Electrical methods - Key points

In this lecture, we discuss the general concepts underlying all electrical methods:

- Similarity and dissimilarity with gravity
- Principle of electrical surveying
- Conduction of current in rocks
  - Charge and polarization
  - Distribution of charge and electrical field
  - Conductivity and resistivity, relation to metallic content and fluids

Electrical properties of rocks will be discussed in the next lecture (to keep videos shorter)

- Reading:
  - Reynolds, Section 7.3 – 7.5, 7.7
  - Dentith and Mudge, Sections 5.1 – 5.3

Electrical phenomena – similarity to gravity

- Electrical phenomena are analogous to gravity, with some important differences
- Similarities:
  - Potential field produced by a "source". For gravity, the source is the mass density ρ, and for electrical field, the source is the electrical charge, q.

    This means that similar to gravity, there exists a scalar function φ called "electric potential," which gives the potential energy of the charge: $U(x,y,z) = q\varphi(x,y,z)$

    so that the electrical field $\mathbf{E}$ is the negative gradient of this function: $\mathbf{E} = -grad\,\varphi$

    and electrical force applied to charge q: $\mathbf{F} = q\mathbf{E}$

    Beware of some ambiguity of notation: in electrical models and the following lecture, "ρ" usually denotes not density but the resistivity of the medium
  - Further similarity is the same Poisson's equation governing the field:

$$\nabla^2\varphi = 4\pi k_e q$$

    where $k_e \approx 8.99 \cdot 10^9\ \mathrm{N \cdot m^{-2} \cdot C^{-2}}$ is the Coulomb's constant
- Consequently, Gauss's law and all the basic solutions we studied for gravity field **g** apply to electrostatic fields

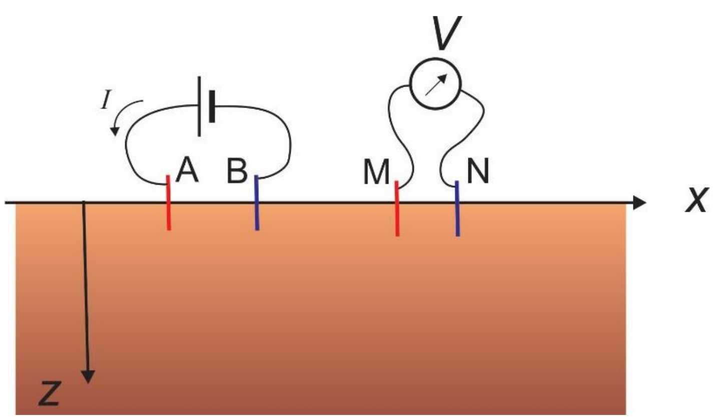
Electrical phenomena – differences from gravity

- However, there also are several major differences:
  - Unlike the density, charge q can be of positive and negative polarities. Charges of the same sign repel from each other, oppositely to gravity
  - Charges move relatively freely through rock, in the form of the current density, denoted by vector $\mathbf{j}$.
  - Practically every molecule of the material contains two opposite charges that are separated spatially. Because of this bipolar structure, the average medium is characterized not only by its charge density $\mathbf{q}$ but also by a new property not found in gravity models - the density of dipole moment. We will denote this quantity (defined below) by vector $\mathbf{p}$.
    - This additional mode of charge distribution $\mathbf{p}$ explains new effects not seen with gravity: the Spontaneous Potential (SP) and Induced Polarization (IP)
- Also fortunately for geologists and engineers, electrical properties of materials vary broadly for different rocks and their physical conditions. Therefore different methods complement each other and provide different types of information.
- Finally, the big advantage of electrical imaging is that unlike gravity, it can be conducted with controlled sources – by injecting charges (currents) at selected points and measuring the potentials at multiple locations
  - Controlled-source acquisition greatly increases the volume and uniformity of coverage and allows obtaining much better constrained images
  - The controlled effects are often much stronger than natural ones and relatively noise-free

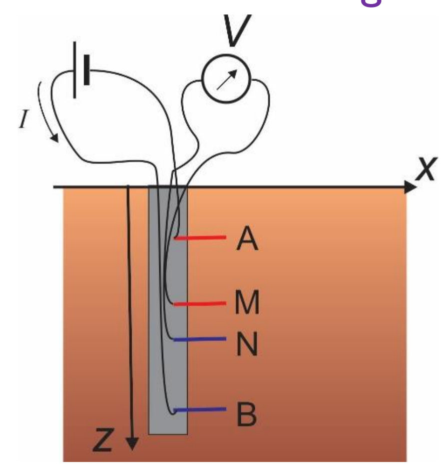
Electrical surveying

- A typical (controlled-source) electrical survey looks like shown in figures below
  - Four electrodes are placed into the ground:
    - Source "current" (in fact, voltage from a sufficiently powerful source) is applied to two electrodes, which are conventionally denoted A and B
    - Voltage V (difference in potential ϕ) is measured between electrodes M and N
  - The results are usually obtained from the ratio of measured voltage and current (resistance): $R = V/I$
  - By varying the spacings between electrodes A-B and M-N and mutual positions of these pairs, different depths and horizontal locations are studied
    - Moving along the horizontal direction X (vertical Z for borehole) is often called "profiling"
    - Expanding the spacing of the array increases its penetration depth and is called "sounding"

