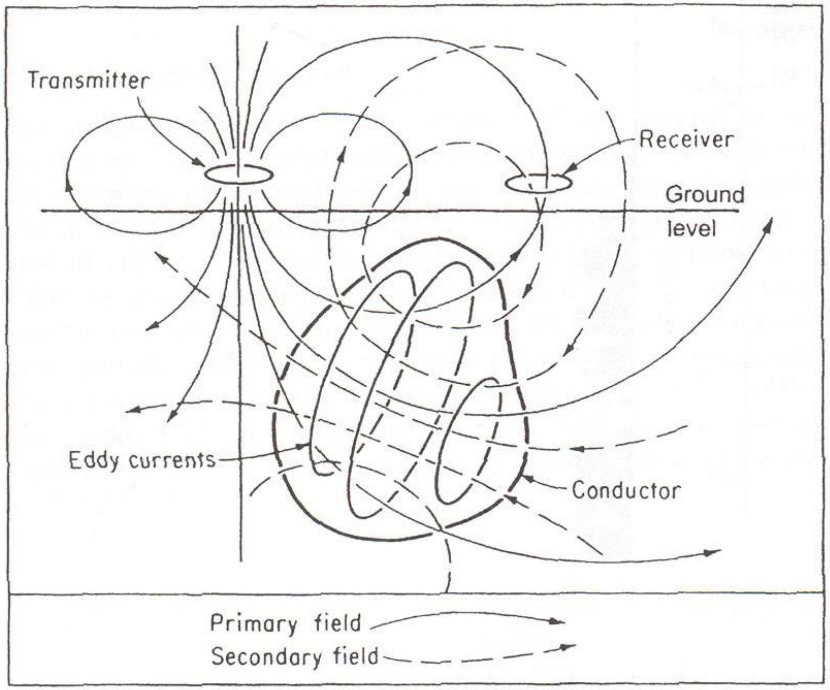
Electromagnetic methods (EM) - Key points

- Principles
  - Induction
  - Response function and characteristic time
  - Basic solutions
  - In-phase and quadrature components of signals
- EM systems
  - Frequency-domain
  - Time-domain
- EM survey styles
  - Long-wire
  - Moving-coil (Slingram)
  - Plane-wave (far-field)
  - Tilt angle
- Effects of ground conductivity on EM waves
  - Skin depth
  - Frequency-dependent apparent resistivity

- Labs # 8, 9, 10
- Reading:
  - Reynolds, Chapters 10 and 11
  - Dentith and Mudge, Sections 5.4 – 5.6

Principle of Electromagnetic (EM) methods

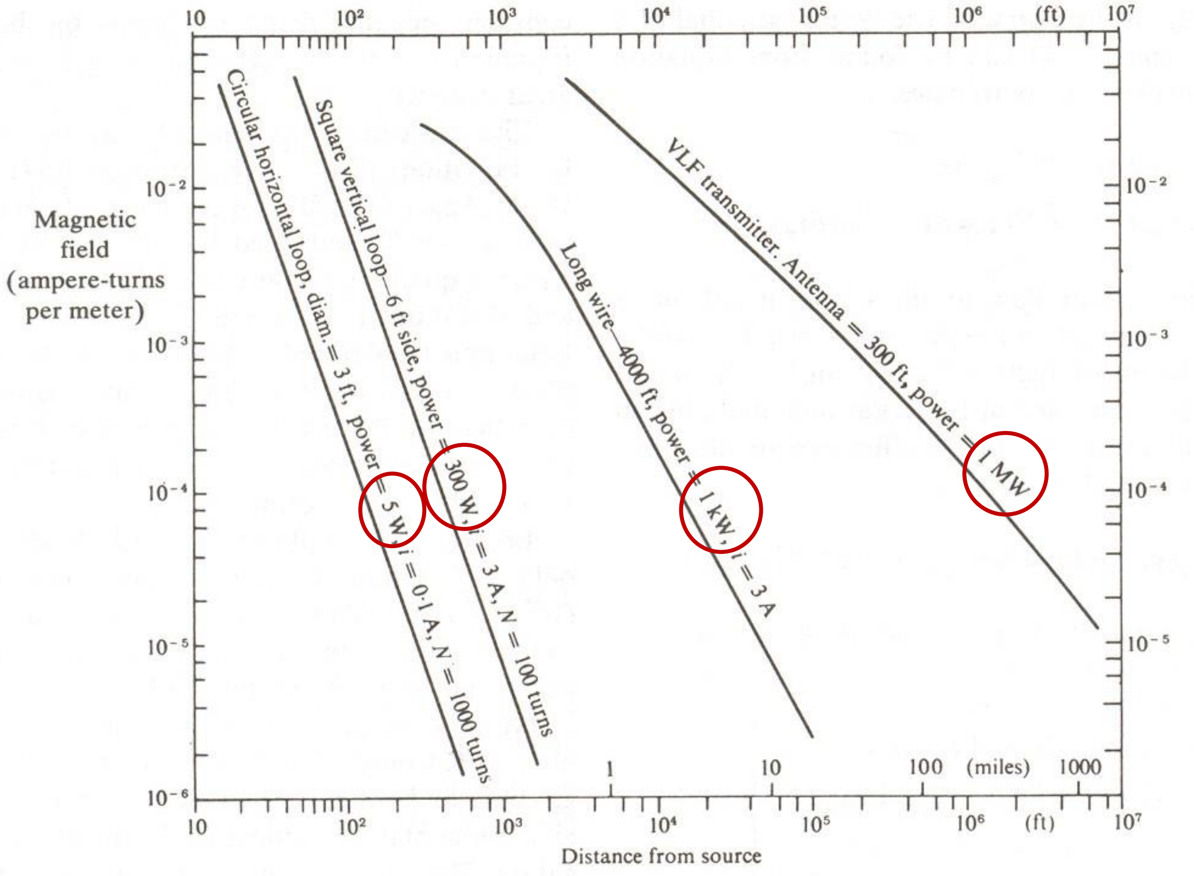
- There exist numerous EM methods utilizing electromagnetic waves for detecting conductive bodies in the ground
  - Frequencies are relatively low (~50 Hz to ~25 kHz) with very large wave speeds ($3{\cdot}10^5$ km/s), and therefore the wavelengths are long (the shortest > 10 km)
  - Within the study area, EM fields look not really like moving waves but as magnetic and electric fields near-synchronously oscillating in time
- Broadly, EM principle can be described as “Induced magnetic Polarization of a conductor” by oscillating magnetic field
  - Time-variant primary magnetic field from a transmitter coil (Tx) induces eddy currents within the conductor
  - Induced currents produce secondary magnetic (and electric) fields recorded by the receiver coil (Rx)
- Two large groups of EM techniques:
  - Time-domain (sharp pulses emitted, rise and relaxation times measured)
  - Frequency domain (sine/cosine signals used; amplitude variations and phase shifts between Tx and Rx measured)



EM survey sizes and depths

- Distance of EM profiling is determined by the power of the source and acquisition style (large Tx loops, moving coils)

- Depth of EM sounding is determined by the frequency of the source and conductivity of the ground
  - Note the sizes of Tx loops, powers of typical sources, and distances of coverage in this diagram



