GEOL 384.3 and GEOL 334.3

Lab #8: Electromagnetic Induction

In this lab, you will study the effect of electromagnetic (EM) induction in the way it is used in geophysical surveys. In the field, EM induction is measured between the transmitter (often denoted Tx) and receiver (denoted Rx) coils. The most common transmitters and receivers in electromagnetics are multi-turn coils of wire wound on small loops. A large oscillating current in the transmitter induces electric fields and currents everywhere around it. In particular, the induced eclectic field produces voltage in the receiver coil. The signal in the Rx coil is a function of the frequency, the current amplitude in the transmitter, the size of the coils, the relative separation between Tx and Rx, and their orientations. The direction in which the current flows through the coils is also important. This direction is indicated by an arrow on the inside of each coil we use in the lab.

This lab will look at how induction occurs between the transmitter and the receiver coils. First, let us consider the transmitter (Tx) itself.

Coil inductance theory in a nutshell

From the viewpoint of its EM properties, the Tx coil is characterized by its inductance *L* and resistance *R*. With respect to its response to an applied electric voltage at frequency *f*, the *L* and *R* are combined in a complex-valued impedance $Z = R + i\omega L$, where $\omega = 2\pi f$ is the so-called angular frequency (measured in radians per second). With the use of this frequency, the voltage applied to the coil is written as the real part of the complex-valued voltage function *V*(*t*):

$$V_{\text{measured}}(t) = V\cos(\omega t) = \operatorname{Re} V(t), \text{ where } V(t) = Ve^{i\omega t}, \tag{1}$$

and *V* is the complex-valued amplitude of the sinusoidal voltage. If we select the time zero (*t* = 0) at the moment at which $V_{\text{measured}}(t)$ peaks, then *V* becomes a real value (also equal the absolute value $V = |V|$), and <u>the time dependence of voltage is a cosine function</u>: $V_{\text{measured}}(t) = V\cos(\omega t)$.

For the current in the coil, similar relations take place. The observed current is understood as the real part of a complex-valued function *I*(*t*):

$$I_{\text{measured}}(t) = \operatorname{Re} I(t), \text{ where } I(t) = Ie^{i\omega t}. \tag{2}$$

Since the time start *t* = 0 is already selected based on *V*(*t*), the amplitude *I* here is complex-valued and can be written as $I = |I|e^{i\phi}$, where $\phi = Arg(I)$ is the <u>phase advance of *I*(*t*) relative to *V*(*t*)</u>. If $\phi < 0$, then it is said that the current *I*(*t*) lags behind the voltage *V*(*t*).

Note about in-phase and quadrature components

The real part of the complex-valued I is called the in-phase component of current amplitude (i.e., the part of $I(t)$ in phase, or synchronous with voltage): $I_{\text{IN-PHASE}} = \operatorname{Re} I = |I| \cos\phi$. The imaginary part of I is called the quadrature component, or QUAD for brevity below, and it equals $I_{\text{QUAD}} = \operatorname{Im} I = |I| \sin\phi$. The quadrature component of any signal has at 90° phase advance relative to its in-phase component.

If the in-phase and quadrature amplitudes are determined, any signal (for example, current $I(t)$) can be presented as a subtraction of cosine and sine functions of ωt:

$$I(t) = |I| \cos(\omega t + \phi) = I_{\text{IN-PHASE}} \cos\omega t - I_{\text{QUAD}} \sin\omega t \,.$$

Thus, the first of these terms with time dependence cos(ωt) has a peak value and zero derivative at $t = 0$. By contrast, the QUAD part (with sin(ωt)) has a zero value and peak negative derivative at $t = 0$.

The nonzero phase lag ϕ is due to the inductance of the Tx coil, L. For a given V, the complex-valued current amplitude is given by the Ohm's law:

$$I = \frac{V}{Z} = \frac{V}{R + i\omega L} \,. \tag{3}$$

Therefore, the phase advance of current is the negative of the complex argument of the impedance: $\phi = -ArgZ = -\arctan(\operatorname{Im} Z / \operatorname{Re} Z)$. From these relations, the amplitude and phase of the current measured in a single coil divided by voltage amplitude in it, are

$$\frac{|I|}{V} = \frac{1}{\sqrt{R^2 + (\omega L)^2}} \,, \qquad \text{and} \qquad \phi = -\arctan\frac{\omega L}{R} \,. \tag{4}$$

Since ϕ is negative, it is said that the current experiences a phase lag relative to the voltage.