Surface recording

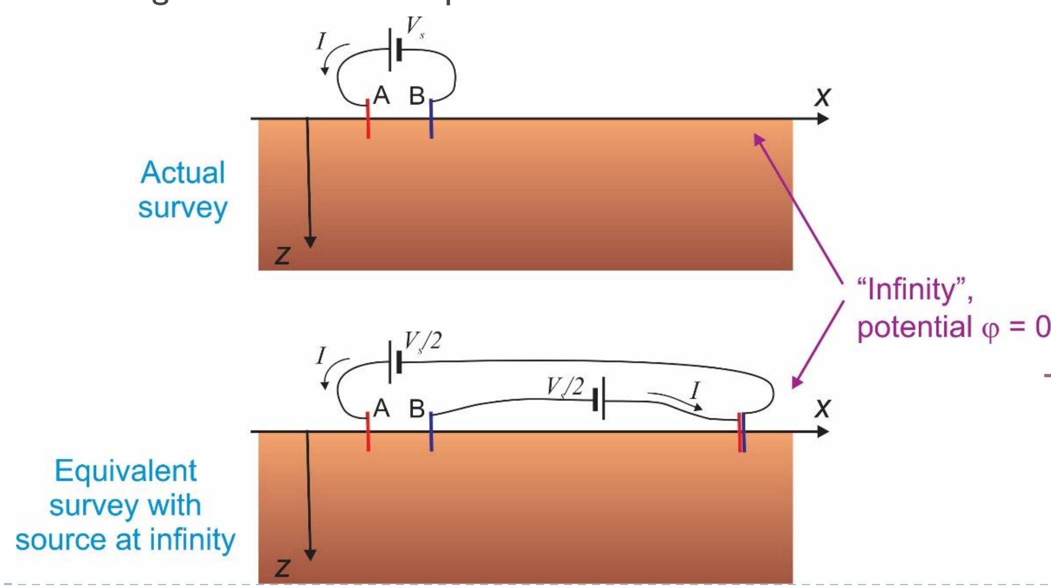


Borehole recording



Principle of superposition and electrodes at infinity

- What happens when the source is applied to electrodes A and B?
  - Due to the principle of superposition (linearity of all equations within most of the study area), the result can be viewed as a combination of two independent experiments using the same current *I*.
    - In each of these experiments, one source electrode is at the infinity (figure below)
    - All charges and currents will be the same everywhere within rock
- Thus, when modeling electrical experiments, it is convenient to think only about point (pole) source (with the second electrode anywhere at the infinity)
  - Similarly, we can place a potential electrode at the infinity
  - The measured voltage will be a sum of potentials measured at electrodes M and N independently

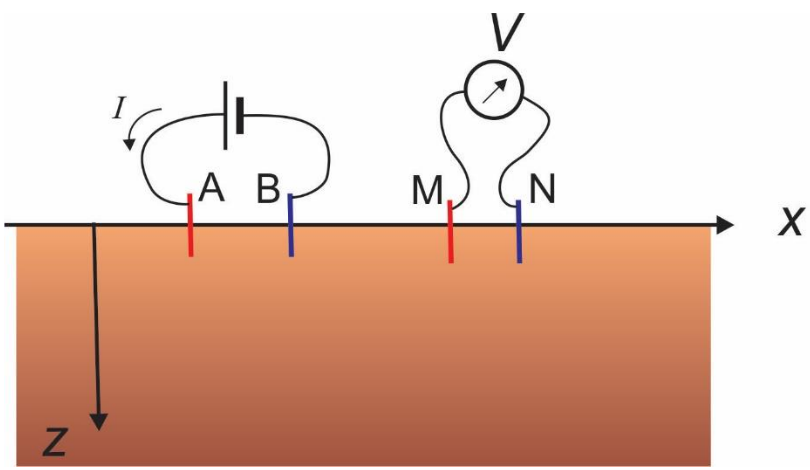


The electrode at infinity can be placed anywhere (to the right, left, or any side of the profile)

Reciprocity

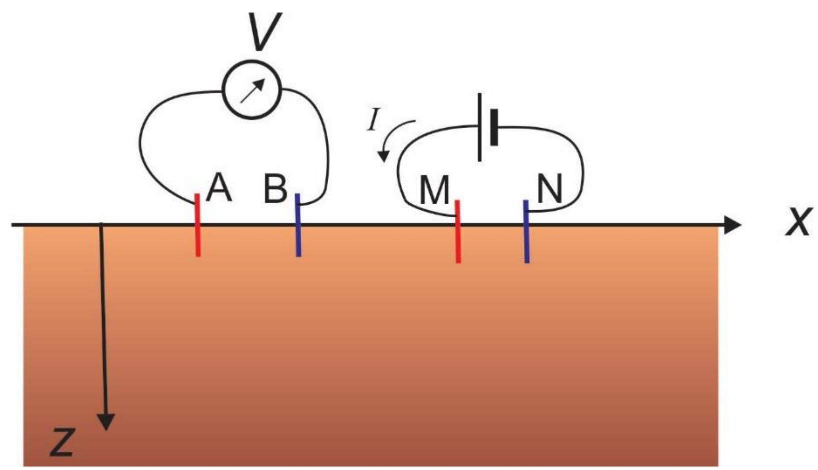
- An interesting property of electrical imaging is its reciprocity:
- If we switch places the current and potential electrodes, the resulting resistance $R = V/I$ will be the same
  - This property is very general. It does not depend on the subsurface structure and represents a fundamental consequence of the existence of the potential function φ for the electric field and bi-directional property of resistance
  - This property can be (relatively) easily shown from the pole-pole reduction of the surveys described in the preceding slide

Ordinary recording



Reciprocal recording

(measuring the same voltage V if injecting the same current I)