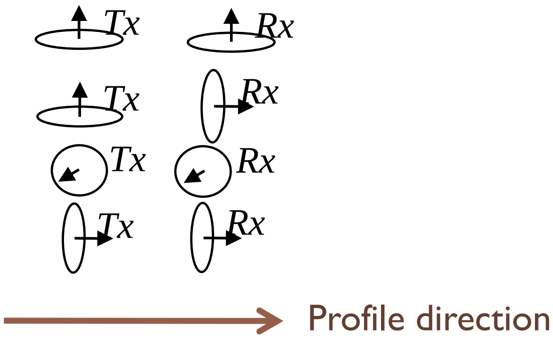
Electrical and Electromagnetic (EM) techniques

(red shows what we mention in this course)

Transmitter type	Receiver type			
	Ground wire	**Wire and small coil**	**Small coil (on ground)**	**Small coil (in air)**
Grounded wire Galvanic	Resistivity, IP		Magnetometric resistivity (MMR), Magnetic IP, Some TEM systems	
Inductive		CSA MT (Controlled-Source Audio-frequency Magneto-Telluric)		
Small loop			Slingram, Horizontal-loop EM, Vertical-loop EM, Tilt-angle method, Ground conductivity meters (GCM), Some time-domain EM (TEM) systems Coincident loop Borehole systems	Airborne EM, Time-domain towed-bird, Helicopter rigid-boom, Drone
Large loop (or long wire)			Large-loop systems, Sundberg method, Turam, Many time-domain (TEM) systems, Borehole systems	
Plane wave Vertical antenna		VLF-resistivity		VLF
Natural geomagnetic field	Telluric currents (MT)			

EM acquisition styles

- Double dipole (dipole-dipole) arrays
  - Convenient (two-person operation)
  - Configurations:
    - Horizontal coplanar
    - Perpendicular
    - Vertical coplanar
    - Vertical coaxial
    - Other angles
- Turam (long wire)
- “Dip-angle” techniques
- Different platforms:
  - Land
  - Airborne
  - Seaborne
  - Borehole



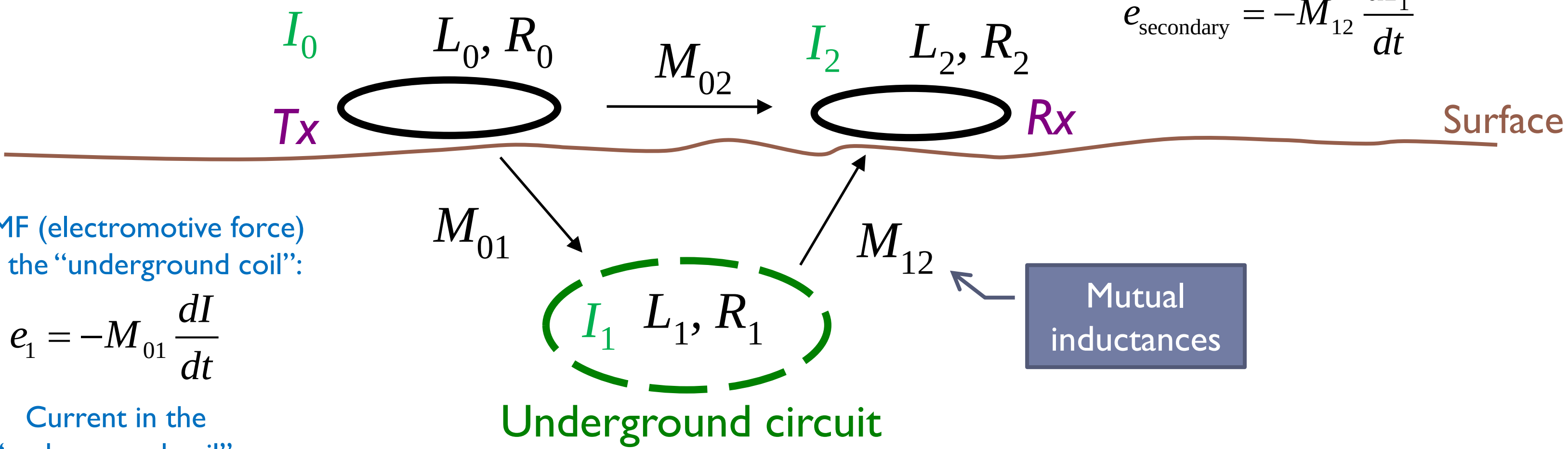
Common principle - Mutual Tx-Rx inductance

- Most EM methods consist in detecting the variation of mutual inductance between the transmitter (Tx) and receiver (Rx) coils
  - This inductance is affected by the presence of the inductance (bodies, conductive "loops") and the level of resistance (conductivity) in the subsurface

Currents in receiver coil:

$$e_{\text{primary}} = -M_{02}\frac{dI}{dt}$$

$$e_{\text{secondary}} = -M_{12}\frac{dI_1}{dt}$$



EMF (electromotive force) in the "underground coil":

$$e_1 = -M_{01}\frac{dI}{dt}$$

Current in the "underground coil":

$$I_1 = \frac{e_1}{R_1 + i\omega L_1} = \frac{-M_{01}}{R_1 + i\omega L_1}\frac{dI}{dt}$$

Note that for any harmonic function f here: $\frac{df}{dt} = i\omega f$

Response function

- The effect of the subsurface (R_1, L_1) is evaluated from the ratio of the secondary to the primary currents in Rx (from preceding slide). This ratio is called the EM response function:

$$F = \frac{e_{\text{secondary}}}{e_{\text{primary}}} = \frac{-i\omega M_{01} M_{12}}{M_{02}\left(R_1 + i\omega L_1\right)} = \frac{-M_{01} M_{12}}{M_{02}} F\left(Q\right)$$

This factor is called "coupling coefficient"

"Response function"

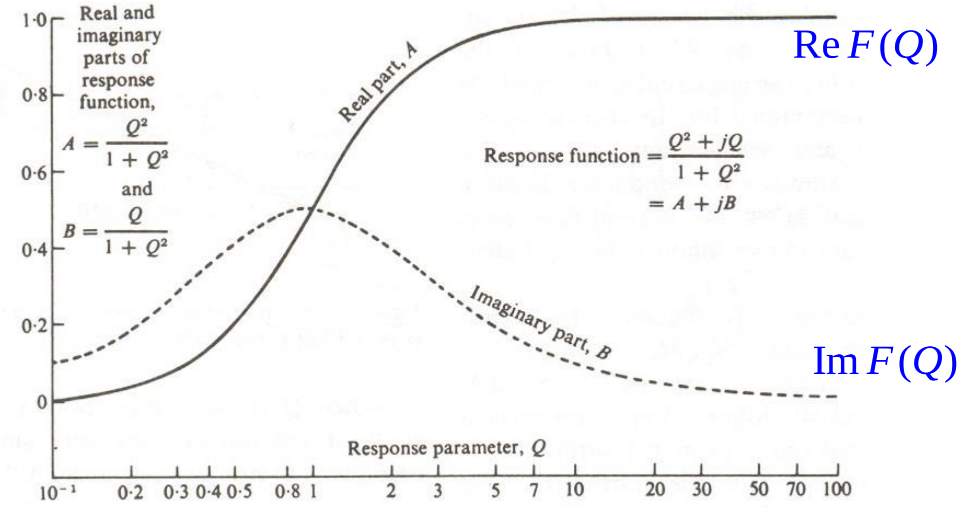
- Above, the normalized response function only depends on a combination of the parameters of the subsurface: $Q = \frac{\omega L_1}{R_1} = \omega \tau_c = \frac{\omega}{\omega_c}$

and equals: $F\left(Q\right) = \frac{iQ\left(1 - iQ\right)}{1 + Q^2} = \frac{iQ}{1 + iQ}$

- Thus, the frequency-dependent EM response of the medium is described by its characteristic time τ_c or frequency $f_c = 2\pi/\tau_c$. See your Lab #8.