The inductance/resistance system like a coil has a characteristic frequency, which is also called "critical" or "relaxation" frequency:

$$\omega_c = \frac{R}{L} \text{ (in radians/sec), or } f_c = \frac{\omega_c}{2\pi} = \frac{R}{2\pi L} \text{ (in Hz).} \tag{5}$$

This frequency provides a natural frequency unit for the coil. Using this frequency, relations (4) can be written as functions of f/f_c only:

$$Z = R\left(1 + i\frac{f}{f_c}\right), \quad \frac{|I|}{V} = \frac{1}{R\sqrt{1 + (f/f_c)^2}} \,, \qquad \text{and} \qquad \phi = -\arctan\frac{f}{f_c} \,. \tag{6}$$

Alternatively, the characteristic frequency can be described by the characteristic time ("relaxation time"): $\tau_c = 1/\omega_c = 1/2\pi f_c$.

From the above relations, note the behaviours of the coil at low and high frequencies:

a) At low frequencies $f \ll f_c$, $\frac{\left|I_{\text{low-frequency}}\right|}{V} \approx \frac{1}{R} = const$ and $\phi \approx 0$. This is the "resistive" regime frequency range. The coil acts like a piece of wire, and the current and voltage are nearly in phase.

b) At high frequencies $f \gg f_c$, the current reduces with frequency as $\frac{\left|I_{\text{high-frequency}}\right|}{V} \approx \frac{1}{R}\frac{f_c}{f}$, and the phase delay approaches $\phi_{\text{high-frequency}} = -\arctan(\infty) = -\pi/2$, that is a 90° phase shift of the current relative to voltage in the Tx coil. This frequency range above f_c is called the "inductive" range.

Graphs of these *I*/*V* responses are shown in Figure 1 in terms of the relative frequency f/f_c and scaled impedance *Z*/*R*.

Now consider the inductance caused by the Tx coil in another Rx coil. If no current is flowing in the Rx coil, then the voltage in it is caused by the time derivative of the current in the transmitter coil:

$$V_{Rx} = i\omega L_{12} I_{Tx} = V_{Tx}\frac{i\omega L_{Tx-Rx}}{R + i\omega L}, \tag{8}$$

where $L_{\text{Tx-Rx}}$ is the mutual inductance of the two coils. Thus, the amplitude of the Rx/Tx voltage ratio as a function of frequency is

$$\left|\frac{V_{Rx}}{V_{Tx}}\right| = \frac{\omega L_{Tx-Rx}}{\sqrt{R^2 + \left(\omega L_{Tx}\right)^2}} = \frac{2\pi f_c L_{Tx-Rx}}{R}\frac{\left(f/f_c\right)}{\sqrt{1+\left(f/f_c\right)^2}}. \tag{9}$$

This function has a "mirror" shape to the one for a single coil, with the plateau located at higher frequencies (Figure 1, upper right). The phase response has a similar shape to self-inductance but changing from 90° at low frequencies to zero at high frequencies (Figure 1, bottom right).

For both self-inductance and induction, the critical frequency f_c corresponds in the response curves to amplitude equal $1/\sqrt{2} \approx 0.707$ of the asymptotic amplitude at the plateau, or phase $\phi = -\arctan 1 = -45°$ for self-inductance and $\phi = 45°$ for Rx inductance (pink arrows in Figure 1).