Resistance matrix

- Electrical sampling of the subsurface using arbitrary electrode arrays can be nicely summarized by the concept of resistance matrix:
  - Assume that we have L electrodes at the surface, apply potentials ϕ_n to each of them simultaneously ($n = 1 \ldots L$), and measure currents I_n in each of them
  - Then, for ordinary cases, the currents will be linearly related to the potentials and vice versa:

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \varphi_L \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & \cdots & R_{1L} \\ R_{21} & R_{22} & R_{23} & \cdots & R_{2L} \\ R_{31} & R_{32} & R_{33} & \cdots & R_{3L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{L1} & R_{L2} & R_{L3} & \cdots & R_{LL} \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_L \end{pmatrix}$$

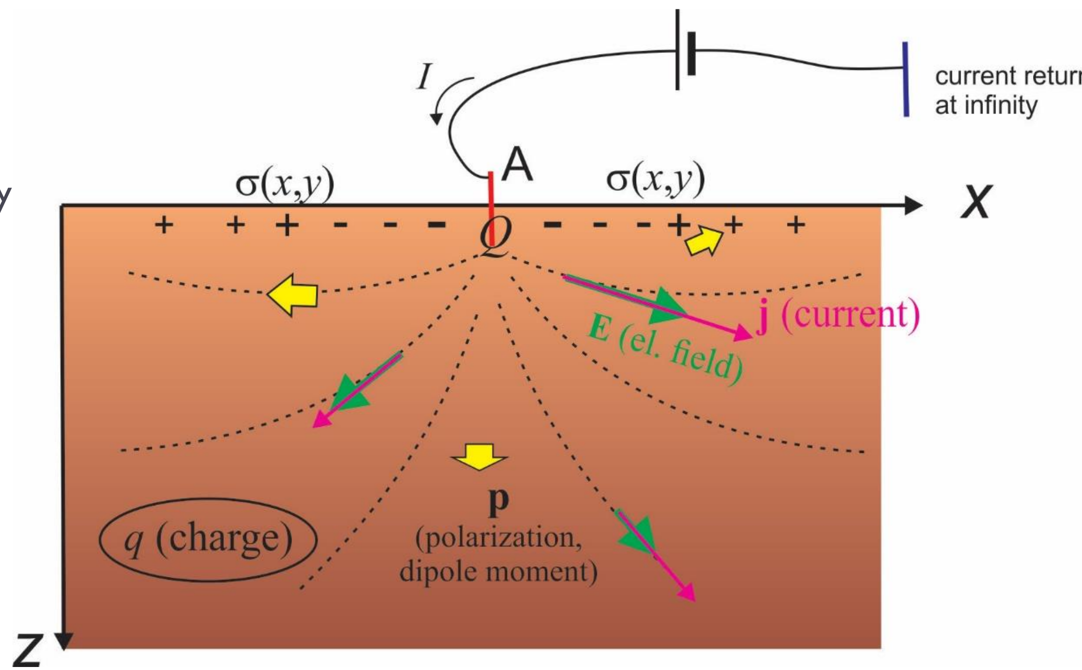
Resistance matrix

(I use 'L' for the number of electrode stations because letter 'N' is often reserved for one of the potential electrode names)

- The resistance matrix is always symmetric: $R_{ij} = R_{ji}$
  - This symmetry represents the reciprocity property above

Effects of electrical source

- What happens when a (single-pole) source is applied to electrode A or B?
  - Charge Q is concentrated near the electrode (figure below)
  - Rock is polarized, with polarization vectors $\mathbf{p}$ dependent on formation and rock properties
  - Induced charges σ appear on the surface and layer boundaries
  - Free charge q may also appear within volume
  - Electrical field $\mathbf{E}$ is created, with streamlines determined by Q and the spatial distributions of q, σ, and $\mathbf{p}$
  - Current $\mathbf{j}$ flows along $\mathbf{E}$, with intensities proportional to conductivity



- Let us consider the meanings of terms current and polarization in the next slides

Current

- Many materials contain electrically charged particles (electrons, ions) that are relatively mobile and can move from one location to another. In the absence of electric field, these particles exhibit Brownian motion, but on average, they stay in place.
- When an electric field $\mathbf{E}$ is applied to a medium, each elementary charge q experiences force $\mathbf{F} = \mathbf{E}q$ and drifts in the direction of the field (for $q > 0$) or against it (for $q < 0$). The velocity of this average drift is proportional to $\mathbf{E}$ and equals

$$\mathbf{v} = \mu\mathbf{E}$$

where parameter μ is a material property called the mobility of charge q.

- If the medium contains N of such charge carriers per unit volume, then the charge transmitted per second through unit area is the "current density", denoted $\mathbf{j}$:

$$\mathbf{j} = Nq\mathbf{v}$$

Note that this is simply (total charge within unit volume)$\times\mathbf{v}$

- Combining the above equations, we see that current density is proportional to the electric field: $\mathbf{j} = \sigma\mathbf{E}$, where σ is the conductivity:

$$\sigma = Nq\mu$$

Thus, "conductivity" is the (charge per unit volume) times the mobility

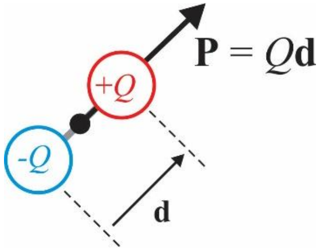
- The inverse of this quantity is called resistivity: $\rho = \dfrac{1}{\sigma}$

Modes of conduction

- There are three general types of mobility μ different modes of current conduction:
  - Electrolytic (mobile ions in pore fluids and some solids). This is the most common mechanism of electrical conduction in rock
  - Electronic (charge carried by free electrons). This mode is common in metals.
  - Dielectric (by alternating polarizations). This mode is significant when using switching or alternating current, such as in Induced Potential measurements. We will talk about polarization in the next slides.
- In all modes, current conduction in the ground is anisotropic. The conductivity is typically lower across layer bedding than along it.
  - Typical anisotropy levels λ are up to 2.

