

GEOL 384.3 and GEOL 334.3

Lab #4: Resistivity of rock core samples

In this lab, you will simulate laboratory measurements of resistivity in rock core samples.

Five rock core samples are supplied (Figure 1). The data on dimensions, densities and porosities are as shown in Table 1 in the attached [worksheet file](#).



Figure 1. Core samples of this lab (Room 265 Geology)

In this Table 1, you can verify the calculation of volumes from the lengths and diameters of cylindrical cores. The porosities (except M1) have been measured by weighing the samples fully saturated and dry. From these measurements, the porosity is

$$\phi = \frac{\text{saturated_weight} - \text{dry_weight}}{\text{volume} \times \rho_{\text{water}}} \times 100 \text{ [\%]}$$

where ρ_{water} is the density of water.

Experiment

The resistivity of the rock (ρ , differential quantity) is measured by first measuring the total resistance of the available cylindrical core sample. A plate electrode is clamped to each end of the cylindrical core and a small low-frequency voltage is applied to the electrodes. This is a two-electrode system, which means that voltage V is measured between the same electrodes injecting current (unlike the four-electrode system with different current and voltage electrodes shown in the lectures).

The sample resistance is the voltage drop (V) across the core divided by the current (I):

$$R = \frac{V}{I}.$$

The resistivity is then found using the following formula:

$$\rho = kR, \quad \text{where} \quad k = \frac{A}{L}, \quad (1)$$

A is the cross-sectional area of the flow path (your core), and L is its length. The factor k transforming the measured resistance (R) into the in-situ resistivity (ρ) is measured in meters and called geometry factor. Similar geometry factors are key in most other resistivity measurements.

When conducting electrical experiments, one important issue needs to be always considered, which is the contact resistance. The resistance calculated from the measured V and I above is the sum of the resistance of the flow path in the rock and the contact resistances at the two ends, where wires are connected to the rock. In order for this arrangement to yield an accurate resistivity, the resistance of the flow path in the rock must be considerably larger than the (unknown) contact resistance at the ends. Therefore, we would need to ensure that the contact resistance is much lower than that of the sample. The smaller the resistance of the core sample, the smaller the contact resistance has to be in order that it can be safely neglected. For example, if the resistivity of sample B1 is about 1000 Ωm , then the resistance through the core is $10^5 \Omega$. If a one percent error in resistivity is desired, then the contact resistance would need to be kept below 1000 Ω .

Contact resistance below 1000 Ω is fairly easy to achieve. The measured contact resistance with this system is essentially zero at high frequencies ($> \sim 100\text{Hz}$ and less than 1000 Ω at frequencies $< 1\text{Hz}$). The contact impedance is in fact due to polarization of the copper end plates. Most of the cores have resistances much larger than this polarisation impedance, and so we do not need to be concerned.

The current density at the contacts (current divided by contact area) may also be important. Very small current densities are to be avoided because they are difficult to measure accurately. Very large current densities are also to be avoided as the contact impedance increases as the current density increases. Very small injection voltages (less than a volt) and very large injection voltages (greater than 100's V) should be avoided. If you keep the injection voltage between 5 V and 100 V, polarization should not be a problem.

Electrode polarization is responsible for the variable contact resistance described above. It is always present and can be a problem in measuring resistivity on core samples. For this reason, a pulsed current is injected to achieve some cancelation of the positive and negative current cycles. Typically, the pattern is a "square wave", that is a positive +V for ~ 1 sec and $-V$ for ~ 1 sec, and this is repeated for at least four cycles. Averaging over all the cycles beats down the noise. A sinusoidal current could be used as well. Typically, polarization is low at frequencies greater than about 100 Hz, and so a sinusoid, say about 100 Hz, could be used as well. In the field, small current densities are typical, and therefore we should try to keep lab current as small as possible but large enough that the signal is much greater than the noise. There is some experience involved in choosing the right current density, but the acceptable range is quite wide. You can use the following rules:

- (1) Current densities larger than $1\text{A}/\text{m}^2$ should be avoided and current densities less than $0.01\text{A}/\text{m}^2$ might also be avoided.
- (2) The current needs to be large enough that the measured voltage is significantly greater than the noise, e.g. 100 mV if the noise is 1 mV.

Another way to keep the influence of polarization small is to have the electrode pads saturated with CuSO_4 in contact with the copper electrodes. Our samples are fairly resistive ($> 1 \text{k}\Omega$), and even with water as

the contact with the copper electrodes, the contact impedance from polarization is unlikely to exceed a few tens of Ohm.

In the experiment, the resistivities of the core samples will be measured using the SYSCAL meter (Figures 2 and 3) with an attachment implementing a two-electrode configuration. Samples should be connected to terminals A and B, and voltage will be measured between terminals M and N through a resistor splitter.

Electrical contact with the rock will be maintained by using copper electrodes with layers of paper saturated with CuSO_4 . The surface of the samples should be kept dry before taking readings, especially if CuSO_4 has dripped down the sides.

