GEOL 384.3 and GEOL 334.3

Lab #1: Analysis of gravity collected in a stairwell of Geology building

In this lab, we will study gravity variation with elevation using a gravity survey performed in the southwest stairwell of the Geology building. Normally, this lab involves a short data acquisition, but this time, we will use the data acquired in 2009. Th gravity meter used in UofS field schools and these labs is shown in Figure 1.

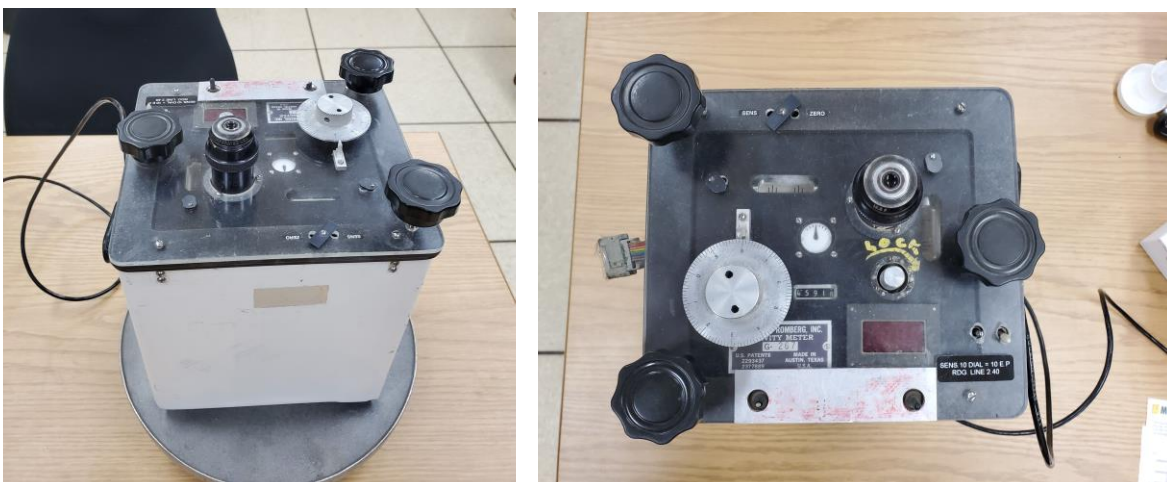

Figure 1. Lacoste G267 relative gravimeter used in this lab

Most if this lab is by Jim Merriam, with some modifications and additional assignments.

Survey

A gravity survey consists of four principal parts:

A) Selecting the locations of the stations at which you will perform measurements and surveying their precise locations by using GPS;

B) Establishing an absolute-gravity station (for calibration of the relative instrument);

C) Establishing a base station visited periodically (which allows correcting for the drift). This station is usually one of the stations of the survey.

D) Performing measurements.

These three parts are described below. First, prior to our survey (years ago), absolute gravity was measured at a point in the loading dock (dot on the floor marked “g”). We refer to this location as the “absolute gravity station”. Absolute gravity stations in Canada are established by Natural Resources Canada.

There are about 5000 stations in the Canadian Gravity Standardization Network. In the Saskatoon area, there is one at the airport, one at City Hall and one at the UofS Seismic vault. Most airports and most city halls in Canada would have an absolute station.

Note that the absolute gravity at the loading-dock station equals 981120.59 mGal.

Second, a base station was established in the sub-basement near the stairs. The base station near the survey area is needed to allow (reasonably) frequent calibration of the measurements. In a field survey, the nearest absolute-gravity station may be many km away, and the instrument and tidal effects will drift during travel.

Gravity readings at this point were tied to absolute gravity by measuring the difference between gravity at the base and gravity at the absolute station. The connection between the absolute station and base survey station is normally repeated at least three times. Typically, we get a difference of 0.49 dial units between these stations, or 0.49*1.04845 = 0.51 mGal. Therefore, to obtain the absolute gravity at our base station near the stairs, we would add 0.51 mGal to the absolute gravity in the loading dock.

Thus, the absolute gravity at the selected base station is:

$$g_{\text{absolute base}} = 981121.10 \text{ mGal}$$

To control the instrument drift during the survey, measurements at the base station were taken:

1) Before the first reading up the stairs,
2) After the last reading and
3) Every hour in between.

For the actual survey, a station was established on each landing from the sub-basement to the fourth floor, such that the first three are below ground surface and the rest above ground surface. The data (time, elevation and dial reading) are in file STAIRWELL.txt.

Most field gravity meters are RELATIVE READING instruments. This means they need to be calibrated so that you know what value of gravity corresponds to a dial reading. A meter should be supplied with a calibration table that allows you to do this. A portion of the calibration table for the Lacoste 267G was given in the lecture and is repeated in Table 1 on the next page.