Polarization

- Polarizable media such as rocks interact with the electric field not only by means of charge density q but also by the "dipole moment" density, $\mathbf{p}$
  - For a single molecule represented by two charges $+Q$ and $-Q$ separated by distance d, the dipole moment is a vector of magnitude Qd directed toward the positive charge. See Figure here:



Here, $\mathbf{d}$ is a vector connecting the charge $-Q$ to $+Q$

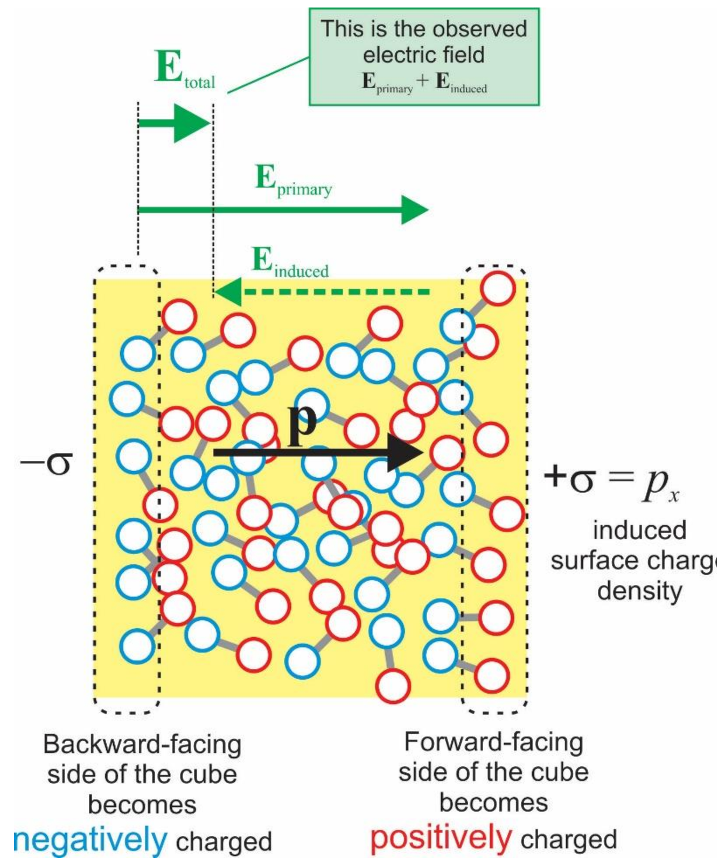
- A sum of vectors $\mathbf{P}$ for molecules within a volume V gives the mean dipole moment density $\mathbf{p}$ for the medium:

$$\mathbf{p} = \frac{1}{V}\sum_{i=1}^{N} Q_i \mathbf{d}_i$$

where the summation takes place over all elementary dipoles.

Meaning of the polarization of the medium

- Imagine that a cube of a polarizable medium is subjected to an electrical field **E**. The molecular dipoles will turn (positive ends move forward and negative – backward relative to the field); see Figure.
- As a result, the interior of the cube remains neutral (equal number of positive and negative charges), but the sides attain surface charge densities $\pm\sigma$ equal the *X*-th component of dipole moment.
- For example, in a typical electrical experiment, the vertical component of the dipole-moment vector is seen as a surface charge induced on the free surface (σ in the cartoon two slides above).
  - Similar surface charges are induced on the boundaries of subsurface layers with contrasting dielectric properties and boundaries of bodies
  - The charge density induced on the surfaces equals minus divergence of **p**: $q_{\text{induced}} = -\text{div}(\mathbf{p})/\varepsilon_0$
- The induced **p** (or surface charges) creates the induced **E** field (dashed green arrow) which largely compensates the primary field by the source (see the callout in the figure)



Electric properties of materials

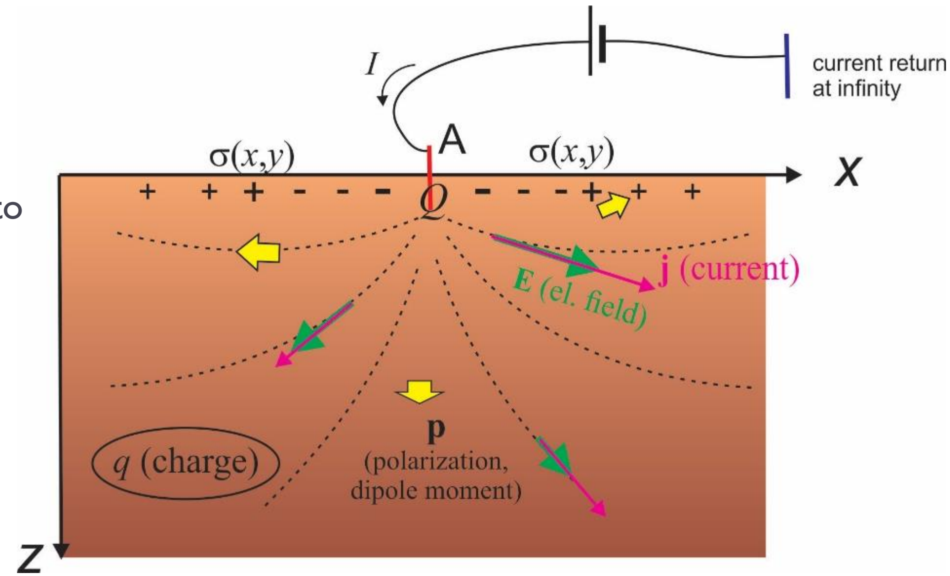
- The distribution of the fields $\mathbf{E}$, $\mathbf{p}$, $\mathbf{j}$, and q within the subsurface (figure repeated below) is determined by rock properties:
  - Conductivity σ (unfortunately, also denoted σ; do not mix it up with surface charge density!) or its inverse, resistivity: $\rho = 1/\sigma$ (do not confuse with mass density!!). For isotropic rock, the relation is (the differential Ohm's law):

