

Digital Filtering

- Convolution of time series
- Convolution as filtering process
- Cross- and auto-correlation
- Frequency filtering
- Deconvolution

- Reading:

- › Telford et al., Sections A.10,11

Convolution of time series

- *Convolution* for time (or space) series is what commonly is multiplication for numbers.
- Example of a ‘*convolutional model*’: rise in lake level resulting from rainfall
 - ◆ Let’s assume that the recorded rainfall over 5 months is: 2, 3, 1, 4, 3 cm, respectively.
 - ◆ The lake level will respond to a 1 cm of rain fall, say, with a 2-cm rise in the first month and 1-cm in the second. This is called the ‘*impulse response*’.
 - ◆ Lake level rises from the different months of rainfall will accumulate *linearly*.

Lake Level Rise

Numerical example

- The resulting lake levels can be calculated by the following procedure, called *convolution*:

1	2	3	4	5	<i>Months</i>
2	3	1	4	3	<i>Rainfall time series</i>
1	2				<i>Lake response (reversed)</i>
4					<i>Rise in the first month</i>
2	3	1	4	3	
1	2				
2+6=8					<i>Rise in the second month</i>
2	3	1	4	3	
1	2				
3+2=5					<i>Rise in the third month</i>
2	3	1	4	3	
	1	2			
1+8=9					<i>Rise in the fourth month</i>
4	8	5	9		<i>Resulting lake level sequence</i>

- The impulse response series is reversed and shifted, and sample-by sample dot product is taken to find the response at any moment

Convolution

General formulae

- The resulting lake levels can be calculated by the following procedure, called *convolution*. Convolution of two series, u_i , and w_i , denoted $u * w$, is:

$$(u * w)_k = \sum_i u_{k-i} w_i$$

- As multiplication, it is symmetric (commutative):

$$u * w = w * u$$

- Note that if we need to multiply two polynomials, with coefficients u_k and w_k , we would use exactly the formula above. Therefore, **in Z or frequency domains, convolution becomes simple multiplication** of polynomials (show this!):

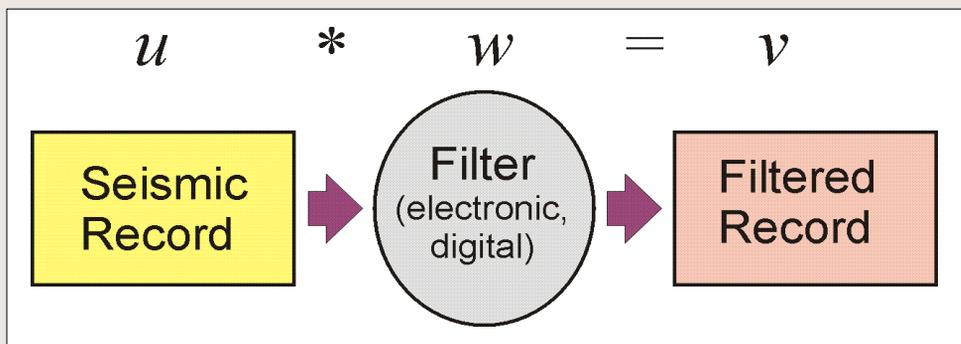
$$u * w \leftrightarrow Z(u)Z(w) \leftrightarrow F(u)F(w)$$

- This is the key property facilitating efficient digital filtering.

