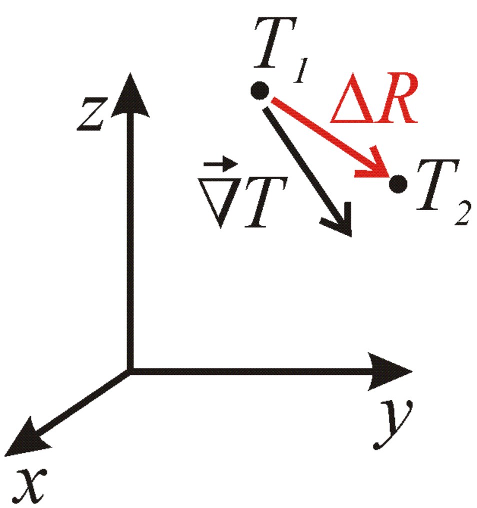
Exercise: evaluate $c_k = \varepsilon_{ijk} \delta_{ij}$

Field

- Physical quantity which takes on values at a continuum of points in space and/or time
  - Represented by a function of coordinates and/or time:
    - Scalar: $f\left(x, y, z, t\right)$ or $f\left(\mathbf{r}, t\right)$
    - Vector: $\mathbf{F}\left(\mathbf{r}, t\right)$ (spatially-variable force)
    - Tensor: $\sigma_{ij}\left(\mathbf{r}, t\right)$ (stress)
  - Always associated with some *source*, carries some kind of *energy*, and often able to propagate *waves*
- Ultimately, *everything in physics is fields!*

Scalar Fields

- Gradient
  - Spatial derivative of a scalar field (say, temperature, $T(x,y,z,t)$)
  - It is a Vector field, denoted ∇T ('nabla' T):



$$\Delta T = T_2 - T_1$$

$$= \vec{\nabla} T \cdot \Delta \vec{R}$$

$$= \frac{\partial T}{\partial x} \Delta x + \frac{\partial T}{\partial y} \Delta y + \frac{\partial T}{\partial z} \Delta z$$

$$\Delta \vec{R} = \vec{i}\,\Delta x + \vec{j}\,\Delta y + \vec{k}\,\Delta z$$

$$\vec{\nabla} T = \vec{i}\,\frac{\partial T}{\partial x} + \vec{j}\,\frac{\partial T}{\partial y} + \vec{k}\,\frac{\partial T}{\partial z}$$

Vector Fields

Differential operations and relations

- Gradient of a vector field is a tensor:

$$\left(\mathrm{grad}U_j\right)_i = \partial_i U_j$$

- Curl operation produces a new vector field:

$$\left(\mathrm{curl}F\right)_i = \varepsilon_{ijk}\partial_j F_k$$

Two Important Relations

- Divergence of a curl is always zero:

  div(**curl**(ψ)) ≡ 0.

  This will be the S wave

- Curl of a gradient is zero:

  curl(**grad**(ϕ)) ≡ **0**.

  This will be the P wave

- These properties are easily verified using Einstein's notation (try this!):

$$\left(\mathbf{grad}U\right)_i = \partial_i U$$

$$\left(\mathrm{curl}\,\mathbf{F}\right)_i = \varepsilon_{ijk}\partial_j F_k$$

Static Fields and Waves

- Fields in geophysics typically exhibit either *static* or *wave* behaviours:
  - Static – independent on time:

$$\frac{\partial T}{\partial t} = 0$$

Stationary temperature distribution (geotherm).

  - Wave – stable spatial pattern propagating with time:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0$$

Acoustic (pressure) wave.

This is the typical form of wave equation; c is the velocity of propagation.

$$p = f\left(x - ct\right)$$

Plane wave propagating along the X-axis.

$f(\ldots)$ is the waveform at time t, it has its "zero" at $x = ct$ and propagates to $x > 0$

Exercise: show this

Signal and Noise

- Geophysical data always contain some SIGNAL and some NOISE
  - Signal - 'deterministic' part that we want to know
    - Consistent with the method employed
  - Noise - anything else mixed into the measurement
- **Sources of noise:**
  - Instrument
  - Geologic sources
  - Too simple theory (*e.g.*, using 2-D sounding in a 3-D Earth)
- **Types of noise:**
  - Coherent (caused by the signal itself, worst of all)
  - Incoherent (random, coming from unrelated sources)
    - Such noise can be reduced by filtering
- Main task of data processing is to increase the signal/noise (S/N) ratio

S/*N* improvement by stacking (summation) of recordings

- "Stacking" (summation) is the most common approach to increasing the Signal/Noise ratio
- To derive the *S/N improvement factor*, consider stacking of *N* records with identical signals and random noise:

$$u_i(t) = s(t) + n_i(t)$$

- Stacked signal amplitude is proportional to *N*:

$$\sum_{i=1}^{N} u_i(t) = Ns(t) + \sum_{i=1}^{N} n_i(t)$$

- Noise power increases $\propto N$ (despite what is commonly said, noise is not "attenuated" by stacking!):