$$\mathbf{j} = \sigma \mathbf{E} = \frac{\mathbf{E}}{\rho}$$

  - Relative electric permittivity (dielectric constant) ε of rock gives its induced dipole moment:

$$\mathbf{p} = \left(\varepsilon - 1\right)\varepsilon_0 \mathbf{E}$$

where $\varepsilon_0 \approx 8.8541878128(13)\times 10^{-12}$ F/m is the "permittivity of free space" (constant simply due to the selected SI unit system), and $\varepsilon \geq 1$ is non-dimensional ($\varepsilon = 1$ for air/vacuum)



- We will discuss these properties in the second part of this lecture

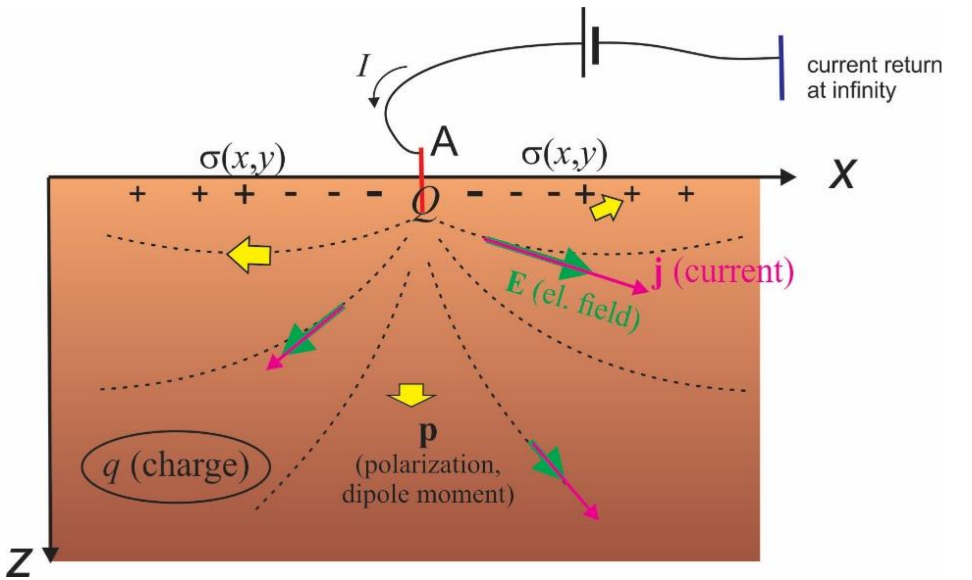
Governing relations for $\mathbf{E}$ and $\mathbf{j}$

- In addition, there are a couple general governing relations for the electric field and current:
  - $\mathbf{E}$ is related to rock charge density (free q and induced $\sigma = -\text{div}(\mathbf{p})$) by Poisson's equation (as in Introduction lecture):

$$\text{div}\mathbf{E} = \frac{q - div\mathbf{p}}{\varepsilon_0}$$

  - Our observations are often stationary (steady currents are measured). Then, the divergence of current equals zero:

$$div\mathbf{j} = 0$$

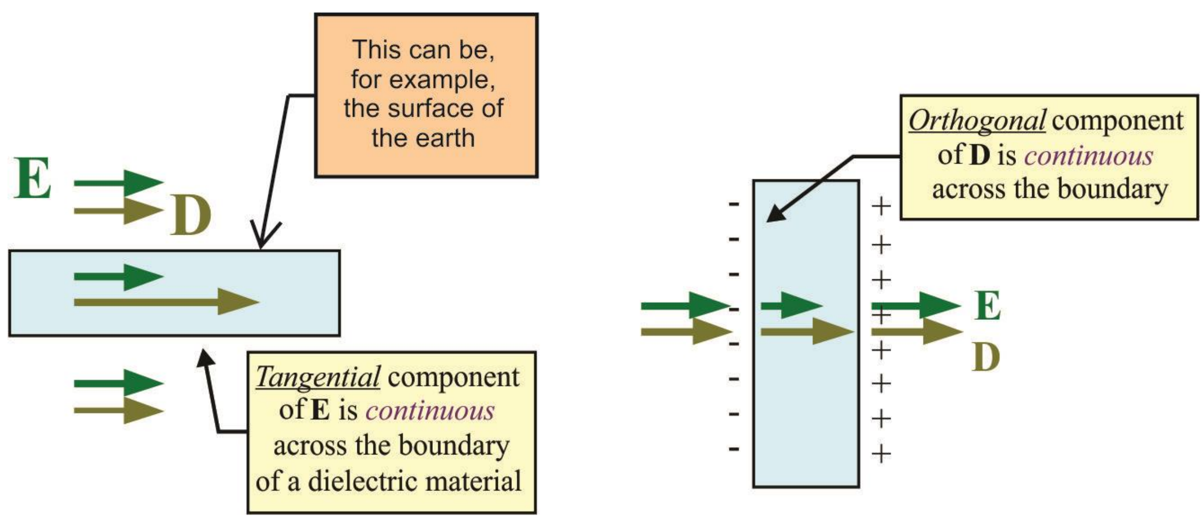


Electric displacement field (**D**)