Characteristic frequency (f_c) and time (τ)

- Thus, for EM method, the key property of the subsurface is the characteristic (critical, relaxation) time τ or frequency $f_c = 1/\tau$
- From response spectra, f_c (point at which $Q = 1$ in figure below) is the frequency at which the measured $\mathrm{Im}F(f)$ peaks and $\mathrm{Re}F(f)$ equals ½
  - You will use these criteria to determine f_c for metal targets in Lab #8
  - f_c is also called the “critical”, or “relaxation” frequency
  - f_c is also the frequency at which EM energy absorption by the subsurface is the strongest (this is how microwave works)



$$Q = \omega\tau_c = \frac{f}{f_c}$$

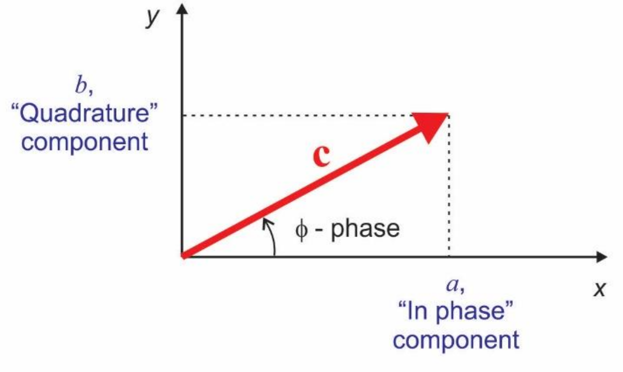
“Basic solutions” for time constants

- For structures of simple shapes, time constants are modeled analytically and numerically. This is similar to “basic solutions” and “form factors” we saw in gravity and magnetics

Conductor shape	**Time constant, τ**
Sphere of radius a	$\frac{1}{\pi^2}\sigma\mu\mu_0 a^2$
Horizontal cylinder of radius a	$\frac{1.71}{\pi^2}\sigma\mu\mu_0 a^2$
2-D conducting plate of thickness h and finite depth extent L	$\frac{2}{\pi^2}\sigma\mu\mu_0 hL$
Thin prism of thickness h and average dimension L	$\frac{1}{10}\sigma\mu\mu_0 hL$

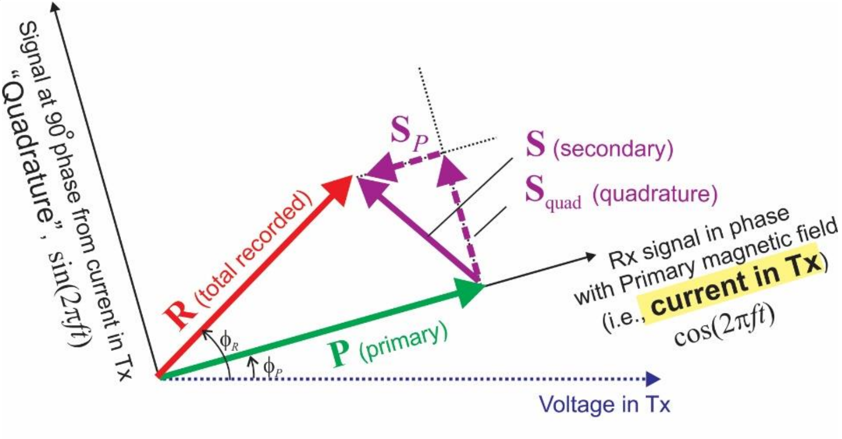
In-phase and Quadrature signals

- The response function contains the imaginary 'i' factor: $F(Q) = iQ/(1+iQ)$. This means that at low frequencies, conductivity anomalies cause voltages in Rx shifted in phase by 90° relative to the primary magnetic field. How to isolate and visualize this 90° delay?
- Harmonic (sin and cos) signals can be represented graphically by vectors on a 2-D plane. For example, vector **c** in the diagram on the right represents the phase-rotated signal



$$A\cos(\omega t - \phi) = a\cos\omega t + b\sin\omega t \quad \text{, where } A = \sqrt{a^2 + b^2}$$

- By using this relation, any harmonic signal can be separated into two components:
  - Varying with time as $\cos(\omega t)$ (amplitude a in the figure). This component is called "in-phase" (with the reference signal used for X axis)
  - Varying with time as $\sin(\omega t)$ (amplitude b in the figure). This component is called "quadrature" and is lagging the in-phase by 90° of phase.
- The second figure shows the in-phase – quadrature decomposition of the EM signal recorded in Rx coil
  - This can be done relative to the primary magnetic field (green) or to the voltage in the transmitter (Tx)
  - See lab #9 for more discussions and exercises



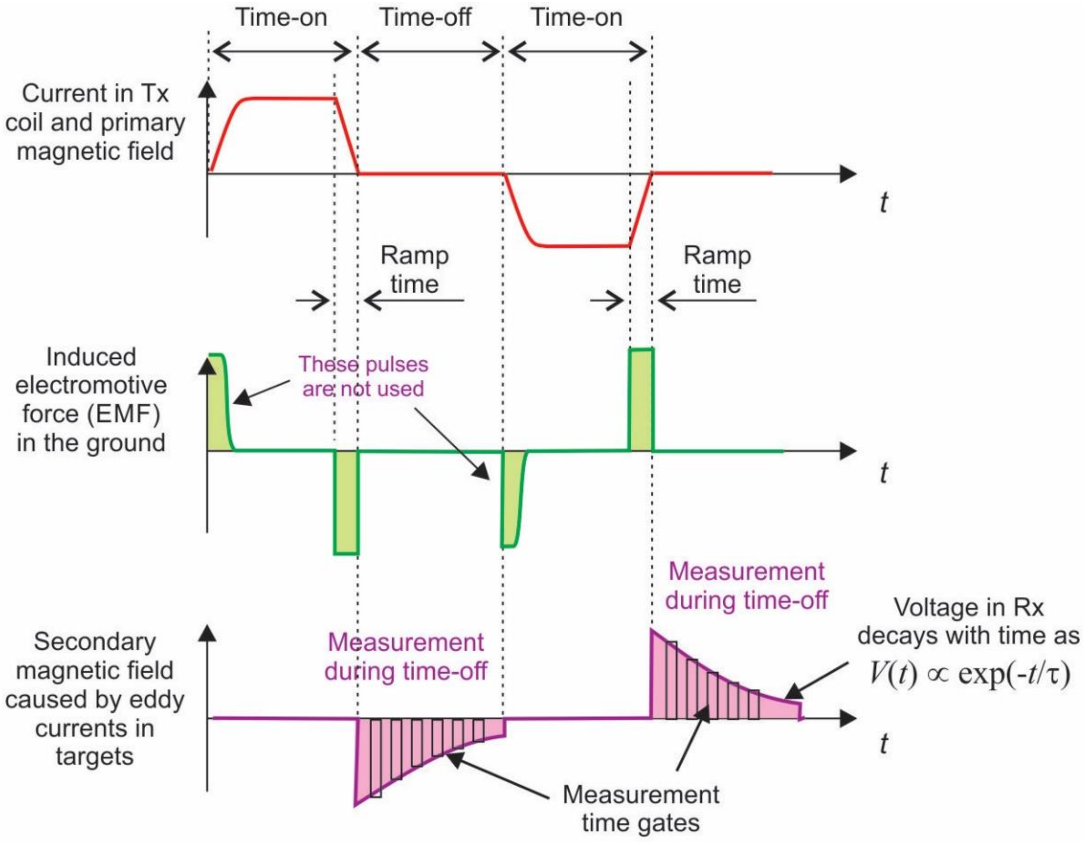
Techniques

- In the following slides, we will consider several EM techniques:
  - Time-domain
  - Frequency-domain:
    - Large, fixed Tx loop (Sundberg, Turam)
    - Moving small Tx coil (Slingram)
    - Tilt-angle

