Magnetic method - Key points

- Magnetic field
- Similarities and differences from gravity
  - Biot-Savart law; magnetic potential
  - Dipoles and monopoles
- Magnetization
  - Magnetic dipole moment
  - Magnetic susceptibility and permeability
- Magnetic induction
- Total field
- Shapes of anomalies
- Transformations
  - Reduction to the pole, pseudo-gravity
- Magnetic field of a line source

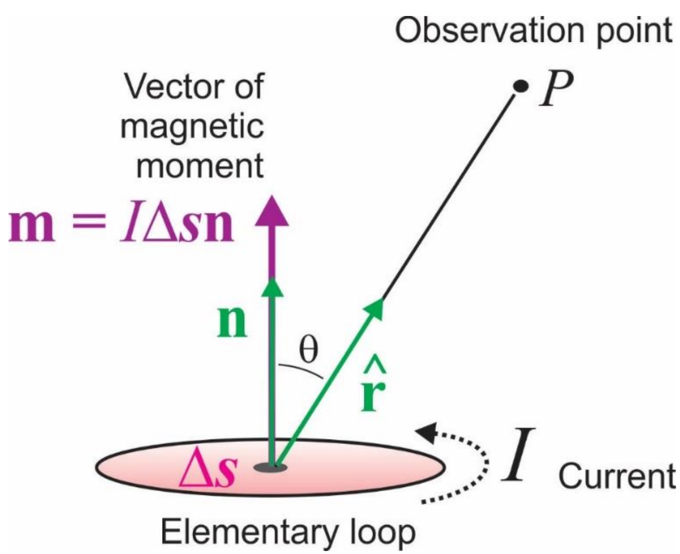
- Labs # 7 and 11
- Reading:
  - Reynolds, Chapter 3
  - Dentith and Mudge, Chapter 3

Uses of geomagnetic surveys

- Magnetic surveys are useful for:
  - Reconnaissance for base metals
    - Concealed dikes
  - Location of alluvial gold and heavy minerals
  - Archaeology
  - Reconnaissance for oil and gas
    - Large-scale geological structures, particularly basement
  - Environmental geophysics
    - Location of ferrous objects (pipes, cables, etc.)
    - Location of buried drums (contaminated waste, etc.)
    - Location of buried military ordnance (shells, bombs, etc.)
  - Identification of the basement and tectonic structure

Magnetic dipole moment

- Magnetics is quite analogous to potential fields (gravity, electrostatics), but also with some differences, which need to be understood carefully
- There exist no magnetic "charges" or "monopoles"
  - …but for static fields (time-independent), they can be constructed mathematically and are useful for understanding observations and models
- The elementary magnetic source is the *dipole* formed by a loop of current:

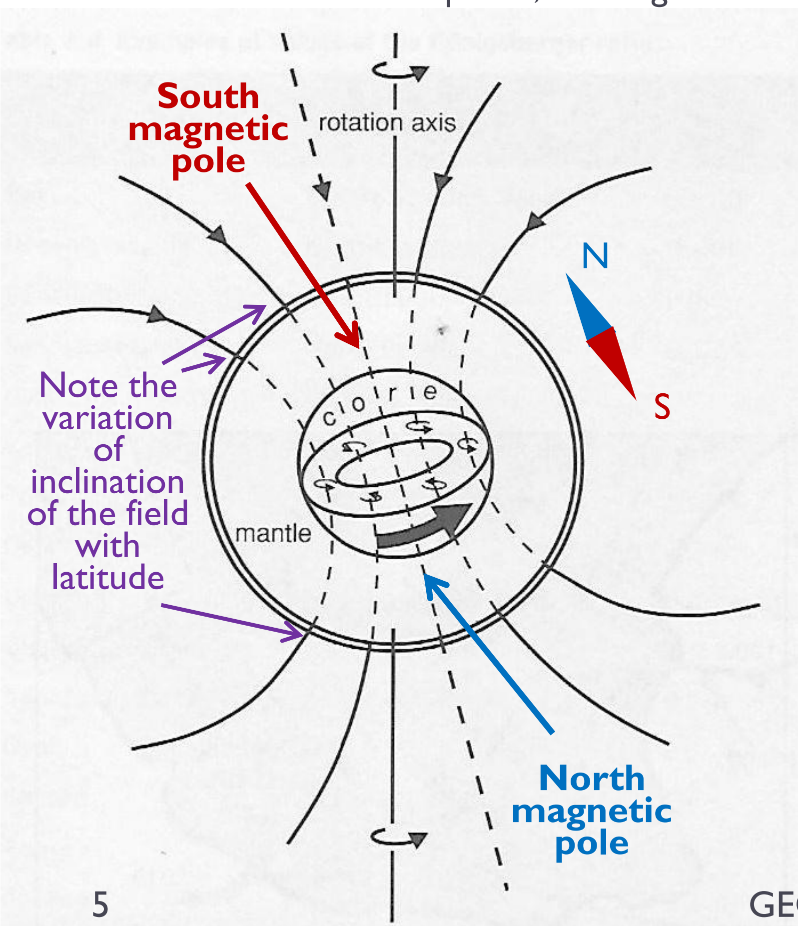


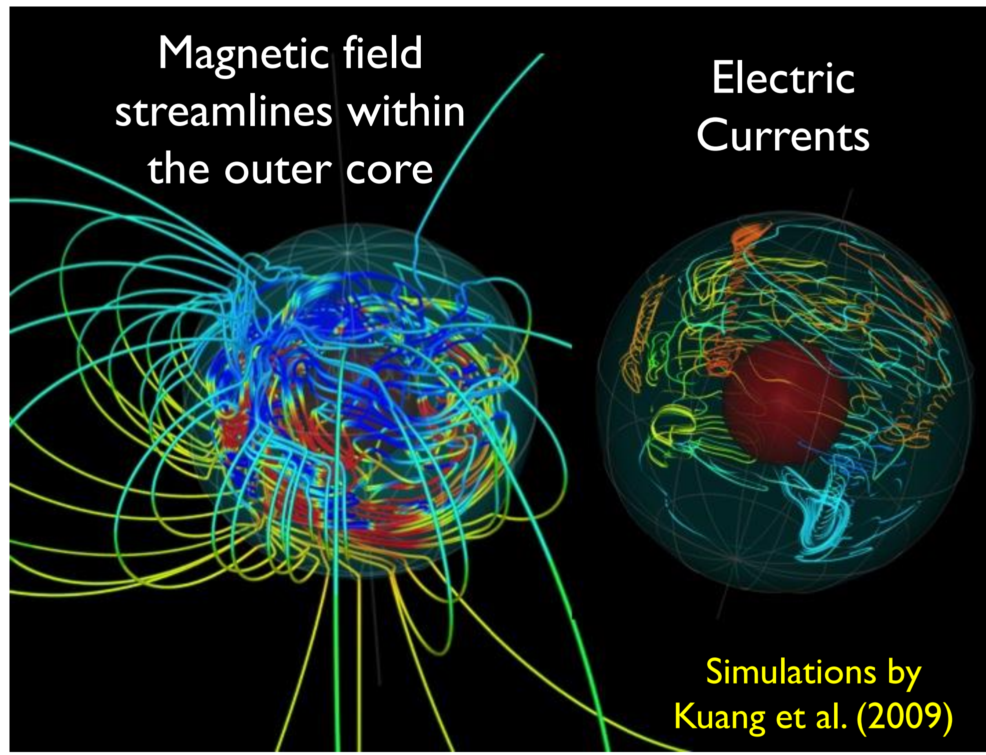
Magnetic field

- Magnetic field is produced by <u>moving electric charges</u> (current $\mathbf{I}$ in the preceding slide) and also acts on <u>moving charges</u>
- The field is described by two mutually related vectors (similar to fields $\mathbf{E}$ and $\mathbf{D}$ in electrostatics):
  - The magnetizing field strength $\mathbf{H}$ (measured in Ampere/meter in SI units)
  - The magnetic induction (or flux density) $\mathbf{B} = \mu\mu_0\mathbf{H}$ (measured in Tesla = Weber/m^2, where Weber is the unit of magnetic flux – product $|\mathbf{B}|\Delta S$ for an area ΔS across $\mathbf{B}$)
    - This field $\mathbf{B}$ gives the actual forces acting on objects like magnetometer
    - μ here is the magnetic permeability
- There are two physical effects caused by magnetic field: $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$
  - <u>Force</u> acting on a line of <u>current</u> or moving particle:
    - This force tries moving the particle <u>in an orthogonal direction</u> to both $\mathbf{v}$ and $\mathbf{B}$
  - <u>Torque</u> acting on a <u>magnetic dipole</u> (like the compass needle): $\mathbf{L} = \mathbf{m} \times \mathbf{B}$
    - This torque tries rotating the dipole $\mathbf{m}$ so that it becomes parallel to $\mathbf{B}$