You will place the results of the measurements in Table 2 in [worksheet](#) file. In that table, I added a column for the “formation factor” F , which is the measured resistivity divided by the resistivity of pore water:

$$F = \frac{\rho}{\rho_w} \quad (2)$$

(do not confuse water resistivity ρ_w with density above!).



Figure 2. SYSCAL meter

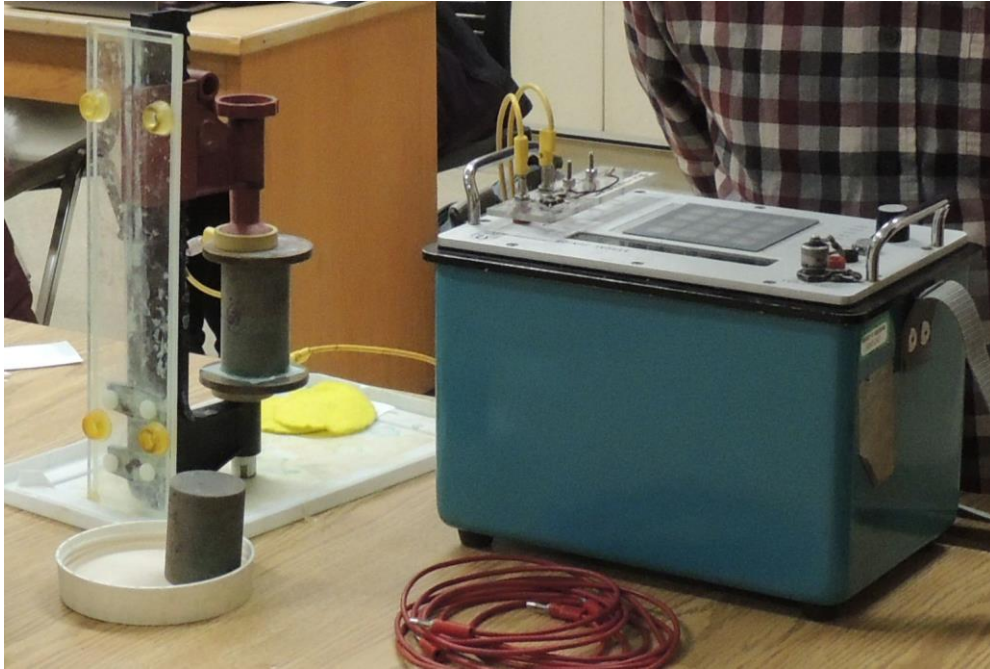


Figure 3. Conducting the measurements (note the sample in the vertical non-conductive clamp on the left)

In all calculations below, pay attention to using consistent units. Better convert all units given in the tables into the SI system:

- Sample dimensions in meters (m);
- Masses (referred to as “weights” in the worksheet) in kilograms (kg);
- Voltages in Volts (V);
- Currents in Amperes (A);
- Resistances in Ohms (Ω);
- Porosities, saturation, and anisotropy parameters – use fractional values (not per cent).

When placing output values back in the tables, make sure to convert the units again as requested.

Assignments

- 1) **Evaluate geometry factors k** and other missing parameters for each sample in Table 1 in the [worksheet](#). Note that k should be output in *meters*. To determine k , you will need to evaluate the cross-sectional areas A first.

- 2) **Perform measurements listed in Table 2.** For each sample, first measure the resistance using the ohmmeter, and then use SYSCAL to measure the voltage and current of the square wave. Use voltages around 50 V and currents around 5-20 mA.

When reading voltages from SYSCAL, note that they are measured between terminals M and N, which receive only a fraction of the voltage between A and B. To obtain the voltage between A and B, **multiply the column of readings by the “A-B voltage factor”** given above Table 2.

When measuring the first sample (e.g., B2), try several lengths of time cycles in SYSCAL. **Do the resistances vary significantly for different cycle times?**

- 3) Using SYSCAL readings, evaluate the resistance R , resistivity ρ , and formation factor F values in Table 2. Formation factor can be obtained from eq. (2), in which water resistivity ρ_w is given in Table below (also Table 3 in the worksheet).

Table of salinity and resistivity of water at 20°C

Salinity (g/l)	Resistivity, ρ_w ($\Omega \cdot m$)
0.07	74.5
0.7	7.8
2.5	2.3
7.0	0.88
70.0	0.11

- 4) Compare the resistances obtained by the ohmmeter (low-voltage DC source) and SYSCAL (higher-voltage square wave). **Comment whether their difference appears significant and discuss reasons for it.**
- 5) **Plot F versus porosity ϕ** in log-log scale. For plotting, use paper or some software. **GEOL334: preferably use Matlab/Octave**, and instead of the `loglog()` function, use `plot()` of the decimal logarithms of ϕ and F .