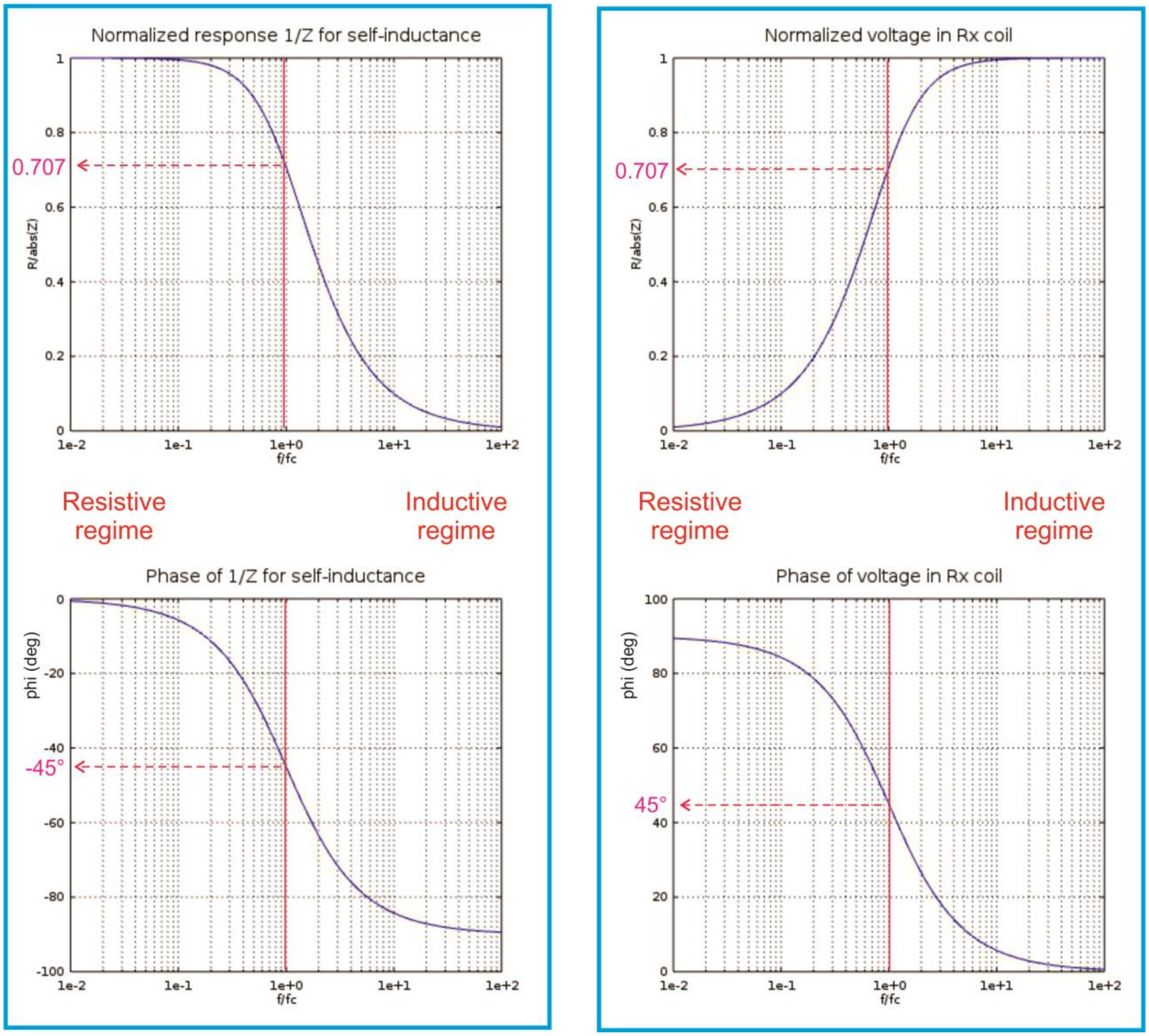

Figure 1. Amplitude and phase of I_{Tx}/V_{Tx} response for self-inductance of the Tx coil (left) and for voltage ratio V_{Rx}/V_{Tx} for a receiver coil (right). Pink lines show how to find the critical frequency f_c. This frequency separates the resistive and inductive frequency ranges.

Measurement procedure

Current in the Tx coil will be created using a function generator, and measurements conducted using the Picoscope (digital oscilloscope) (Figure 2). Connect the current in the transmitter Tx coil to channel A, and the measured voltage (in Tx or Rx) to channel B in the instrument.