The use of this calibration table simply consists in interpolating its second column by matching your dial readings with the first column. To perform this interpolation:

A) For your dial reading *d* from the instrument, find the row in the table with *Dial reading* nearest but less than *d*;

B) Evaluate the difference between *d* and the table value found in step A;

C) Multiply this difference by the *Factor for the interval* and add the result to the mGal value obtained in step A

For example, if the dial reading is *d* = 4650.32, then the calibrated value of gravitational attraction is

$$4817.47 + 50.32\times1.04845 = 4870.23 \text{ mGal}$$

Table 1. Calibration table for UofS Lacoste Romberg 267G

Dial reading	*Value (mGal)*	*Factor for interval*
4300	4502.91	1.04853
4400	4607.77	1.04853
4500	4712.62	1.04848
4600	4817.47	1.04845
4700	4922.31	1.04844
4800	5027.16	1.04848

Assignments

1) **Familiarize yourself** with the contents of file STAIRWELL.txt. Note the columns with the time of each reading and coordinates, in particular the elevations. As explained in the lectures, elevations should be known precisely (within ±3 cm) when using gravimeter, and times (within minutes) is important for drift corrections.

   Note the columns STN and FLAG. STN is the sequence station numbers, with number 0 reserved for the base station. This number does not have to be 0 in all cases. In the field, it is convenient to start station numbers from 100 or 101 and increase them in north or east directions, so that you would know where to look for the next station. Starting from 100 or 101 allows extending the line in the opposite direction if necessary, without running into difficulties with negative numbers in some software.

   Column FLAG marks whether the reading is done at the base or "field" station (B or F) or is a repetition of the preceding measurement (repeated measurements are performed to estimate errors).

Further, you can work with the same table or make a new table containing time, station, height, raw dial reading and gravity. In this table, convert the times into hours from some convenient reference. For example, you can use 2 pm on the day of this survey.

You will be working by adding columns to this table and making plots of its columns.

2) **Calibrate each of the dial readings** as described above. Add the calibrated recordings as a new column in the table.

The next step is to drift correct the gravity data. Drift refers to a continuous change in the reading level of the instrument dependent on time only. Part of this is a real change in gravity due to the earth's tides, and part is due to a change in the instrument – a stretch of the spring, thermal expansion, or some other mechanical change. The tidal effect on gravity can be calculated reasonably well and subtracted, or it can be considered as part of the instrumental drift, as you will do it below.

Drift correction is based on the idea that with no drift (instrumental or tidal), the instrument would read a constant value at any given station at all times. In particular, all the base station readings would be the same. The drift correction should therefore result in corrected base station readings that are nearly constant in time.

The drift correction consists of calculating a time-only function approximating the readings at the base station and subtracting this function from all data. This time function would be the model of drift, which would tell you how the instrument was drifting and how gravity was changing due to the earth tides in this region.

3) **Perform drift correction**. To do this, you need to plot the values of base station gravity vs time, as shown by symbols in Figure 1 on the next page. You do not need to plot error bars; just keep in mind that the errors are estimated as 0.01 mGal.

Next, draw either a straight line, smooth curve, or several connected straight-line segments approximating the base-station gravity these points. This is your drift model.

GEOL384 students – **use any software (Excel?) or hand** for plotting and data fitting; **GEOL334** – **use Matlab or Octave.**

The resulting plot should look (but not exactly) like the figure on the next page. In this figure, a linear drift with time is chosen, because that seems to be all that is required. In this example, there are five base-station readings and a straight-line drift model has been computed. Note that the drift model does not exactly fit the base readings and their error bars, and so a drift model with some curvature could be added as well.

GEOL334 students – try function *polyfit*() for fitting polynomial functions of orders 1 and 2 and discuss the difference between the resulting drift models.

Select some time as your reference time – for example, the time of the last base reading before the first field reading. Mark the measured gravity at this reference time as the "reference level" (red in the figure, 4780.37 mGal). Any other value could have been chosen and they would all have resulted in the same result for tied-to-absolute gravity.

Then, **correct whatever the instrument was reading at each time** to the reference level. The correction consists in adding to each point in the data table the difference between the reference level (red line in the figure) and the drift-model level (blue line) at that time.

For example, suppose that at the time of a particular field reading, the drift curve (blue) indicates that the gravimeter was reading 0.02 mGal below the reference level (red). Then this field reading needs to be increased by 0.02 mGal. At 2.4 hours in the figure, the meter was reading 0.04 mGal lower than the reference level, so 0.04 mGal would have to be added to the field reading taken at this time.