Convolution

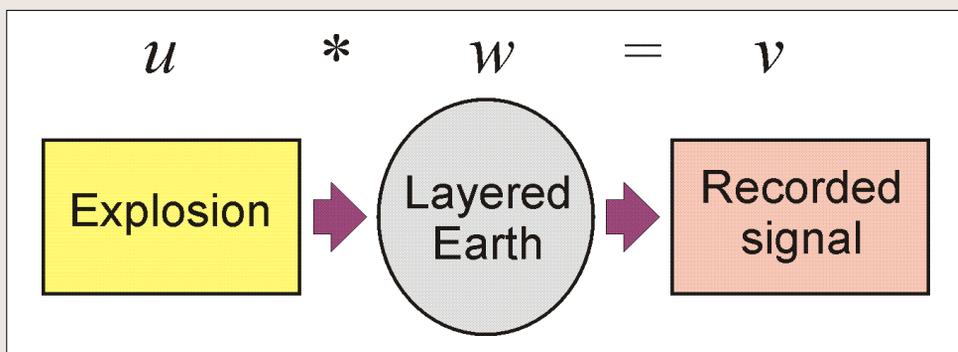
Two important cases of interest

- **Digital signal filtering**



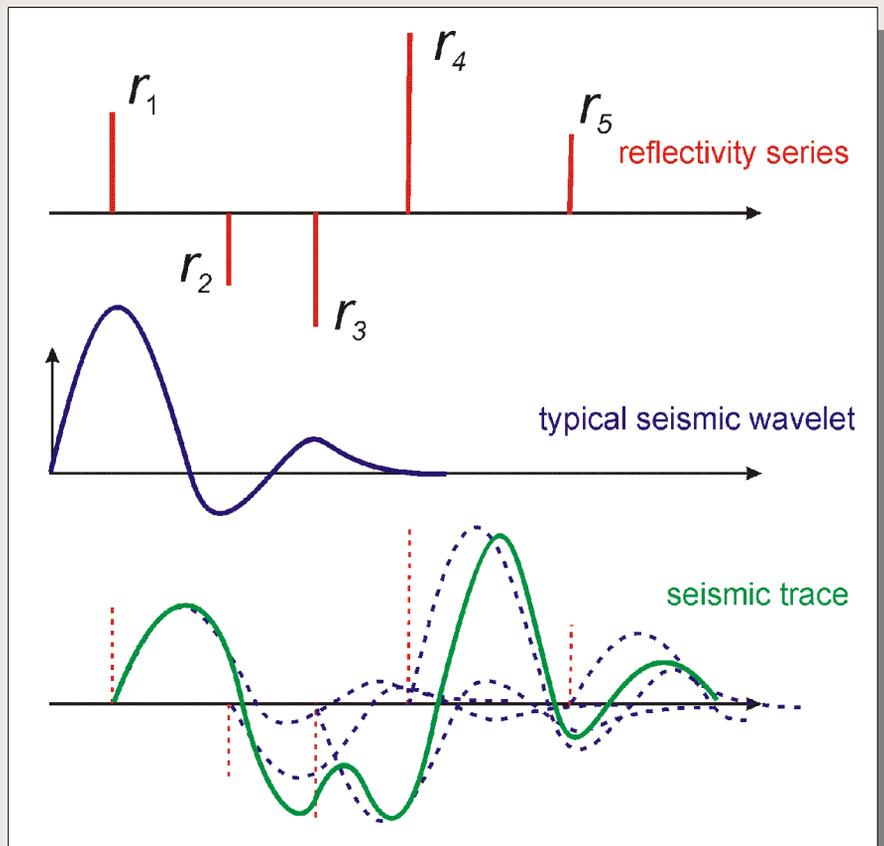
- **Earth's response** is also a filter. Note that in this case, the *impulse response* is unknown and is of primary interest

- ♦ Hence reflection processing deals with *inverse filtering*... (i.e., finding the filter)



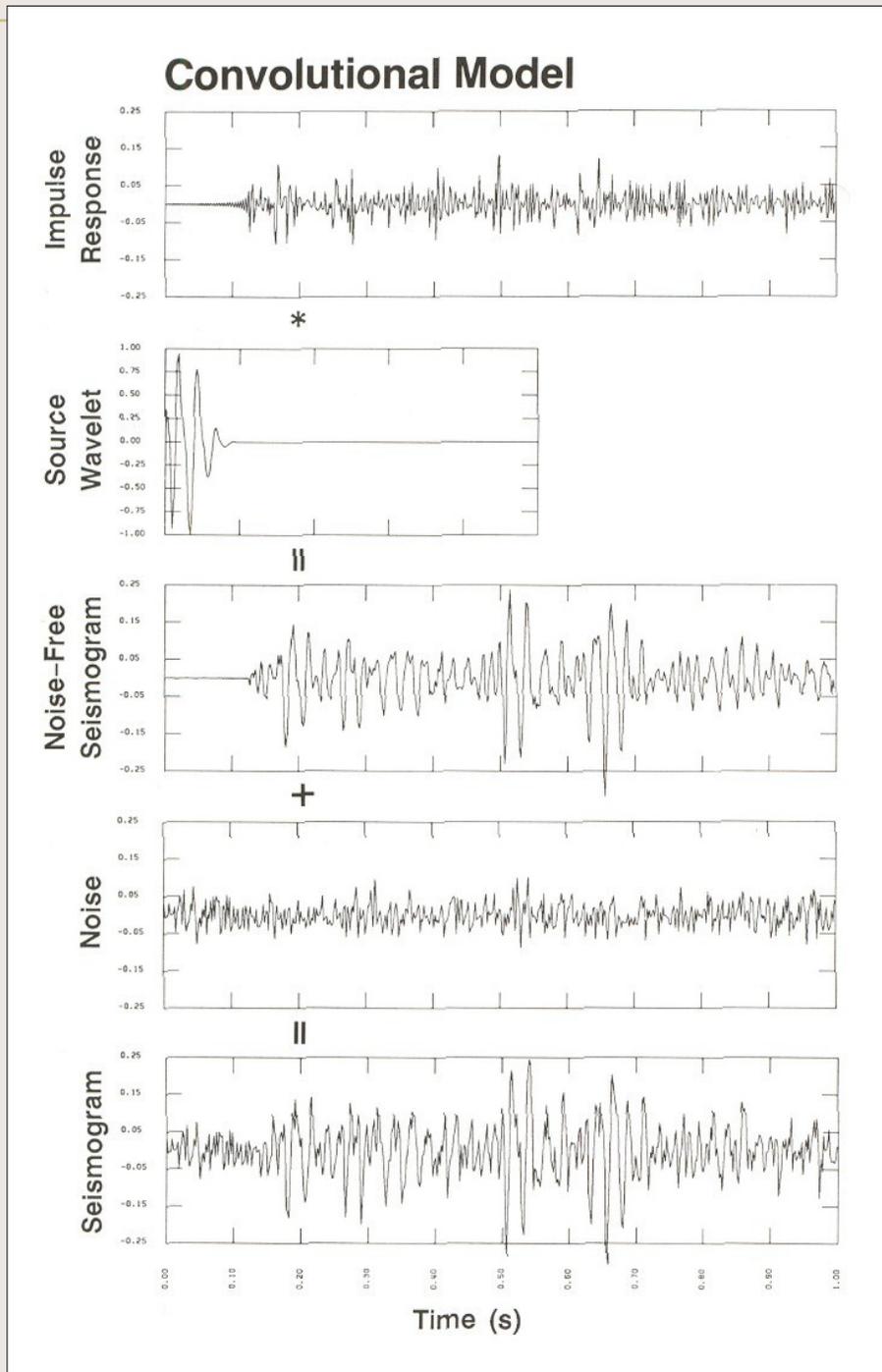
Convolutional model of a reflection seismic record