From the screen of an analog oscilloscope or waveform display of the Picoscope (Figure 2), the phases can be read as follows. We should measure the time Δt from a rising (or falling) crossing of channel A signal to the nearest rising (or falling, respectively) crossing of the signal in channel B. If the signal in channel A is cos(ωt), then the signal in B is $const \times \cos\left[\omega\left(t-\Delta t\right)\right]$. The phase difference in radians is then $\phi = -\omega\Delta t$, or in degrees ϕ(deg) = $-360° f \Delta t$, where f is the frequency in Hz, and Δt is in seconds.

However, instead of these measurements of Δt we will use the complete shapes of recorded signals. During each measurement, the Picoscope outputs a several Matlab (.mat) files containing digitized segments of records from its channels A and B. These files are uploaded to a computer using the USB interface and processed in a GNU Octave or Matlab program to obtain the amplitude and phase values. This measurement is shown in Figure 3.

If the time zero of the record is selected at the peak of voltage in channel A, then the amplitude V_A of signal A is obtained directly from the maxima of the cosine function, and the phase of this signal equal zero. To determine the amplitude V_A and phase ϕ_B of signal B, this signal can be written as a combination of cosine and sine functions of ωt:

$$u_B\left(t\right) = V_B \cos\left(\omega t + \phi_B\right) = a\cos\omega t + b\sin\omega t\,, \tag{7}$$

where $a = V_B \cos\phi_B$ and $b = -V_B \sin\phi_B$ are constants. Parameters a and b are found by least-squares fitting of the measured $u_B(t)$ dependence (red line in the left Figure 2), and from them, $V_B = \sqrt{a^2 + b^2}$ and $\phi_B = -\arctan\left(b/a\right)$ are obtained.

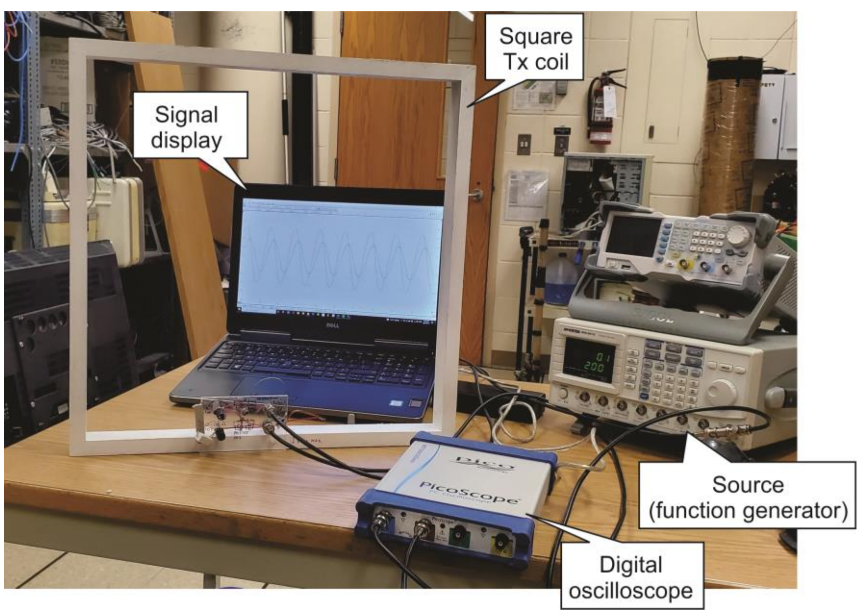

Figure 2. Measurement of coil self-induction.