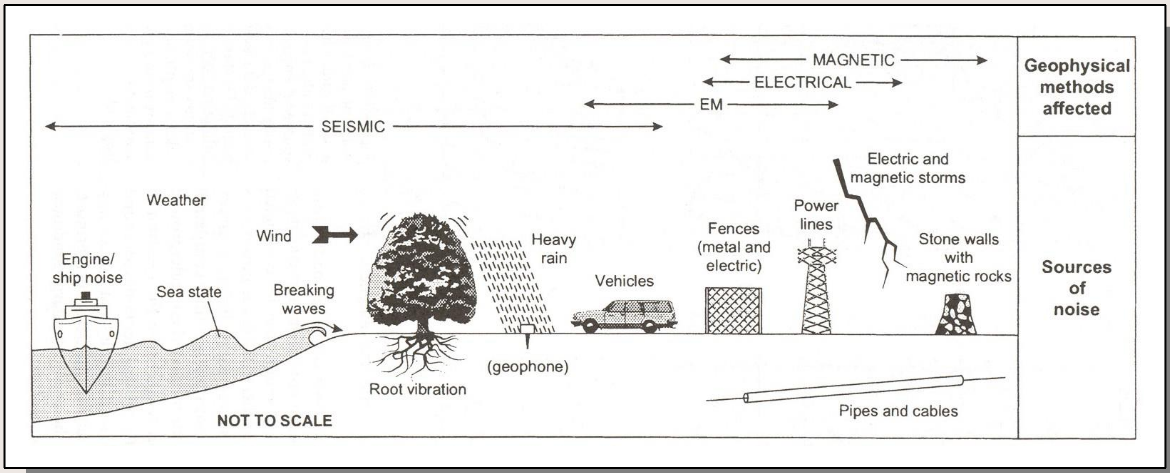
$$\left[\sum_{i=1}^{N} n_i(t)\right]^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} n_i(t)\, n_j(t) = Nn^2(t)$$

- Therefore:

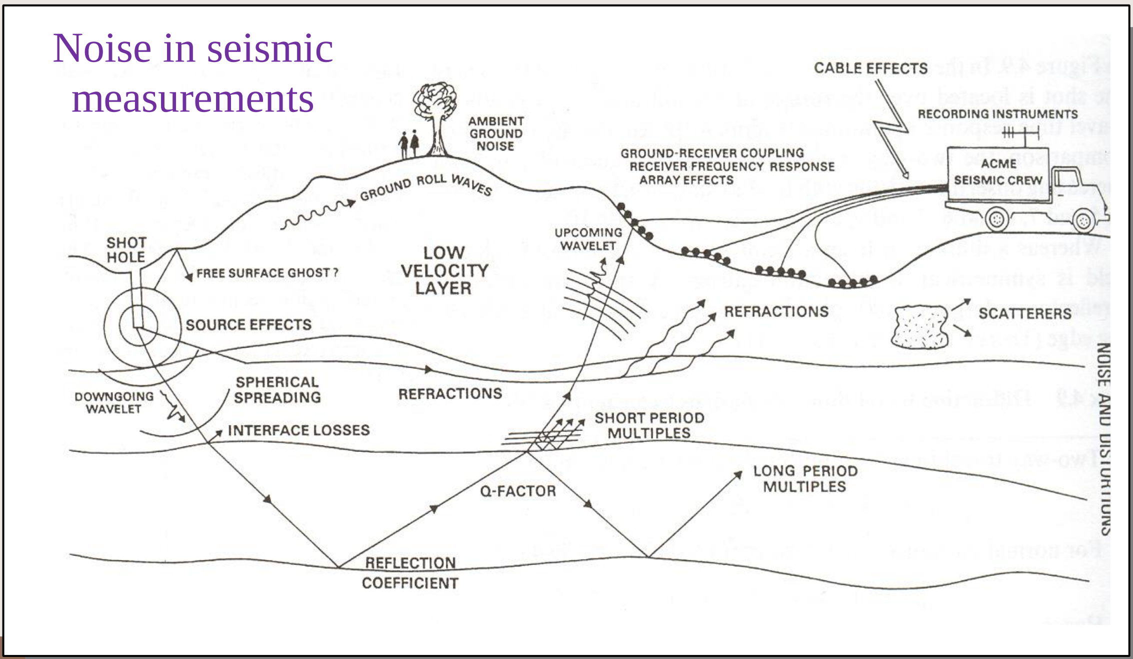
$$\frac{S}{N} = \sqrt{N}\,\frac{s}{n}$$

S/*N* ratio increases as $\sqrt{N}$

Noise in Geophysical Measurements



- For seismics, the *signal* is represented by reflections and refractions
  - For 2D, also only those coming in-plane.
- Several factors cause degradation of the seismic signal:



From Reynolds,1997