- Reflection seismic trace is a convolution of the source wavelet with the Earth's 'reflectivity series'



Convolutional model

A more realistic example



Cross- and Auto-correlation

- *Cross-correlation* gives the degree of similarity between two signals:
 - ◆ For each value of a 'lag' i :
 - ◆ Shift the second trace by the lag
 - ◆ Calculate dot product:

$$\text{cross}(u, w)_k = \sum_i u_{k+i} w_i$$

- The lag for which the cross-correlation is largest gives the time shift between the two records
 - ◆ A most important application – pre-processing of Vibroseis recordings
- *Auto-correlation* of a record is its cross-correlation with itself
 - ◆ It is symmetric in terms of positive and negative lags
 - ◆ It indicates the degree to which the signal repeats itself.

Linear Filtering

- Most operations with seismic signals can be represented by a convolutional operator:

$$v = F u$$

- It is *linear*:

$$F(u_1 + u_2) = F u_1 + F u_2$$

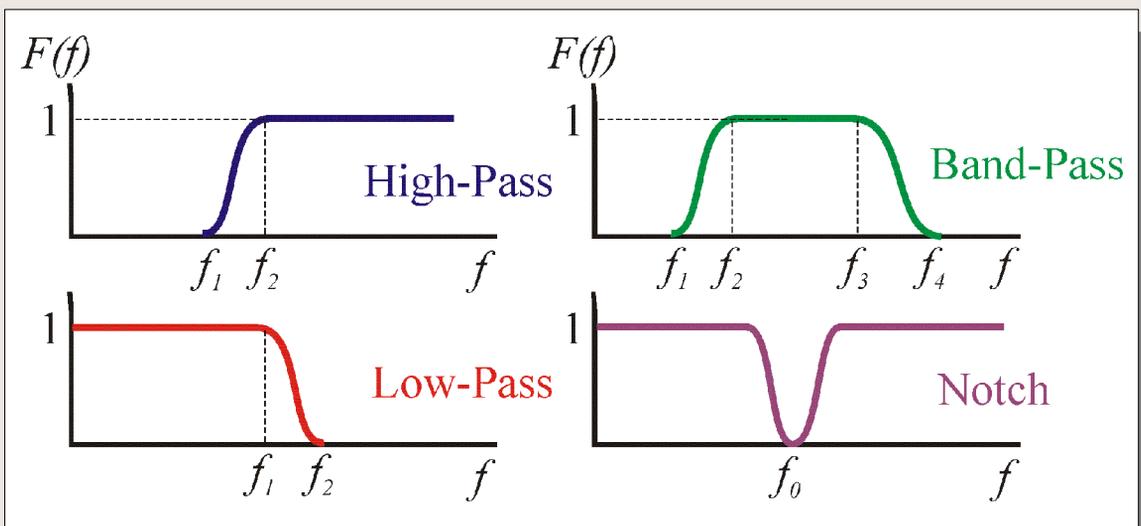
- It is *translationally (time-) invariant*:

$$F \text{ time shifted}_t(u) = \text{time shifted}_t(F u)$$

- The filtering operator is represented differently in different domains:
 - Convolution in time domain
 - Complex-value multiplication in Z- and frequency domains
 - This allows easy frequency filtering (selective enhancement or suppression of harmonic components in the signal)