If there is no clay in the samples, the $\log(F)$ vs. $\log(\phi)$ data (or F vs ϕ on a log-log scale) will fall on a straight line corresponding to the functional relationship (Archie’s law):

$$\frac{\rho}{\rho_w} = F = a\phi^{-m}, \quad \text{or} \quad \log_{10} F = \log_{10} a - m \log_{10} \phi. \quad (3)$$

where m is the slope in units of [decades/decade] called the cementation exponent. The value of m ranges from about 1.3 to 2.5, and so you should see a slope in this range. Dimensionless parameter a is called “tortuosity” and represents the hypothetical limit of the formation factor when $\phi = 1$ (the whole rock consists of pores). Note that a is usually close to but above 1.0.

Values of a and m vary from rock to rock, and consequently they can be different for our samples. However, the four samples labelled A or B seem texturally very similar although the A samples still look different from the B. Therefore, the values of $\log_{10}F$ should all line up on a fairly straight line vs. $\log_{10}\phi$.

6) **Verify whether you can make the above observations from your plot.** The sample M1 is clearly different from the others, and it will probably plot off the trend of the other samples. **Comment whether this is so in your data.**

7) **For samples labelled A and B, evaluate ρ_w and salinity of the pore water in each sample.** This can be done as follows.

Plot a straight line approximating the $(\log_{10}\phi, \log_{10}F)$ points for the four samples A and B from the measurements. From this line, determine parameters a (intercept in eq. (3)) and m (negative slope). We will assume that the obtained Archie's law (eq. 3) applies to all four samples.

Once a and m are known, ρ_w can be determined for each sample from eq. (3):

$$\rho_w = \rho \frac{\phi^m}{a}.$$

By using this ρ_w , determine the salinity by interpolating values from the water resistivity table above (also Table 3 in the worksheet).

Place the results for ρ_w and salinity in two Archie's law columns of Table 4 in the [worksheet](#). **Why are the salinity values not exactly 2.5 g/l** as in Tables 1 and 2?

8) An alternative method to estimate the salinity is from the so-called Hashin-Shtrikman theory. This theory approximates the pore space by packed spheres. This model predicts that

$$\rho = \rho_w \frac{3 - \phi}{2\phi},$$

and therefore inversely,

$$\rho_w = \rho \frac{2\phi}{3 - \phi}.$$

Add these results for ρ_w and salinity into Table 4 in the [worksheet](#). Again **note the similarity or any difference**.

9) The next test examines the anisotropy of resistivity. **Copy from Table 2 the results for the the resistivities for each direction ρ_{L1} , ρ_{L2} , and ρ_T .** The two resistivities measured for current flow parallel to bedding (ρ_{L1} and ρ_{L2}) are longitudinal resistivities and should be fairly similar. The resistivity measured with current across bedding (ρ_T) is the transverse resistivity, which should be much larger.

The ratio

$$\lambda = \sqrt{\frac{\rho_T}{\rho_L}}$$

is called the coefficient of anisotropy. **Calculate and add this ratio in the first two rows of Table 5.**

The coefficient of anisotropy for rock is usually between 1 and 2. Perhaps the highest known λ is for pyrolytic graphite at $\lambda \approx 50-70$, or a ratio of resistivities $\frac{\rho_T}{\rho_L} \approx 3000 - 5000$.

The last experiment studies the dependence of resistivity on the saturation of the pore space. Table 6 in the [worksheet](#) shows parameters of two more samples B1 and B4. These samples (as well as all other we used) are kept in water, and so we assume their saturation to be 100%. Measure the weights of the samples and enter them in Table 7. Then measure the resistivity using SYSCAL, enter the intermediate values and work out the resistivities in Table 7.

After the measurement is completed, put both samples in a microwave for about 5 min to dry and repeat the weight and resistivity measurements. As the samples dry up, their wet masses and saturations will decrease and resistivities increase. Repeat this procedure until the mass decreases no further.

- 10) **Calculate the saturation values in Table 7.** Saturation is the fraction of the pore space occupied by fluid. Therefore, it can be calculated from the average wet and dry densities:

$$s = \frac{\rho_{\text{wet}} - \rho_{\text{dry}}}{\phi \rho_{\text{water}}}$$

(here, symbols ρ denote densities, not resistivities!).

In Archie's law, resistivity is proportional to a power of the saturation:

$$\rho \propto s^{-n}, \quad \text{or} \quad \log_{10} \rho = \text{const} - n \log_{10} s.$$

Plot the s and ρ data from Table 7 in log-log scale (or better plot $\log_{10}(\rho)$ vs. $\log_{10}(s)$ on a linear scale) and estimate the exponent n in the power-law dependence for each sample.

GEOL334: use function *polyfit()* to obtain the value of n .

- 11) **GEOL334 students:** Look carefully at how s and ρ vary when the samples are losing water. You will see that the above measurement procedure has a flaw. **What is this flaw?**

Hints: does the salinity of pore water remain constant during desaturation? Does it reduce or increase? Can you see this from data points?

Hand in:

Brief answers to the questions highlighted in **bold** above with figures embedded in a Word or PowerPoint document by email.