- When solving various problems in electrostatics, it is often convenient to replace the dipole moment field **p** with an "electric displacement" field **D** defined by

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{p} = \varepsilon_0 \varepsilon \mathbf{E}$$

- This simple proportionality $\mathbf{D} \propto \mathbf{E}$ (for isotropic rock) is actually why the dielectric constant ε is defined that way
- Field **D** is only sensitive to free charges such as produced by the source: $\mathrm{div}(\mathbf{D}) = q$
- Fields **E** and **D** possess simple boundary conditions on material contrasts and layer boundaries:



Basic case #1:
point source in an unbounded space

- The most basic case of a field is the field produced by a point source (point charge Q in electricity). The expression for potential φ can be easily understood if keeping in mind its key principles:
  1. Since the field satisfies the Poisson's equation (above), then similar to gravity (see gravity lectures), φ should behave as $1/r$, where r is the distance from the source.
  2. Also, unlike gravity, the potential should also be positive if $Q > 0$ (to provide repulsion force for charges of the same sign).
  3. The field $\mathbf{E}$ (and therefore φ) is also proportional to Q and inversely proportional to ε (dielectric constant of the medium)

- From these tips, the expression for electric potential is (ε_0 is just the usual constant occurring in SI units):

$$\varphi = \frac{Q}{\varepsilon_0 \varepsilon} \frac{1}{r}$$

, and the radial component of electric field equals

$$E_r = -\frac{\partial \varphi}{\partial r} = \frac{Q}{\varepsilon_0 \varepsilon} \frac{1}{r^2}$$

- However, Q for an electrode is unknown but current I is measured instead. From the above relations, note that $E_r = \varphi/r$ and current density $j = E_r/\rho$ (Ohm's law). Therefore:

$$I = 4\pi r^2 j = 4\pi r^2 E_r / \rho = 4\pi r\, \varphi / \rho$$

- Thus, the electric field is proportional to the injected current and resistivity <u>irrespectively of the dielectric constant</u> ε of the medium:

$$\varphi = \frac{I\rho}{4\pi} \frac{1}{r}$$

> Exercise: therefore, what is the charge Q of the electrode injecting current I?

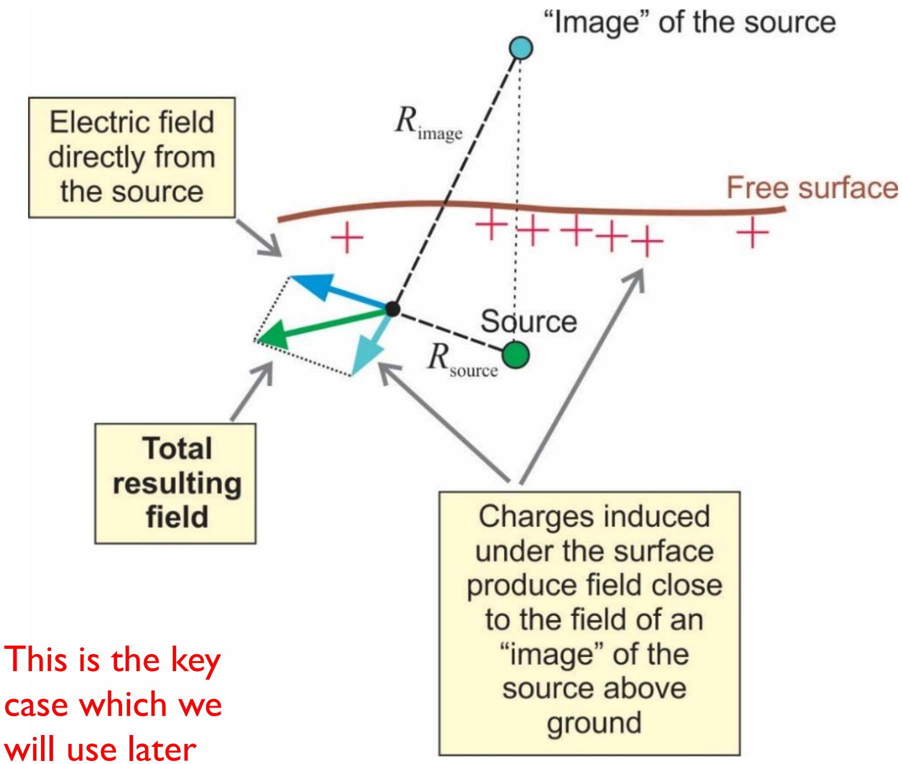
Basic case #2: point source below surface

- This is the key case for most practical measurements
- When conducting measurements on or below the earth's surface, the surface becomes charged and affects the measured potentials
- Let us consider a point source I (or sink, $I < 0$) of current below a near-horizontal surface. The electric field below ground (at black dot in the figure) can be approximated as a sum of fields from the source and its "electrical image" above the ground:

$$\varphi = \frac{I\rho}{4\pi R_{\text{Source}}} + \frac{I\rho}{4\pi R_{\text{Image}}}$$

