Sampling

- Discrete sampling of continuous signals
- Binary representations of data
- Aliasing
- Dynamic range

- Reading:
  - Telford *et al*., Sections 4.7.2-6

Discrete Sampling of Signals

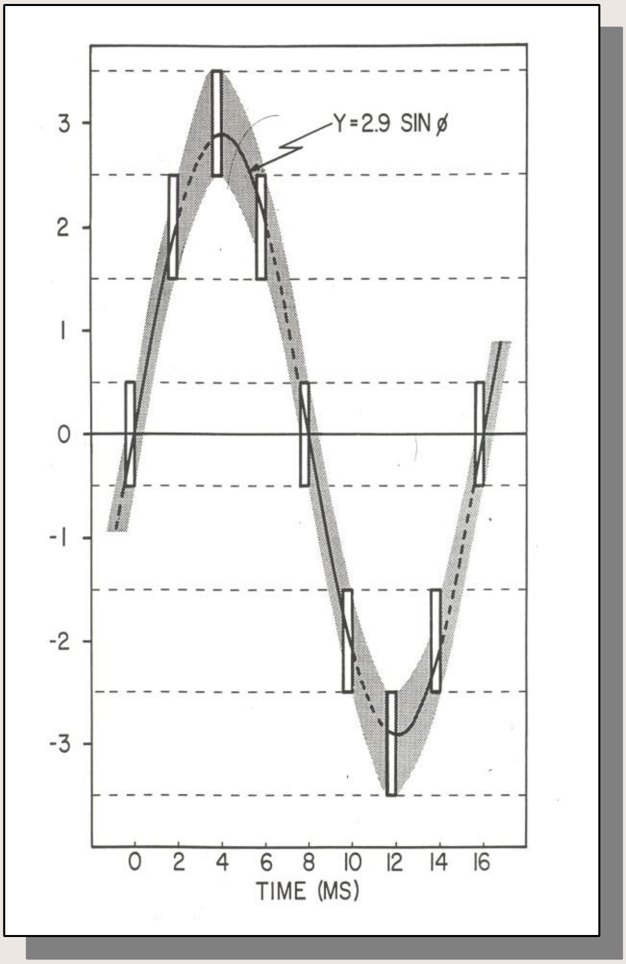
- Suppose we need to digitize a continuous signal (vibration caused by a seismic source, gravity or electrical field, *etc*.).
- To design an Analogue-to-Digital (AD) converter, we have to answer two key questions:

1. Choose the *sampling interval*. How dense the sampling should be?
2. Choose the *dynamic range*. How *deep* should the amplitude measurement be? How many possible values should the discrete output contain?

   - In practice, these questions reduce to choosing a '8-bit’, '16-bit’, or ’24-bit’ AD converter.

Sampling and Quantizing Process

- Analog-to-Digital (AD) converter compares the levels of the signal to a set of predefined levels;
- Integer values are used as output;
- Need to have sufficient sampling density in *time* and *amplitude*.