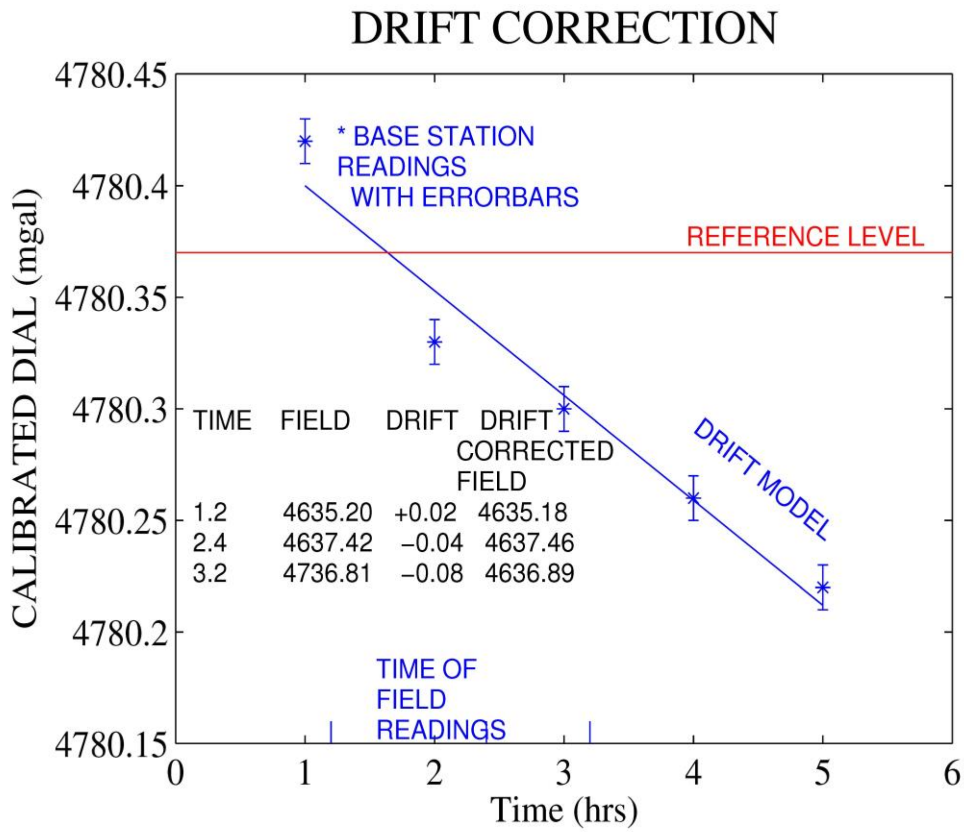

Fig. 1. Base station readings with estimated error ±0.01 mGal (blue error bars) and the fitted drift model (blue line). Black table shows subtraction of this model from the field data (labeled F in the data file).

In your output table, add a column for drift-corrected data and put into it all the field and base-station readings adjusted like this.

Make a separate copy of the plot like Fig. 1, in which you plot the drift-corrected data. Plot the base station data points in blue and field station data in black colors.

Comment whether the variations of the drift-corrected gravity <u>at base station</u> appear random or systematic with time.

4) **Measure the repeatability (random errors) of the data**. By using the repeated readings (marked by FLAG = R in the data file), evaluate the repeatability by the following formula:

$$\text{Repeatability} \equiv \sqrt{\frac{\sum_{j=1}^{N_{\text{pairs}}} \left(g_j - g_{\text{preceding to } j \text{ measuremermt at the same station}} \right)^2}{N_{\text{pairs}}}},$$

where N_{pairs} is the number of repeated pairs of measurements (three in your data file), and all g_j values are of course <u>drift-corrected</u>. **Include this repeatability value in your report.**

GEOL334 – also evaluate the same measure for the drift-corrected base-station readings. How do they compare to the above survey repeatability?

Your calibrated and drift-corrected stairwell gravity is determined by the reference level and should be around 4800 mGal. This is not absolute gravity. Therefore, you need to …

5) **Put your field data on an absolute scale**. To do this, you need to add to each point the difference Δg_{abs} between the absolute gravity and the calibrated and drift-corrected gravity at the base station:

$$\Delta g_{\text{abs}} = g_{\text{absolute at base}} - g_{\text{calibrated at base}},$$

where $g_{\text{absolute base}}$ is given at the beginning of this lab, and $g_{\text{calibrated base}}$ is the reference level you selected in step 3).

6) **Evaluate the latitude correction**. Because all data points are at the same latitude, only one value needs to be calculated. In the lectures, I gave a differential formula convenient for measurements in the field, and here let us use another expression for the World Geodetic System (1984) version of the reference Earth's ellipsoid. This expression gives the gravity at colatitude θ as

$$g_{\text{lat}}(\theta) = 978032.67714 \frac{1 + 0.00193185138639\cos^2\theta}{\sqrt{1 - 0.00669437999013\cos^2\theta}} \text{ mGal}.$$

For the location of this survey, colatitude $\theta = 90° - 52.2° = 37.8°$.

After subtraction of the gravity at the reference ellipsoid, the reduced gravity data equals

$$\Delta g_j = g_j - g_{\text{lat}}(\theta).$$

This quantity should be near –150 mGal.

Make a plot of drift- and latitude-corrected absolute-gravity observations Δg_j vs. height (elevation) and explain why it varies in the way it does.

7) **Apply the free-air correction** (add 0.3086 times the elevation in meters to each Δg_j) and plot the free-air gravity vs height. **Explain** why this graph looks the way it does.

Hand in:

Brief answers to the questions highlighted in **bold** above with figures embedded in a Word or PowerPoint document by email.