Time-domain (pulse-transient) EM

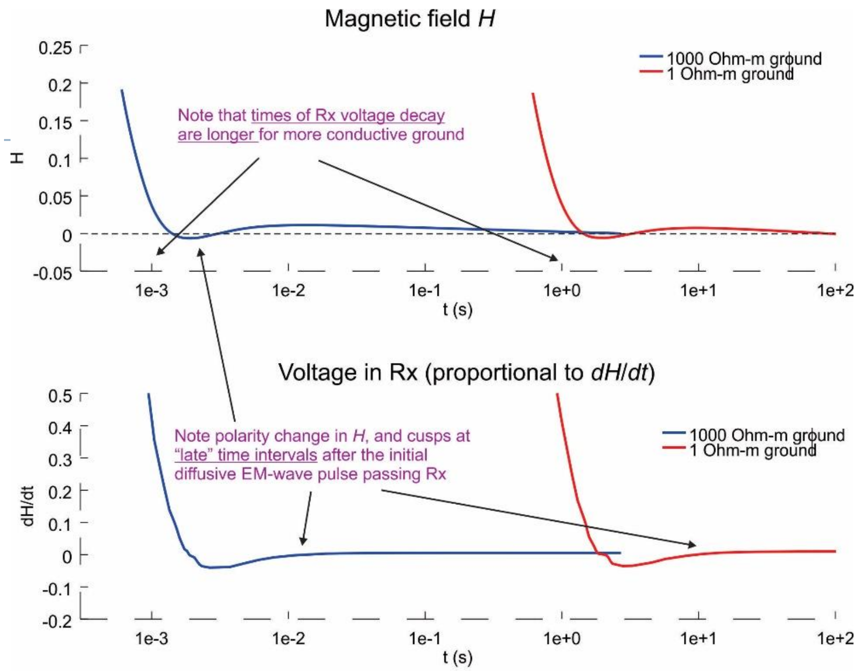
- The characteristic time τ is measured directly in the "Pulse-transient" (TEM) or "Time-domain EM" (TDEM) method

- Similar to time-domain IP
  - τ is analogous to "chargeability" there
- Tx current is switched on and off, with ramps producing pulses of induced EMF
- Eddy currents are produced and decay gradually, with time exponent τ
  - Lead to decaying secondary currents in Rx coil
- Time decays of secondary magnetic field (and Rx currents/voltages) are measured by using a set of time gates (figure on the right)



More detail on TEM

- Also similar to IP, voltage decrease $V(t)$ in the receiver is more complex than a single exp() dependence
  - Magnetic field decrease by diffusive EM waves spreading away from the transmitter
  - The wave speed increases and $V(t)$ decays quicker with higher resistivity of the ground (blue and red curves o the right)
    - The slopes $d[\lg V/dt]$ can be used to derive the apparent resistivity ρ_a of the subsurface
  - There are also additional pulses of $V(t)$ at late times when the second swing of the wave passes the Rx. These pulses are useful for detecting anomalies.
- It is useful to plot profiles of Rx voltage recorded in each time gate (recording channel) individually (bottom figure)
  - Early channels are often affected by the resistivity of the overburden
  - Good anomalies due to conductive target should appear in all channels



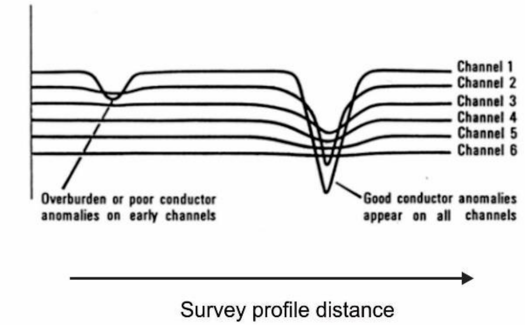
Magnetic field *H*

Note that times of Rx voltage decay are longer for more conductive ground

Voltage in Rx (proportional to *dH*/*dt*)

Note polarity change in *H*, and cusps at "late" time intervals after the initial diffusive EM-wave pulse passing Rx

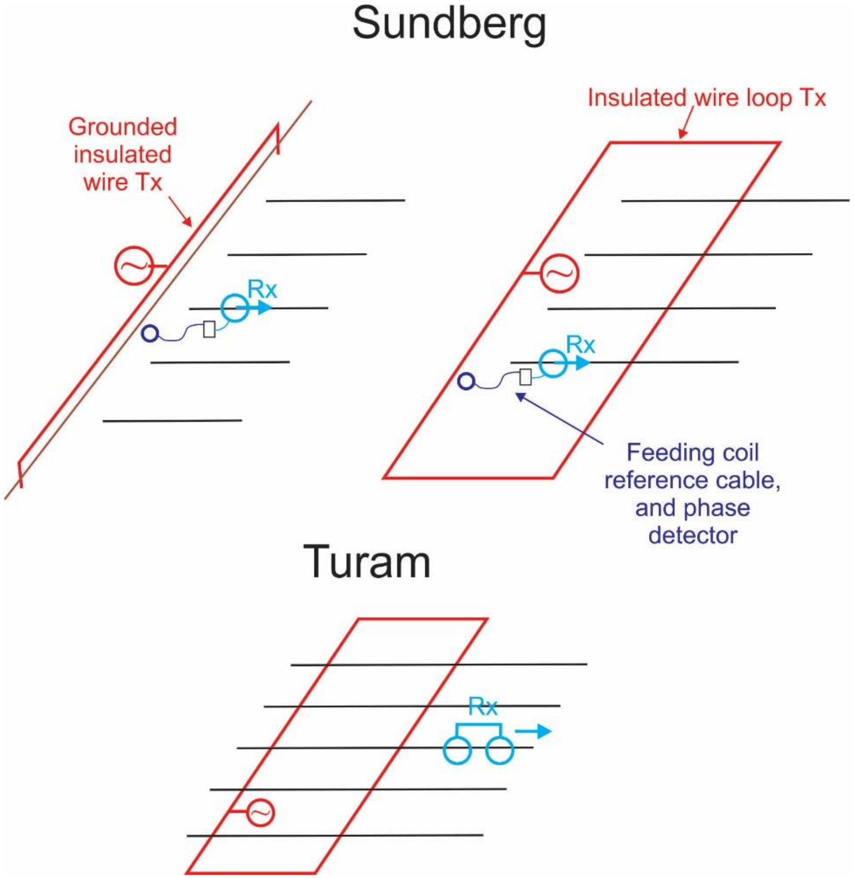
Profiles of Rx voltage within different time gates (channels)



Long-wire methods (Sundberg and Turam)