- If the electrode is located on the surface, the image coincides with the source, and the potential and electric field from the preceding slide are doubled:

$$\varphi = \frac{I\rho}{2\pi R_{\text{Source}}}$$

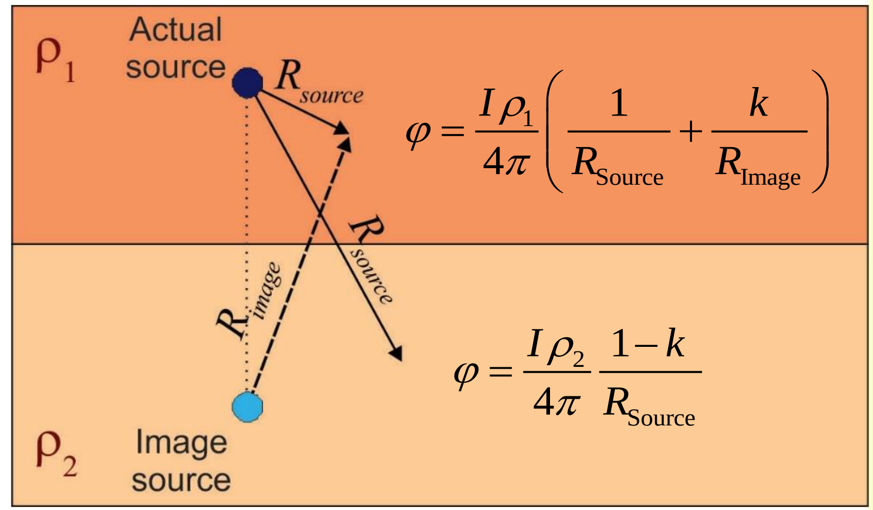


This is the key case which we will use later

Basic case #3: two conductive half-spaces

- If we have two welded, conductive half-spaces:
  - Within the half with the source, the electric field is a sum of the field from the source and field from the image, weighted with some weight k (see figure)
  - Within the second half, only the source field is used with weight $(1-k)$
- Weight k can be found by ensuring two conditions on the boundary:
  - The potential φ is continuous (this is satisfied automatically with any k)
  - The vertical current density j_z is continuous (total charge does not change)
- From these requirements, k equals (verify if you are interested)

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$



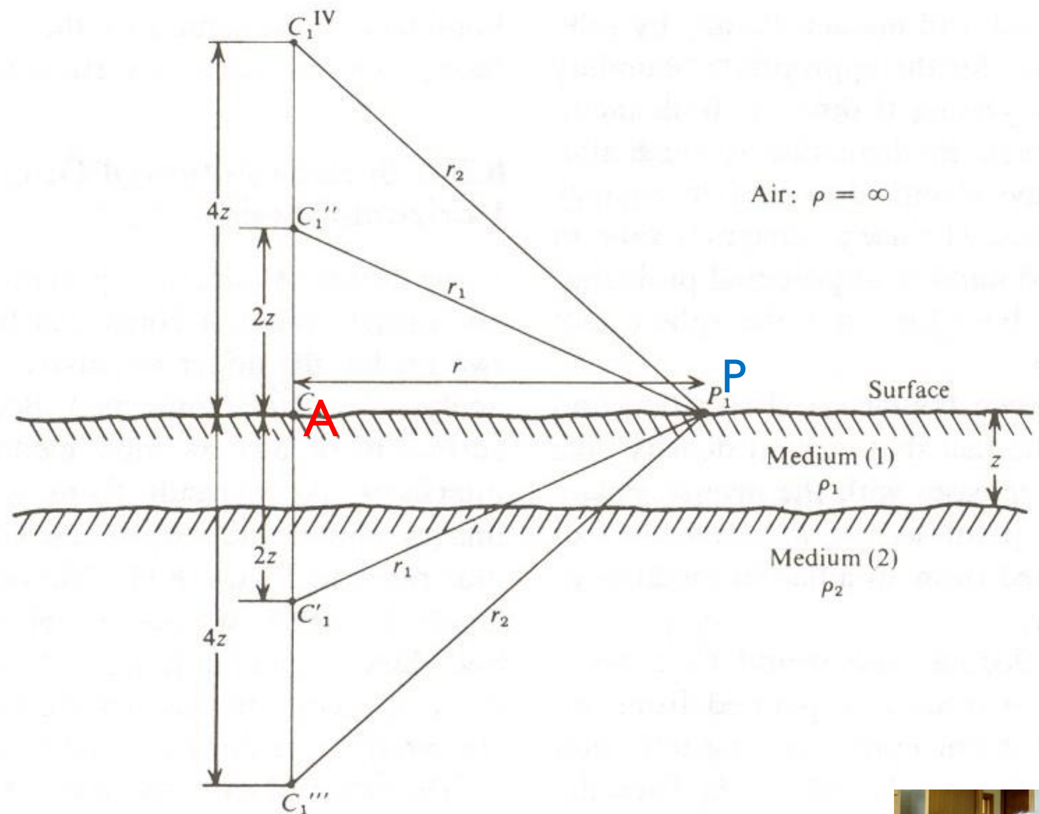
More complex case #4: constant-resistivity layer below surface

- Consider a current source at A and an electrode at point P on the surface, with a conductive or resistive layer below the surface and a half space below it (figure)

- The electric field can be represented as an effect of an infinite series of mirror images reflecting alternatively in both boundaries of the layer:

$$\varphi(r) = \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + \sum_{i=1}^{\infty} \frac{2k^i}{r_i}\right)$$

(distances r and r_i are shown in the figure)