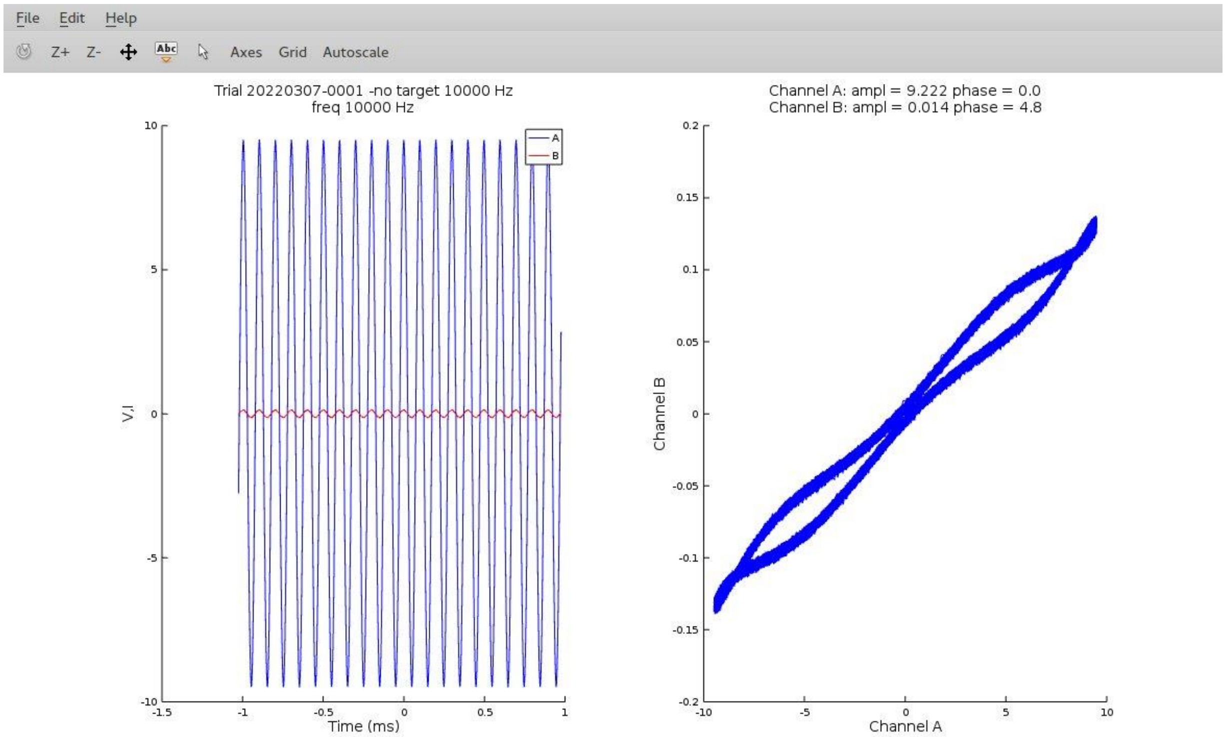

Figure 2. Signal measurements from Picoscope outputs. *Left*: outputs from channels A (blue) and B (red) vs. time. *Right*: cross-plot from these outputs (Lissajous curve) from which the amplitudes and phase are measured (see the header of this panel).

Assignments

Self-inductance test

Start with a test of self-inductance of the transmitter using a single square coil (Figure 2). Connect the current to channel A and voltage (not to exceed 2 V) to channel B in the Picoscope.

1) **Measure** the voltage and current in the coil as a function of frequencies from 20 Hz to 60 kHz. Place results in Table 1 in the worksheet.

2) **In Table 1, fill the column for the absolute value of the 1/impedance**, $1/|Z| = |I|/V$.

3) **Plot |*I*|/*V* and the phase difference ϕ as a function of frequency.** Use log-log scales for |*I*|/*V* and 'semilogx' (logarithmic in frequency only) for phase.

   GEOL334 students: also plot the absolute value of phase vs. frequency on log-log scales. You will see that in this form, the phase curve will be close to a straight line in the region of f_c.

In the plot, **identify the low-frequency and high-frequency regimes** described in eqs. (4) and (6) and Figure 1 above. Do they show the expected trends with frequency?

4) **Determine the coil resistance *R*** from the plateau of the |*I*|/*V* at low frequencies.

5) **Determine the critical frequency f_c.** Try doing this in two ways (Figure 1, right):

I) Locate the –45° level on your plot of the phase.

II) Estimate f_c by finding the frequency at which the current amplitude $|I|/V$ drops to $1/\sqrt{2} \approx 0.707$ of the "resistive" (plateau) level at low frequencies.

Compare the two estimates of f_c.