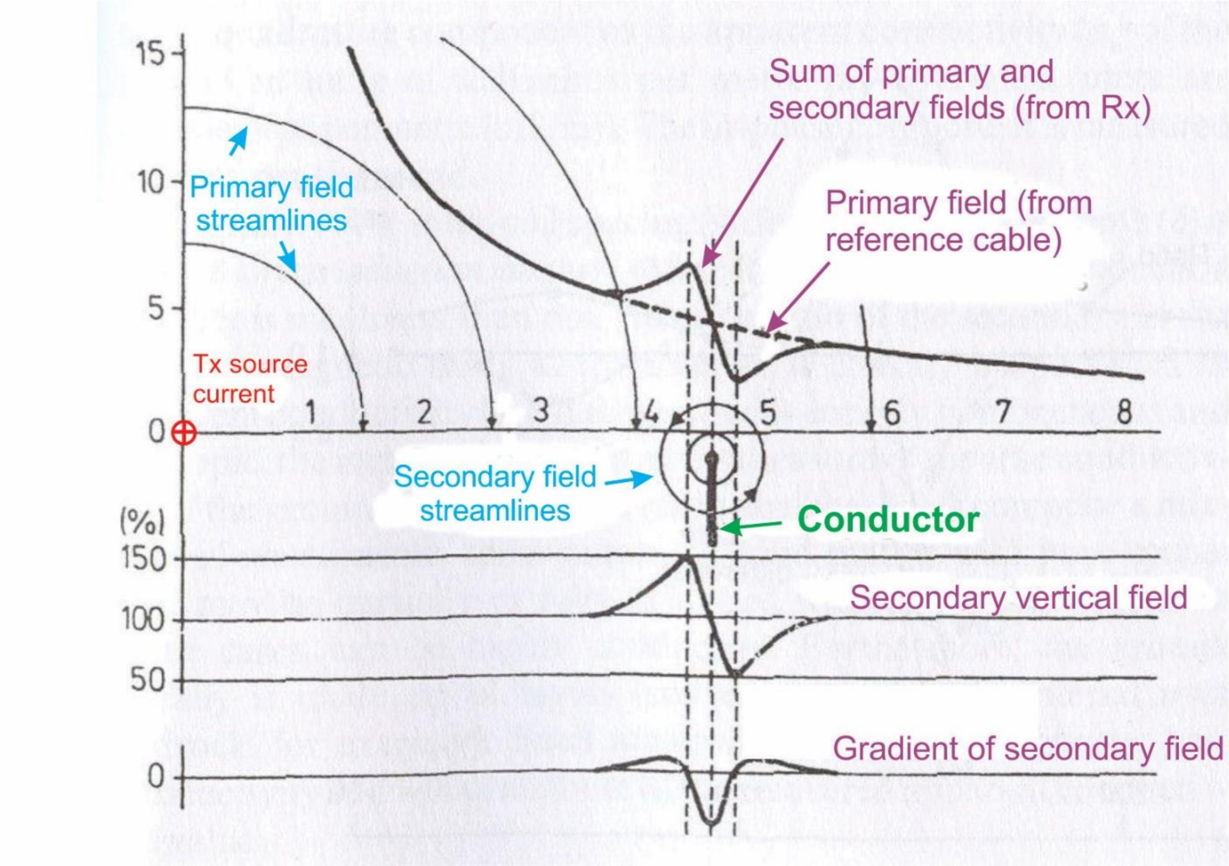
- Large loop of a long grounded wire (~1200 m by 400 m) is laid out along geological strike

- Rx scans along survey lines perpendicular to the wire
  - Phase lags relative to a fixed reference Rx ("feeding coil") are measured
- In Turam method, by using two Rx loops at fixed spacing, gradients of phase lags are measured
  - The feeding coil and long reference cable are not needed



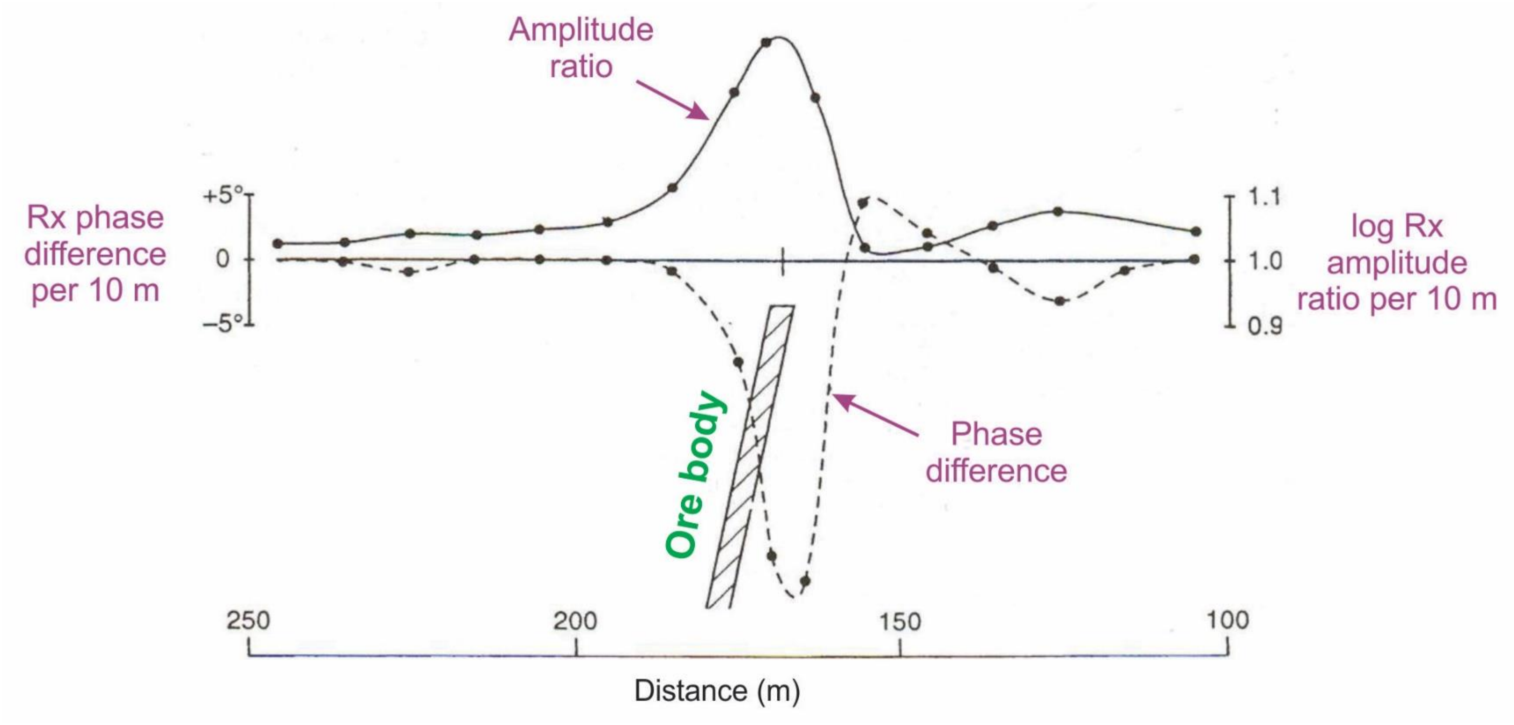
Conductive anomaly in Sundberg method

- A conductive anomaly (across the direction of profiling) is identified by:
  - Zero vertical component of secondary field, with an antisymmetric anomaly across the target
  - Symmetric negative peak of the gradient



