Formula reminder sheet for GEOL384.3 and 334.3

You are expected to be able to give, *explain* and *use* the following equations. Most formulas do not need to be memorized in their exact forms but still need to be understood and interpreted. What physical quantities do they relate? What processes do they describe and what are their consequences?

General

Relation between (gravity or other) field strength and potential: $\mathbf{g} = -\vec{\nabla} U \equiv -\frac{\partial U}{\partial \mathbf{x}}$

Laplace equation: $\nabla^2 U \equiv \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$

Poisson's equation (for gravity): $\nabla^2 U = 4\pi G \rho$

Gauss's law: $\oiint\limits_{\text{closed surface}} \mathbf{g} \cdot \mathbf{n} dA = -4\pi G \int\limits_{\text{volume}} \rho dV$

Relations between wave length λ, period T, velocity c, frequency f, angular frequency ω, and wavenumber k:

$$\lambda = VT, \quad f = \frac{1}{T}, \quad \omega = 2\pi f = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}.$$

Phase (wave) velocity: $V = \frac{\omega}{k} = \frac{\lambda}{T}$

Equations for plane waves:

general: $u(x,t) = f\left(t - \frac{x}{V}\right)$ (arbitrary function)

cosine form: $u(x,t) = A\cos(\omega t - kx)$

complex exp form: $u(x,t) = Ae^{i(\omega t - kx)}$

attenuating wave like magnetic field within a skin layer: $H(z,t) = Ae^{-z/\delta}e^{i(\omega t - z/\delta)}$

Magnitude (absolute value, A) and phase (argument, ϕ) of a complex number (e.g., apparent resistivity, response function): $Z = Ae^{i\phi}$

Standard deviation: $\sigma \approx s_{N-1} \equiv \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(g_i - \bar{g}_N)^2}$

Gravity

Newton's law of universal gravitation: $F = G\dfrac{m_1 m_2}{r^2}$

Universal gravitational constant: $G \approx 6.6726 \cdot 10^{-11}\mathrm{m^3/kg/s^2}$

Gravity above a sheet (thickness h, density ρ, density per unit surface σ): $g = 2\pi G\sigma = 2\pi G\rho h$

Gravity of a point sphere of mass M at depth H below surface: $g = \dfrac{GM}{H^2}\cos^3\alpha$

(angle α is the angle between the direction to the body and the vertical direction at the observation points)

Gravity of a horizontal band of surface density σ): $g = 2G\sigma\Delta\alpha$

($\Delta\alpha$ is the angle subtended by the band from the observation point)

Gravity corrections:

Latitude: $g_{\mathrm{lat}} = -0.8118\left[\dfrac{mGal}{km}\right]\times s\left[km\right]$ (s is the southward distance)

Free-air: $g_{\text{free-air}} \approx -0.3086\left[\dfrac{mGal}{m}\right]\times h\left[m\right]$ (h is the elevation)

Bouguer: $g_{\mathrm{Bouguer}} = 2\pi G\rho H \approx 0.1119\left[\dfrac{mGal}{m}\right]\times h\left[m\right]$

Electrical and magnetic methods - general

Relation between resistivity and conductivity: $\rho = \dfrac{1}{\sigma}$

Archie's law: $\rho = a\rho_w\phi^{-m}$ (ρ_w is the resistivity of pore fluid, a is the tortuosity, m – cementation exponent)

Archie's law with partial saturation s_w: $\rho = a\rho_w\phi^{-m}s_w^{-n}$

Electric field at distance r from point charge Q: $\varphi = \dfrac{Q}{\varepsilon_0\varepsilon}\dfrac{1}{r}$ (potential) and $E_r = \dfrac{Q}{\varepsilon_0\varepsilon}\dfrac{1}{r^2}$ (strength)

Electric field at distance r from injection point of current I at the surface: $\varphi = \dfrac{I\rho}{2\pi r}$ (potential; r is the resistivity of the ground)

Maxwell's and electrostatics equations:

Electric displacement field: $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}$

Magnetic induction field: $\mathbf{B} = \mu \mu_0 \mathbf{H}$

Faraday’s law of induction: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

Ampère’s law: $\nabla \times \mathbf{H} = \mathbf{j} + \dfrac{\partial \mathbf{D}}{\partial t}$