6) **This question for GEOL334 only:** yet another way is to obtain f_c from the low-and high-frequency limits of the current amplitude ($|I|/V$) plots. At low frequencies, the scaled current equals $\left|I_{\text{low-frequency}}\right|/V \approx 1/R$ (case a) above), and at high frequency, the slope with inverse frequency equals (case b)):

$$\text{slope of } I_{\text{high-frequency}} = \frac{d\left(\left|I_{\text{high-frequency}}\right|\right)}{d\left(\frac{1}{f}\right)} \approx \frac{V}{R} f_c \,.$$

Thus, **measure the slope of the high-frequency current and divide it by the low-frequency current**. The ratio should equal the characteristic frequency:

$$\frac{\text{slope of } I_{\text{high-frequency}}}{\left|I_{\text{low-frequency}}\right|} \approx f_c \,.$$

Unfortunately, you may find that the available frequencies are not high enough points in order to constrain the high-frequency asymptote.

7) **Using the measured f_c and resistance of the coil** R, **determine its self-inductance L** from eq. (5): $L = 2\pi f_c / R$. In SI units, L is measured in Henry (symbol H).

Mutual inductance between the transmitter and receiver

Next, set up the two round coils on the table spaced co-axially, with their axes coincident, and at about 1 m apart (Figure 4, left). You do not need to place the foil between the coils shown in Figure 4.

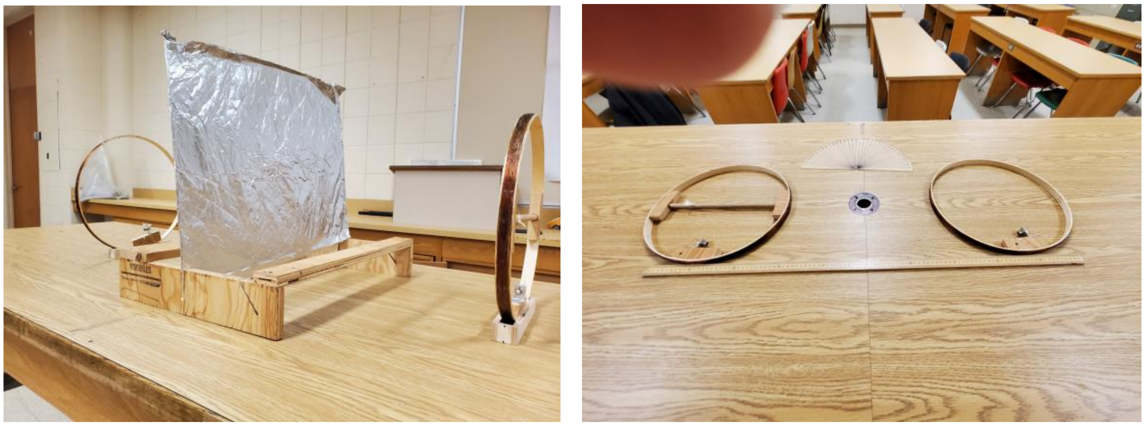

Figure 4. Transmitter and receiver coils used in the measurements, in vertical (left) and horizontal (right) arrangements.

8) **Connect the current Tx reading to channel A** of the Picoscope and **voltage in Rx coil to channel B**, and **perform a similar series of measurements**. Adjust he signal strength in the transmitter so that the current is large but not so large as to heat up the coil. Vary the frequency in the transmitter from 20 Hz to 60 kHz in the function generator and record the signal strength in the transmitting coil and in the receiving coil, as well as the phase difference. **Place the results in Table 2** in the worksheet.

9) **Evaluate the Rx/Tx amplitude ratio,** note that Tx voltages are quite constant, and so the change from Rx column will be small.

10) **Evaluate the in-phase Rx/Tx and "quadrature" Rx/Tx responses in the columns in Table 2**.

    The in-phase and QUAD part are the |Rx/Tx| ratio times $\cos\phi$ or $\sin\phi$, respectively, where ϕ is the Rx-Tx phase difference (make sure to correctly use degrees as argument of the sine function). We will further discuss such responses in Lab #9.