Conductive anomaly in Turam method

- In Turam (gradient) method, conductive structures are located by:
  - Relative amplitude variations between the two Rx coils
  - Negative peak in the phase difference between Rx coils



Moving transmitter coil methods (Slingram)

- Most common in environmental and groundwater work
  - Allows covering long profiles with fixed Tx - Rx arrangement
  - Used on land, airborne, and seaborne
  - Ground Conductivity Meters (CGM) are designed in a similar way (discussed in IP lecture)
- The goal of profiling is in mapping a skin-depth thick layer below the surface
- Source/receiver coupling is determined by "induction number":

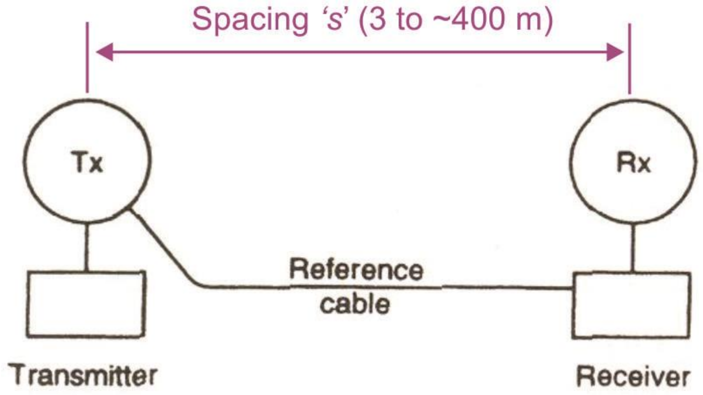
$$B = \frac{s}{\text{skin depth}}$$

- If working at low induction numbers ($B \ll 1$), the ratio of secondary and primary magnetic fields is:

$$\frac{H_{\text{secondary}}}{H_{\text{primary}}} \approx \frac{iB^2}{2} = \frac{\omega \mu_0 \sigma s^2}{4}$$

- Therefore, apparent conductivity is obtained as:

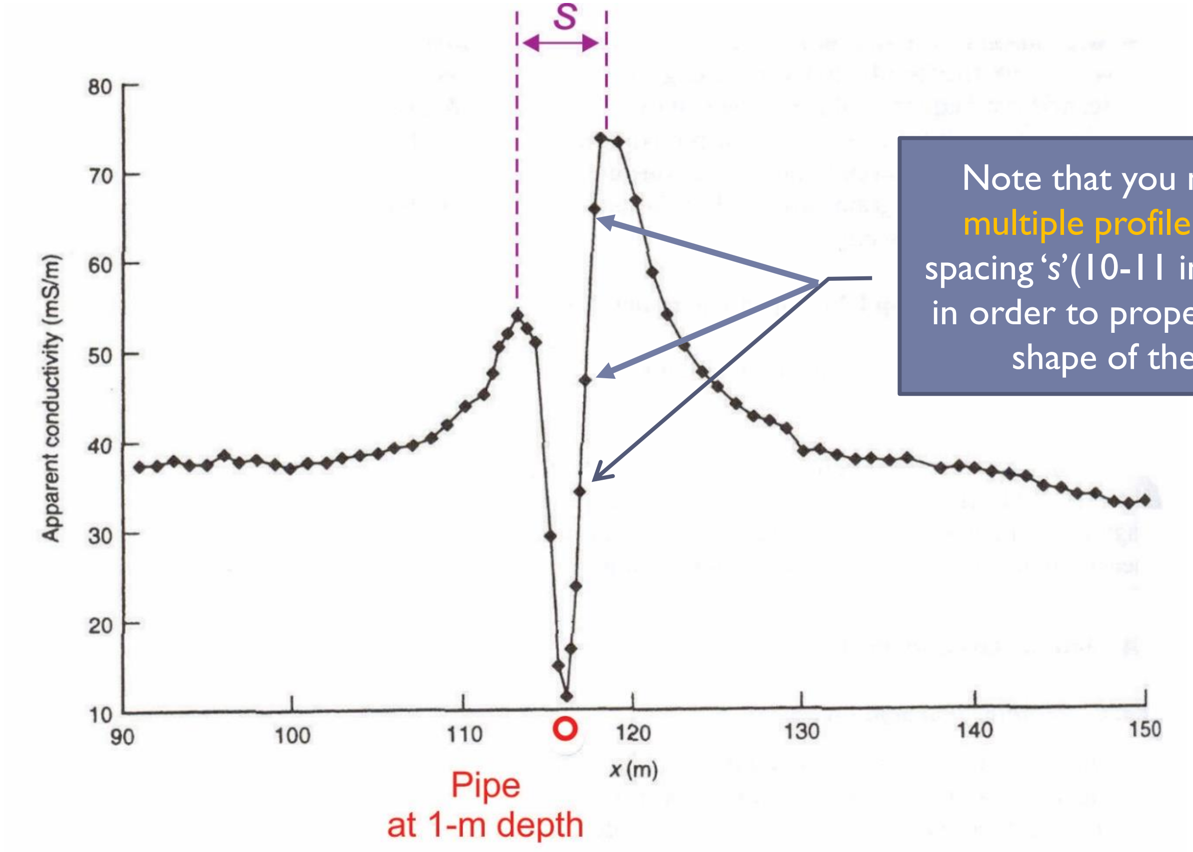
$$\sigma_a = \frac{4}{\omega \mu_0 s^2} \frac{\text{Quad}\left(H_{\text{secondary}}\right)}{H_{\text{primary}}}$$



'*i*' there means that "quadrature" component of the secondary field *H* should be compared to the primary *H* in receiver

Conductive anomaly in Slingram recording

- Note two peaks occurring when either Rx or Tx passes over the target
  - Need to check their spacing, which should equal ‘s’ (Tx – Rx separation) for a valid target



Note that you need to have multiple profile stations per spacing ‘s’(10-11 in this example) in order to properly sample the shape of the anomaly

Moving-coil systems on land (Geonics)

EM34; Allows two coil orientations

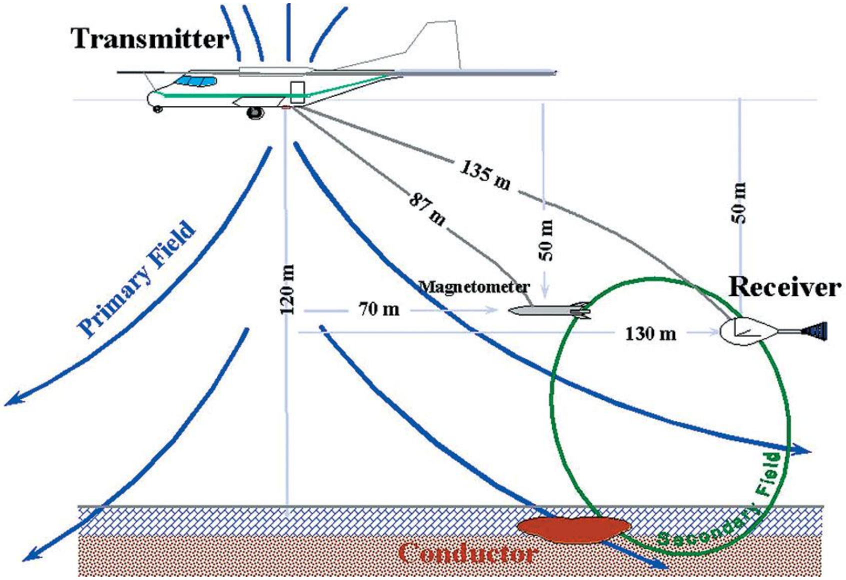
EM38; measuring soil salinity
(by apparent resistivity, as in your lab #4)