Frequency Filtering

- **Key element** of seismic and GPR processing.
- **Low-pass** (high-cut), **Band-pass**, **High-pass** (low-cut), **Notch**
 - Suppressing the unwanted (noisy) parts of the frequency spectrum.
 - Usually *zero-phase*, to avoid phase (travel-time) distortion. This means that the filter does not change the phase spectrum.



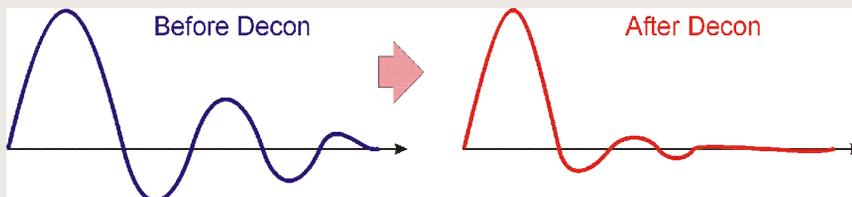
Filter panels



- Trial band-pass filter panels are designed in order to determine the best frequency range for data display and analysis. For a final display, *time-variant* filters are used.

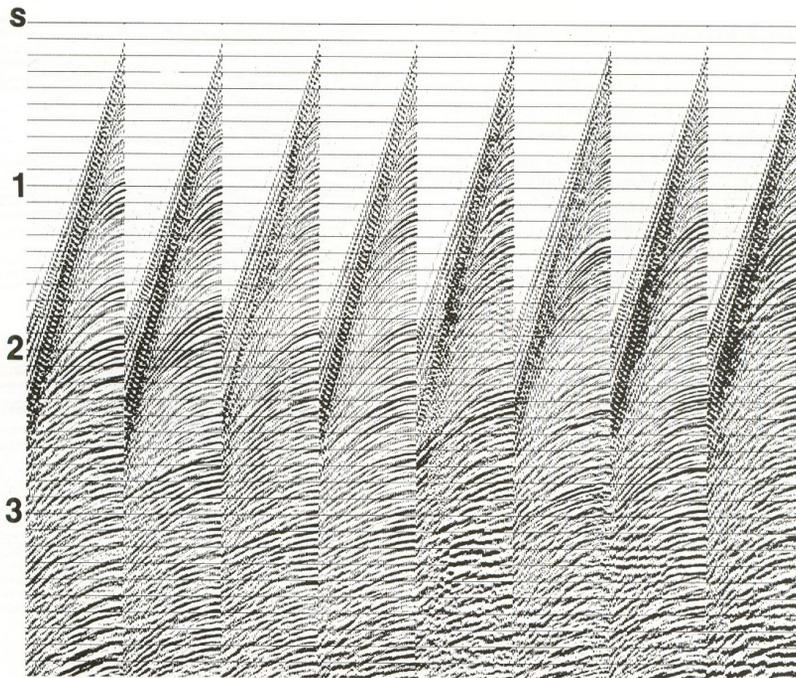
Deconvolution

- **Deconvolution (inverse) filters**
 - To remove (reduce) the effects of wavelet's complexity on the resulting image.
 - Based on the known (or estimated) wavelet shape, an *inverse* filter is designed with the objective to compress this wavelet in time:

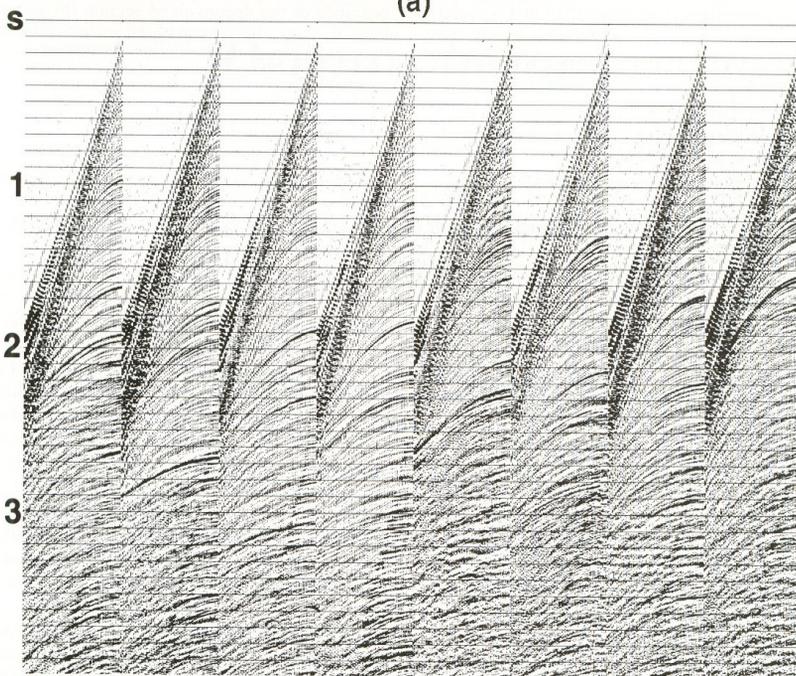


- Numerous deconvolution techniques are available
 - Performed in time or frequency domains.

Deconvolution (shot gathers)



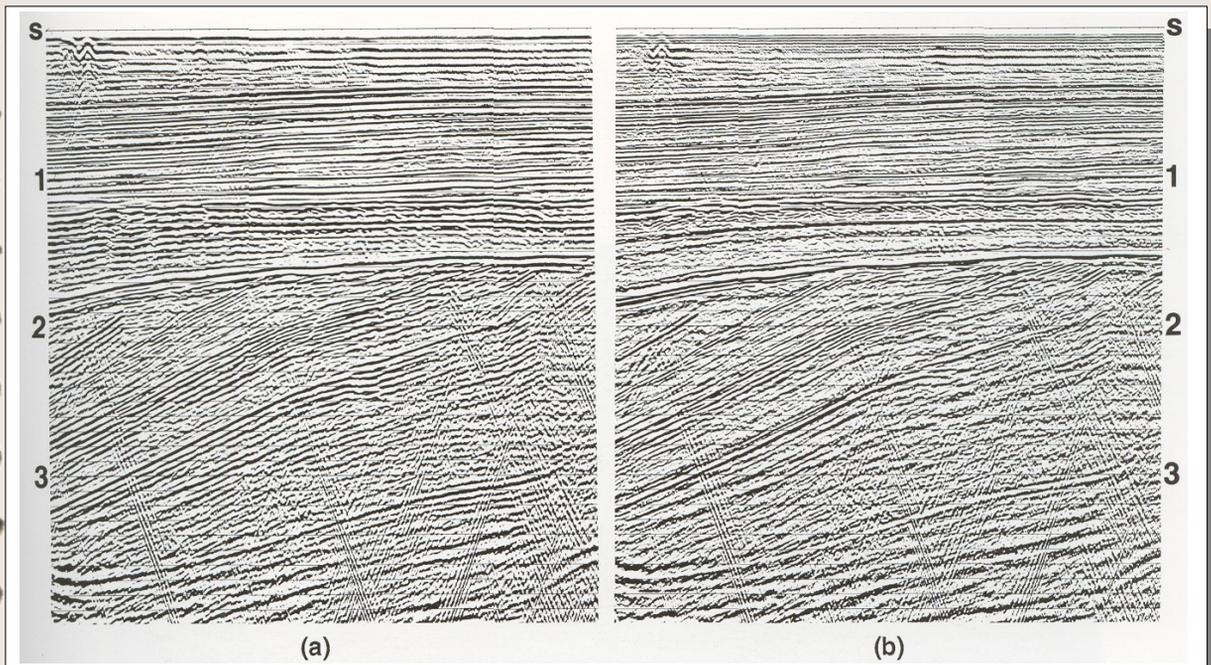
(a)



(b)

From Yilmaz, 1987

Deconvolution (stack)



From Yilmaz, 1987

- Interpreters certainly prefer working with the crisper, high-resolution image (b) that uses deconvolution.

Signal/Noise Ratio enhancement *by filtering*

- This is an example of the most common *multichannel filter*.
- Records are summed ('*stacked*') in order to increase the S/N ratio:
 - ◆ Signal is assumed the same in all channels, therefore, its **amplitude** is increased $\propto N$ (the number of records);
 - ◆ For *incoherent noise*, the **energy** becomes proportional to N , and so the amplitude increases as \sqrt{N} ;
 - ◆ Therefore, the S/N ratio $\propto \sqrt{N}$.
 - ◆ **Note:** *coherent noise* cannot be suppressed by stacking!