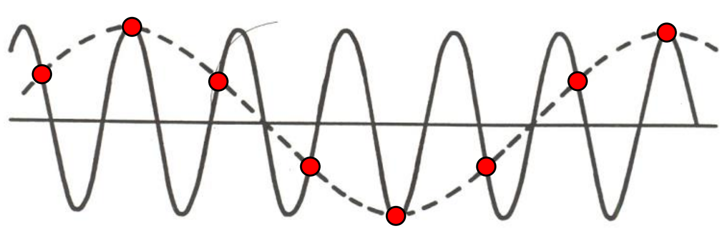


Nyquist Frequency

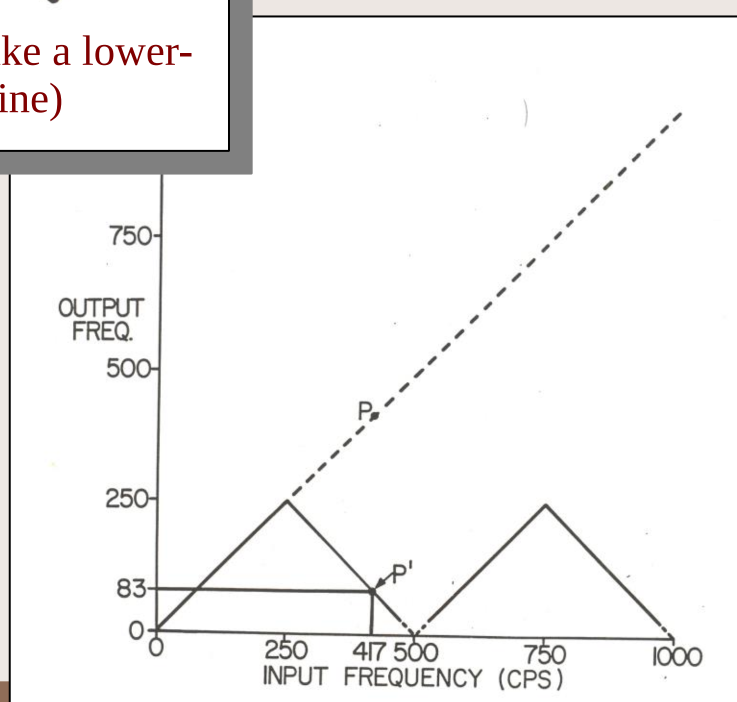
- Consider a seismic record sampled using N samples at a sampling interval of Δt. The *fundamental frequency* (the frequency of sampling) is then $1/\Delta t$
- However, it turns out that if we use less than two points per period $T = 1/f$, the signal can also be ambiguously represented by frequency $f_{aliased} = 1/(2\Delta t) - f$ (see next slide)
- Consequently, only waves below frequency $f_N = 1/(2\Delta t)$ are unambiguously recoverable after discretization. This is called the *Nyquist frequency*
- Thus, **the rule for choosing the sampling interval is: *the shortest period of interest should include at least 2 samples*.**
    - In practice, twice faster sampling is typically used

Frequency Folding
(Aliasing)

- If sampling is attempted at frequency less than twice the frequency of the signal, distortion occurs (called *aliasing*)
  - After playback from discretized records, signal at too high frequency $f_N + \delta f$ looks like low-frequency signal at low frequency $f_N - \delta f$
    - This phenomenon is called **frequency folding**



Aliased readings (red dots) look like a lower-frequency signal (dashed line)



Aliasing

- Inadequate sampling rate results in *aliasing*: the signal above the Nyquist frequency appears as a distorted low-frequency signal
- It is generally very difficult or impossible to clean up records contaminated with aliasing noise
- To avoid aliasing, low-pass (called *anti-aliasing*) filters are built into the electronics of data loggers
  - The analog anti-aliasing filter removes all input signals at frequencies below about $0.9f_N$, so that this signal does not get frequency-folded as shown in the preceding slide

Binary representation of values

- All digital systems use binary system of representation of integer values
  - Floating-point values are represented as three integers: *sign*, *mantissa*, and *exponent*.
    - Example: -314.15 = -0.31415·10^3.

Exponent
Sign
Mantissa

- The binary scale uses only two digits, 0 and 1 (corresponding to a digital circuit states '*on*' or '*off*'). One element of this scale is called *bit*
  - A series of 8 bits is called *byte*, bytes are arranged into *words.*
  - Typical AD converters output 1-, 2-, 3-byte (8-, 16, 24-bit) words.
- Each additional bit doubles the range of possible output values
  - Here is how the decimal value of 101 is represented by a 8-bit binary word 01100101:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	1	1	0	0	1	0	1
0	64	32	0	0	4	0	1

Dynamic Range

- The 'depth' of magnitude recording is measured by its *dynamic range*, expressed in decibels (dB)
  - Dynamic range measures the ratio of the largest and smallest amplitudes that are (or can be) correctly recorded

$$\left(\frac{A_1}{A_2}\right)_{\text{in dB}} = 20\log_{10}\left(\frac{A_1}{A_2}\right)$$

- In a digital recorder, the dynamic range is controlled by the *number of bits* used to store or output the values
  - Each additional bit allows doubling the recorded values. Thus, one byte in a recording system gives $20\log_{10}2 = 6$ dB of dynamic range
  - Modern data loggers use 24-bit AD converters. This gives about 140 dB of dynamic range, which is ample for most applications.