Ohm’s law: $\mathbf{j} = \sigma \mathbf{E}$

Resistivity

Relations between source voltage V, current I, resistance R, geometry factor k, and apparent resistivity ρ_a:

$$R = \frac{V}{I}\ , \qquad \rho_a = Rk$$

Geometry factor for an arbitrary electrode array (A and B are the source electrodes, M and N are potential electrodes): $k = 2\pi \left(\dfrac{1}{r_{\mathrm{AM}}} - \dfrac{1}{r_{\mathrm{BM}}} - \dfrac{1}{r_{\mathrm{AN}}} + \dfrac{1}{r_{\mathrm{BN}}} \right)^{-1}$

Geometry factors for common arrays:

Wenner: $k = 2\pi a$ (a is the spacing between M and N)

Schlumberger: $k \approx \dfrac{\pi}{2l} \dfrac{\left(L^2 - x^2\right)^2}{L^2 + x^2}$ (L is the <u>half-spacing</u> between A and B, l is the <u>half-spacing</u> between M and N, x is the offset of the center of MN relative to the center of AB)

Pole-dipole: $k = 2\pi \dfrac{ab}{b-a}$ (a is the spacing between A and M, b is the spacing between A and N; B at infinity)

Double-dipole: $k = 2\pi l \left(n-1\right) n \left(n+1\right)$ (l is the <u>half-spacing</u> between M and N, n is the number of MN increments between A and M)

Induced polarization

Integral chargeability: $M = \dfrac{\sum_{\text{gates } i} V\left(t_i\right) \Delta t_i}{V_0}$ $[\mathrm{ms}]$

Complex apparent impedance (resistivity): $Z(\omega) = \dfrac{V(\omega)}{I(\omega)}k$ (k is the geometry factor for the electrode array)

Frequency effect: $FE = \dfrac{\rho_a(f_0) - \rho_a(f_1)}{\rho_a(f_1)}$ [unitless or %]

Metal factor: $\text{q Metal Factor} = \dfrac{2\times 10^5 \pi}{\rho_a(f_0)} FE = 2\times 10^5 \pi \left[\sigma_a(f_1) - \sigma_a(f_0)\right] \left[\dfrac{\text{Siemens}}{m}\right]$

Magnetic methods

Biot-Savart law: $\mathbf{B}(\mathbf{r}) = C_m I \oint \dfrac{d\mathbf{r}' \times \hat{\mathbf{r}}''}{r''^2}$

Magnetic field of a line of current: $|\mathbf{B}(r)| = \dfrac{\mu\mu_0 I}{r}$

Total field: $\Delta T = |\mathbf{B}_{\text{ambient}} + \Delta\mathbf{B}| - |\mathbf{B}_{\text{ambient}}|$

Electro-magnetic methods

Response function: $F(Q) = \dfrac{iQ}{1+iQ}$, where response parameter $Q = \dfrac{\omega L_1}{R_1} = \omega\tau_c = \dfrac{\omega}{\omega_c}$

Skin depth: $\delta = \sqrt{\dfrac{2}{\omega\sigma\mu\mu_0}} = \sqrt{\dfrac{\rho}{\pi f \mu\mu_0}}$

Induction number: $B = \dfrac{\text{coil spacing}}{\text{skin depth}}$

Apparent conductivity: $\rho_a = \dfrac{1}{\omega\mu_0} \dfrac{\langle E_H^2 \rangle}{\langle H_H^2 \rangle}$

Seismic refraction

Headwave travel time (linear moveout) equation, zero dip: $t(x) = t_{\text{intercept}} + \dfrac{x}{V_2}$

Intercept time: $t_{\text{intercept}} = \frac{2h\cos\theta_c}{V_1}$

Relation for critical angle: $\sin\theta_c = \frac{V_1}{V_2}$

Seismic reflection:

Seismic Impedance: I (also denoted Z) = ρV

Reflection coefficient: $R_{PP} = \frac{A_{P_{reflected}}}{A_{P_{incident}}} = \frac{I_2 - I_1}{I_2 + I_1}$

Vertical reflection resolution: $\delta z = \frac{\lambda}{4}$

Horizontal reflection resolution at depth H: $\delta x \approx \sqrt{\frac{1}{2} H\lambda}$

Reflection travel time (normal moveout) equation $t(x) = \sqrt{t_0^2 + \left(\frac{x}{V}\right)^2} \approx t_0 + \frac{1}{2t_0}\left(\frac{x}{V}\right)^2$, with $t_0 = \frac{2h}{V_1}$

GPR reflection impedance $Z = \sqrt{\frac{\mu}{\varepsilon}} \approx \sqrt{\frac{1}{\varepsilon}}$, reflection coefficient: $R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$.