11) **Graph the |Rx/Tx| amplitude ratio, in-phase, and QUAD responses** on log-log scale **and phase vs frequency** on semilogx scale (on paper or by software).

Note that no current is flowing in the receiver (open circuit voltage is measured in Rx), and so we are actually measuring the inductance of the transmitter and its critical frequency. The voltage in the receiver is given by eq. (9) and has the "mirror" shape shown on the right in Figure 1. **Verify from your log-log plot whether this is so**.

12) **From the |Rx/Tx| ratio graph, determine the critical frequency f_c** (by any method) and mark it on each of the above graphs. You may get a slightly different critical frequency compared to the previous value. **GEOL 334 students: try explaining the reason for this difference**.

Mark on this figure where the resistive (low-frequency) and inductive (high-frequency) limits are. There is a phase shift of exactly −90° between these limits.

Receiver signal vs. coil separation test

Next, set the coils up on the table so they are co-axial and close but not touching.

13) Using a high frequency such as 10,000 Hz to get a good response, **measure the transmitter and receiver signals while moving them apart** in 10-cm intervals. Put into Table 3 (RECEIVER SIGNAL vs COIL SEPARATION) in the worksheet the Rx voltages normalized to Tx voltages equal 10 V (that is, put there values $(V_{Rx}/V_{Tx})\times 10\text{V}$ which are output by our data acquisition program).

14) **Plot the receiver signal amplitude as a function of distance between coil centers**. You should find that the signal falls off rapidly with distance.

Once the transmitter receiver separation is larger than a few coil radii, the signal falls off as $(a/r)^3$, where a is the coil radius and r the separation. This means that for field work, the transmitter and receiver must be fairly close together, unless the transmitter power is high.

Receiver signal vs. height difference between coils

Set the transmitter coil on the table 1 meter from the receiver location and with its axis horizontal. Set the receiver up coplanar with the transmitter, that is, with its axis horizontal. We raised the receiver above the plane of the transmitter and recorded how the signal in the receiver changes. This configuration is exactly the same as having the transmitter and receiver axes oriented vertically but located at different heights. This mutual position of the coils commonly occurs in measurements over significant topography.

15) Using frequency 10,000 Hz, **measure the transmitter and receiver signals while moving the Rx coil upward** in 10-cm intervals. Put the 10-V normalized voltages in Rx into Table 4 (HEIGHT DEPENDENCE) in the worksheet.

16) **In Table 4, fill the columns *Slope in radians* and *Slope in degrees*** by using the appropriate formula from trigonometry. These slopes are simply the geometrical slopes of the line connecting the Tx and Rx centers.

The magnetic field of the transmitter is symmetric, and so it does not matter whether the receiver is above or below the transmitter; only the height difference is important. How much the signal changes with height difference also depends on the distance between transmitter and receiver, or the slope of the line joining the transmitter and receiver. In the field, we would also like to have the transmitter and receiver at the same height. However, if there is some topography, this is impossible, and so we need to do a correction. The manufacturer calibrates the instrument so that if the coils are *x* meter apart and used on a slope of *y* degrees, a known correction is applied to the data.

17) From Table 4, **if the transmitter is uphill or downhill from the receiver on a 10° slope** (maintaining the same separation) **what is the percent change in the response?**

Receiver signal vs. coil tilt test

Finally, set the transmitter with its axis horizontal and the receiver 1 meter away from Tx and coplanar. We rotated the receiver slowly about an axis orthogonal to the separation of the transmitter and receiver and recorded what happens to the receiver signal. This configuration is exactly the same as having the axes vertical and tilting one or the other. In the field, we would try to keep the transmitter and receiver in the same orientation, e. g. both with axes vertical, by using a level bubble.

18) Using frequency 10,000 Hz to, **measure the transmitter and receiver signals while tilting the Rx coil**. Put the 10-V normalized voltages in Rx into Table 5 (TILT WORK SHEET) in the worksheet.

19) **Summarize how does the receiver signal vary with tilt**. **How large a tilt can you have before the response changes by one percent**?

Hand in:

Brief answers to the questions highlighted in **bold** above with figures embedded in a Word or PowerPoint document by email.