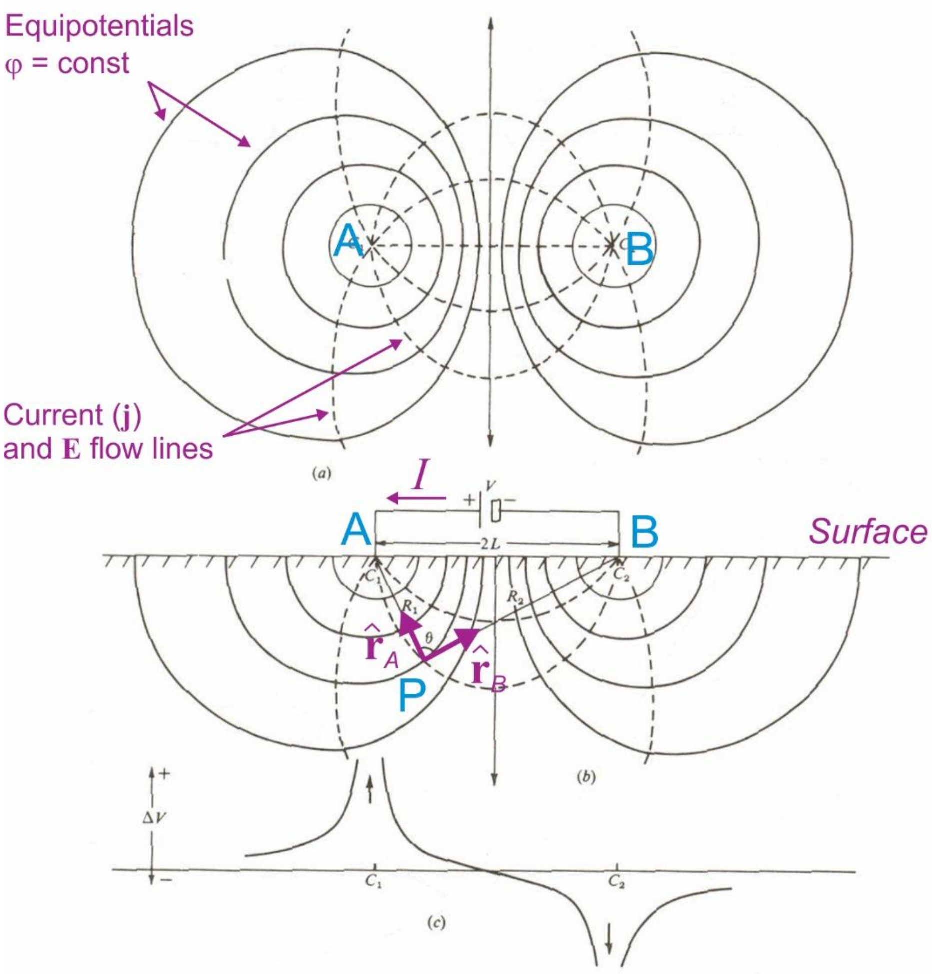
Electrical field of a current dipole

- In a typical resistivity experiment, current I is driven through the ground by a pair of electrodes (denoted A and B)
- From our "basic solution #2", the resulting electric field is:

Potential:
$$\varphi = \frac{I\rho}{2\pi}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

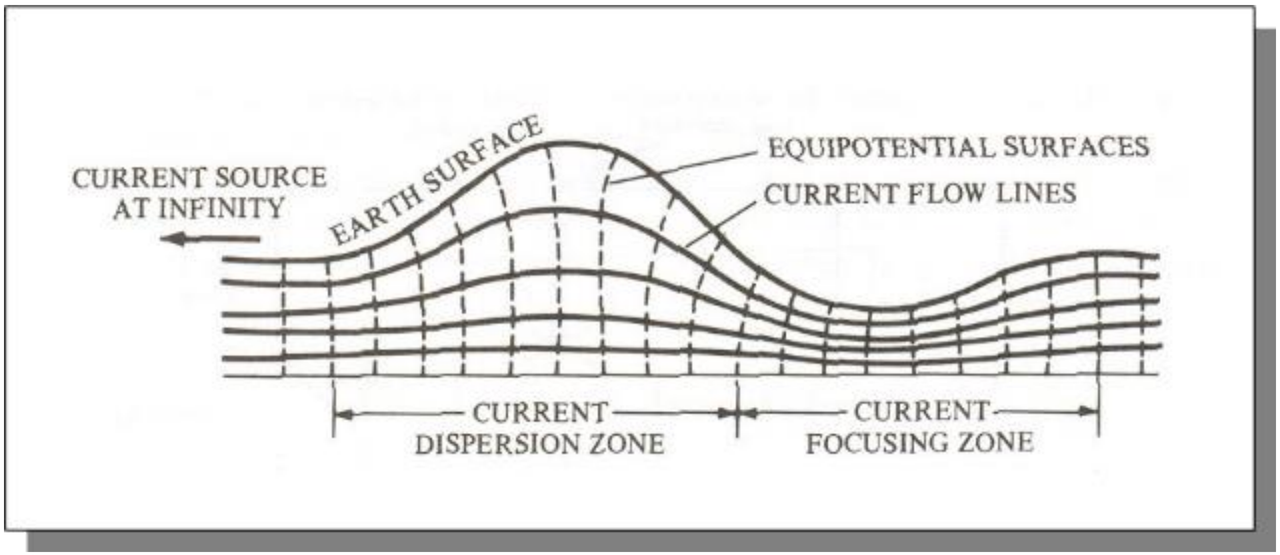
Strength:
$$\mathbf{E} = -\frac{I\rho}{2\pi}\left(\frac{\hat{\mathbf{r}}_A}{r_A^2} - \frac{\hat{\mathbf{r}}_B}{r_B^2}\right)$$

- where r_A and r_B are the distances from the observation point P to the electrodes (see figure), and $\hat{\mathbf{r}}_A$ and $\hat{\mathbf{r}}_B$ are unit vectors in these directions.



Effect of the free surface

- Horizontal and non-horizontal bedding may lead to complex electrical images
- In particular, surface topography affects distribution of induced charge and leads to current dispersion (current decreases within hills) and focusing (current increases within valleys):



- These effects are very important for self-potential (SP measurements)