EM31:
Sounding depth ~6 m
Intercoil spacing 3.6 m,
Frequency 9.8 kHz

Airborne EM systems

- Moving-coil systems are used on low-flying aircraft (figures)
    - Efficient coverage of large areas
    - EM coils may be combined with multiple magnetometers
- Rx coils are usually in a towed “bird”
- Tx coils in a loop towed, mounted on a fixed-wing aircraft, or also in the bird

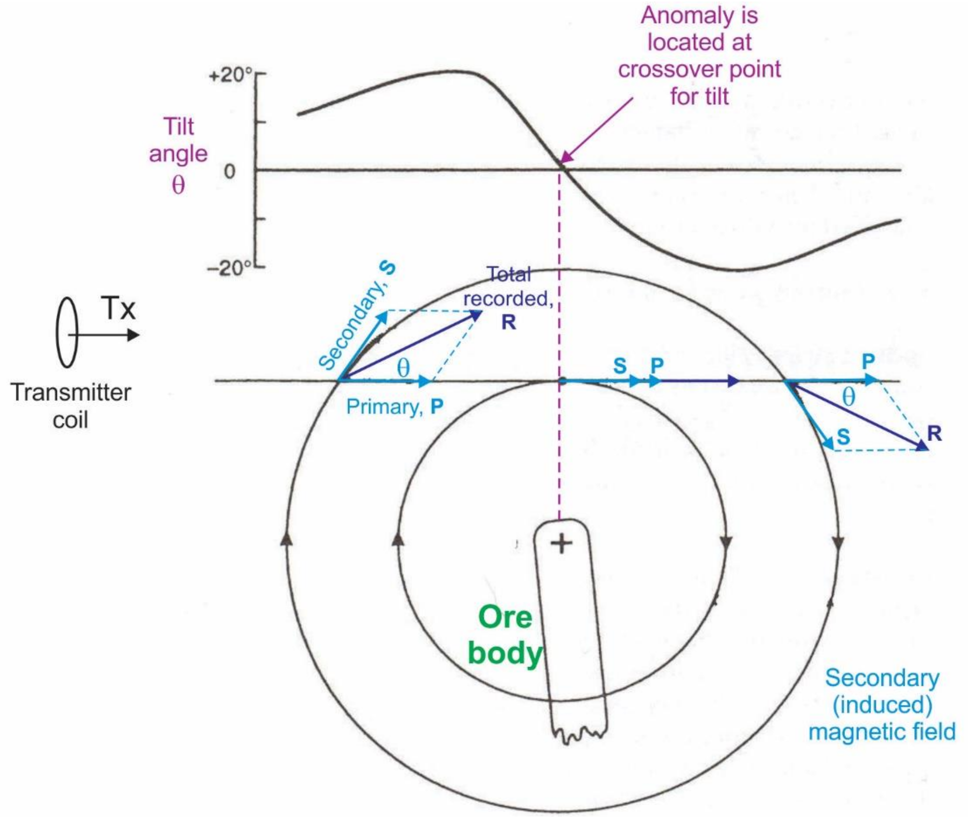
Rigid-coil Frequency-domain system (Fugro)



MEGATEM (Time-domain, Fugro)

Tilt Angle interpretation

- If recording vertical and horizontal components of the magnetic-field vector (which is usual), spatial tilt of the vector gives a convenient way for detecting conductive anomalies
- This is used in long-wire, VLF, and moving-transmitter (Slingram) surveys
- For horizontal primary field (vector **P** in the figure), angle of the field measured at Rx (vector **R**, angle θ) is zero far away from the anomaly and right over it
  - Replacing the profile of θ with its spatial derivative $d\theta/dx$ is called “tilt-angle filtering” or “Fraser filtering”
  - The derivative shows a peak over the conductive anomaly



Plane-wave (“far-field”) EM methods

- When using EM sources located at large distance, the primary field near the receiver is close to a plane wave and nearly spatially-uniform (at low frequencies used in EM work)
  - In a plane EM wave, the electric field **E** is orthogonal to **H**
  - Field **H** is always horizontal
    - This is because waves with vertical **H** are quickly absorbed by the surface
  - Depending on the direction to the source (wave propagation), field **E** can be polarized in two ways:
    - For vertical wave propagation, **E** is horizontal (magneto-telluric case)
    - For horizontal propagation, **E** is vertical (VLF case)
- In the remaining slides, we consider two concepts related to the effects of ground conductivity on such plane EM waves on the surface
  - Skin layer (skin depth)
  - Apparent resistivity

Effect of ground conductivity on EM fields

- How does ground conductivity affect the (time-variant) EM fields?
- EM fields are described by Maxwell's equations:

$$\mathbf{B} = \mu\mu_0 \mathbf{H}$$

Magnetic induction field (**B**)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's law of induction and Lenz law – varying magnetic field causes a curl of **E** (e.m.f.)

$$\nabla \times \mathbf{H} = \mathbf{j} + \varepsilon\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Ampère's law (current creates a curl of magnetic field)

$$\mathbf{j} = \sigma \mathbf{E}$$

Ohm's law (free current is proportional to **E** and conductivity)

- The "displacement current" in Ampère's law above (caused by variable polarization of the medium) is usually much lower than the free ("galvanic") **j:**

$$\varepsilon\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \ll \mathbf{j}$$

and therefore (from Ampère's law above), the magnetic field behaves as a "static" field produced by current:

$$\nabla \times \mathbf{H} \approx \mathbf{j}$$

Effect of ground conductivity on EM fields

- How do fields **E** and **H** (**B**) look near a conductive surface like an ore body or ground?
- Consider an EM field oscillating at relatively low frequency $\omega = 2\pi f$, dependent only on z, and with field **H** polarized along axis *X*:

$$\mathbf{H} = \begin{pmatrix} H_0 e^{i\omega t - \kappa z} & 0 & 0 \end{pmatrix}$$

κ is the logarithmic decrement of amplitude with depth

  - Taking a horizontally-oriented **E** and expressing **E** and **H** from Maxwell's equations above, we have their three components:

$$\mathbf{E} = \begin{pmatrix} 0 & -\dfrac{\kappa}{\sigma} H_0 e^{i\omega t - \kappa z} & 0 \end{pmatrix} \qquad \mathbf{H} = \begin{pmatrix} \dfrac{\kappa^2}{i\omega\sigma\mu\mu_0} H_0 e^{i\omega t - \kappa z} & 0 & 0 \end{pmatrix}$$

- Comparing the first and second expressions for **H**, you can see that κ is complex-valued:
  - what does this complex κ mean?

$$\kappa = \sqrt{i\omega\sigma\mu\mu_0} = (1+i)\sqrt{\frac{\omega\sigma\mu\mu_0}{2}}$$

Skin depth

- The complex-valued κ in the preceding slide means that the amplitude depends on depth as a product of exponential decay and cosine (wave-like) variation:

$$H_x = \mathrm{Re}\left(H_0 e^{-\kappa z}\right) = H_0 e^{-(\mathrm{Re}\,\kappa)z} \cos\left[\left(\mathrm{Im}\,\kappa\right) z\right]$$

  - The first exp() factor here shows exponential decay of all amplitudes with depth:

$$|\mathbf{H}| \text{ and } |\mathbf{E}| \propto e^{-\frac{z}{\delta}}$$

where the “skin depth” δ is:

Skin depth is the depth at which the amplitude drops by $e^{-1} \approx 0.37$

$$\delta = \frac{1}{\mathrm{Re}\,\kappa} = \sqrt{\frac{2}{\omega\sigma\mu\mu_0}} = \sqrt{\frac{2\rho}{\omega\mu\mu_0}}$$

  - The second cos() factor above means a phase shift relative to the phase measured on the surface. If we look at one skin depth (δ) deeper in the ground, the phase will be lagging (in radians) by:

$$\delta \cdot \mathrm{Im}\,\kappa = \frac{\mathrm{Im}\,\kappa}{\mathrm{Re}\,\kappa} = 1$$

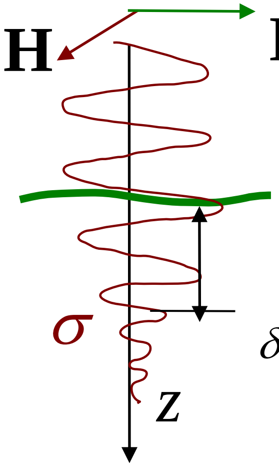
which is about 57°. You should see both of these effects in Lab #10

Skin-layer thickness

- The skin-layer (skin-depth) phenomenon occurs everywhere an oscillating wave is in contact with a conductor
  - The field amplitude exponentially decreases into the conductor as $\exp(-z/\delta)$
  - At high frequencies and in conductive materials, the skin layer is thinner.
    - The thickness of the near-boundary layer penetrated by the field is proportional to

$$\delta \propto \frac{1}{\sqrt{\text{frequency} \times \text{conductivity}}} = \frac{\sqrt{\text{resistivity}}}{\sqrt{\text{frequency}}}$$

  - This is why sampling of the ground (soil, ore bodies, etc.) by EM is limited to 1-2 skin-depth layers under their surface
- Sketch on the right shows that the same formula for skin depth holds for short wavelengths used in Ground-Penetrating Radar (GPR) measurements



Skin depth:

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu\mu_0}}$$

Apparent resistivity from EM measurements

- From the solutions for the horizontally-polarized fields **E** and **H**, their averaged amplitudes are related by:

$$\frac{\left\langle E_x^2 \right\rangle}{\left\langle H_y^2 \right\rangle} = \frac{\left\langle E_y^2 \right\rangle}{\left\langle H_x^2 \right\rangle} = \frac{\left\langle E_x^2 + E_y^2 \right\rangle}{\left\langle H_x^2 + H_y^2 \right\rangle} = \left(\frac{\kappa}{\sigma} \right)^2 = \omega\mu\mu_0\rho$$

- Most near-surface materials are non-magnetic ($\mu \approx 1$), and therefore the (apparent, as usual) resistivity is obtained from the ratio of average or peak horizontal-component amplitudes of the electric and magnetic fields:

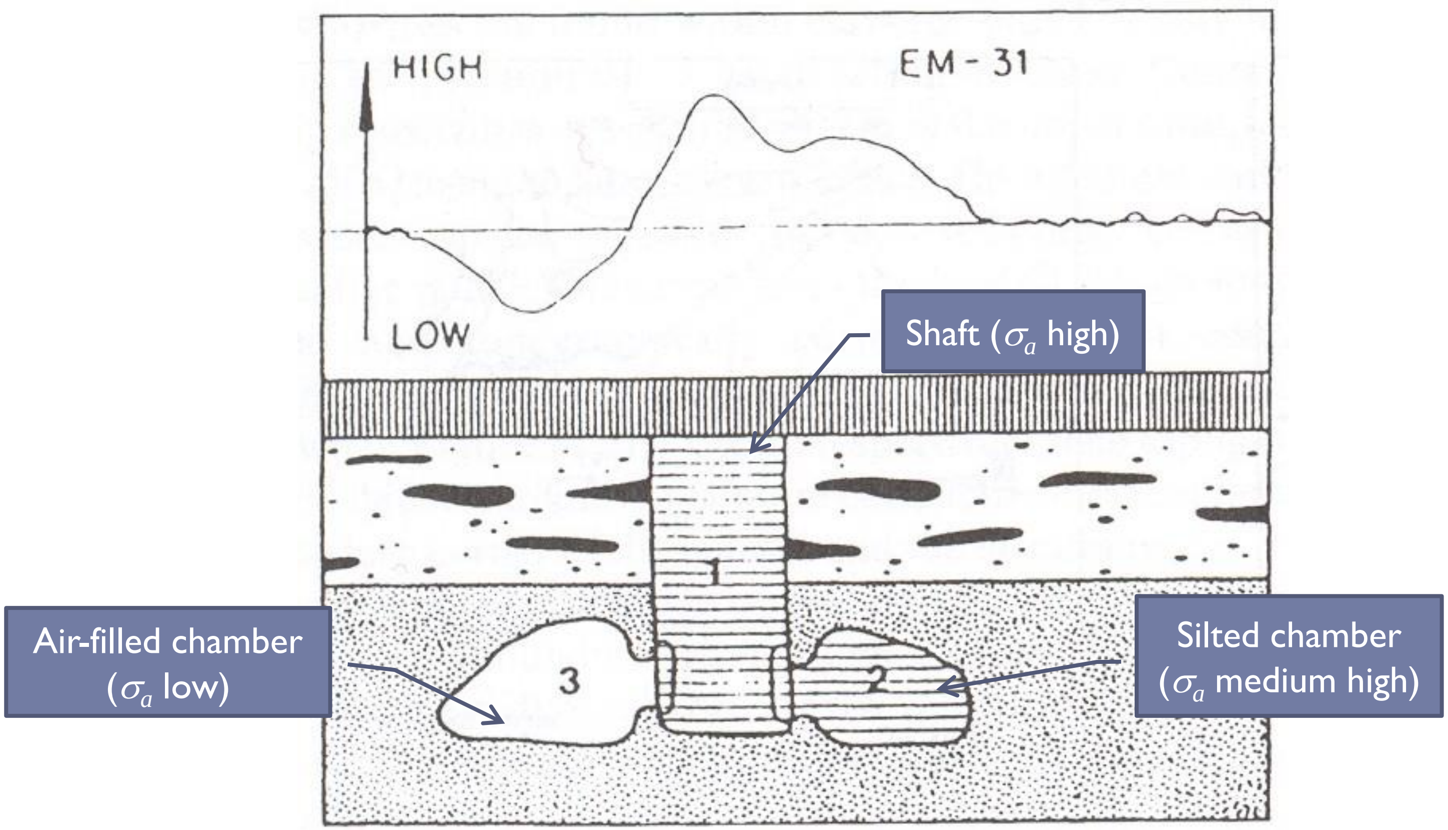
$$\rho_a = \frac{1}{\omega\mu_0} \frac{\left\langle E_H^2 \right\rangle}{\left\langle H_H^2 \right\rangle}$$

subscript '*H*' here simply stands for "horizontal"

- This measure of apparent resistivity is used in magneto-telluric (MT) and the "very low frequency" (VLF) ground-resistivity measurements

Example (apparent conductivity)

- Mapping excavation sites with EM31
- Apparent-conductivity profile ($\sigma_a = 1/\rho_a$) over a tomb at Bab-ed-dhra (Jordan) (Frohlich and Lancaster, 1986)



Shaft (σ_a high)

Air-filled chamber (σ_a low)

Silted chamber (σ_a medium high)