Geomagnetic field

- The magnetic field of the Earth is caused by the geodynamo – currents flowing within the liquid iron outer core of the rotating planet (plot on the right)
  - Its principal part is the dipole field with axis inclined relative to the rotation axis (left)
  - In northern hemisphere, the magnetic field is directed downward, in the southern - upward





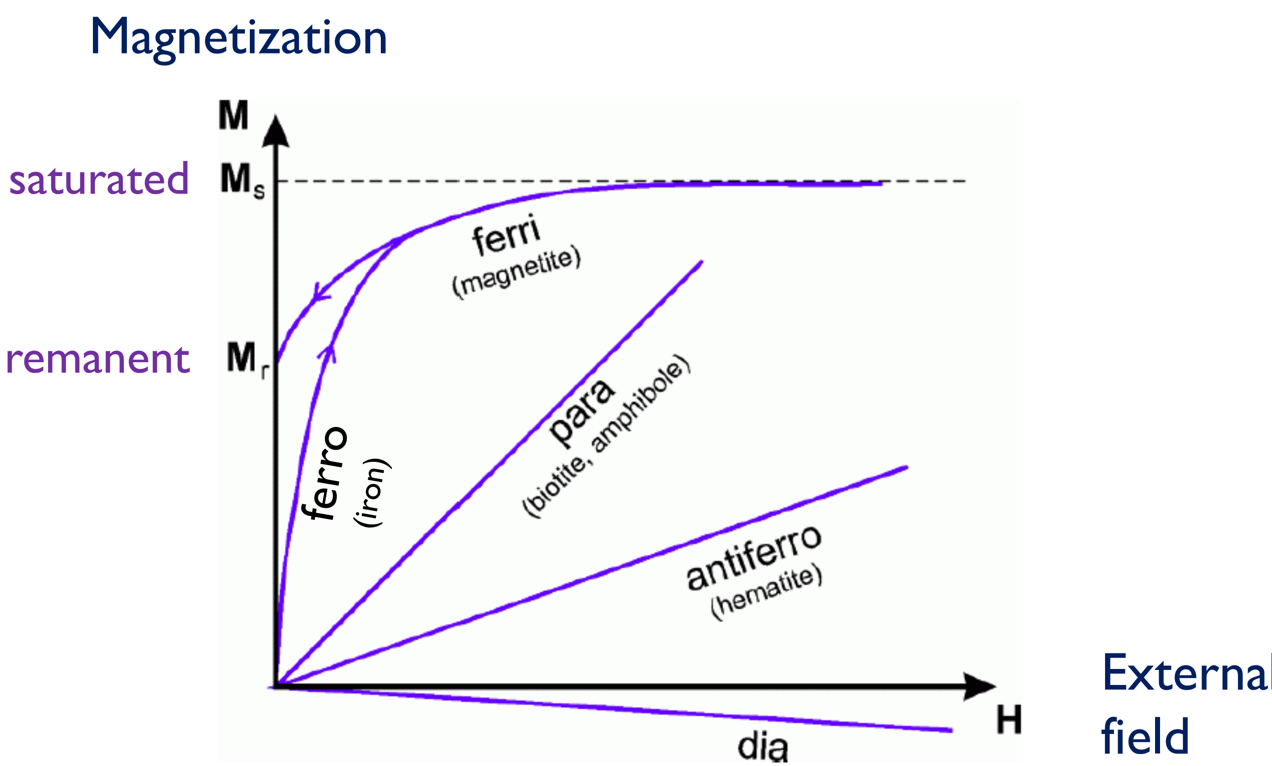
Magnetic properties of rocks

- There are two rock properties used in magnetic surveying
  - Remanent magnetization (the magnetic moments are 'frozen' into the rock independently of the ambient geomagnetic field)
    - This effect can be compared to self-potential (SP)
    - It represents the magnetization in the past history of the Earth. The magnetization was frozen in at the time of rock formation.
  - Magnetic susceptibility (production of magnetization anomalies induced by the present-day ambient magnetic field)
    - Magnetic susceptibility is analogous to density for gravity surveys or dielectric susceptibility for electrical IP imaging

Types of magnetic susceptibility

- There are three distinct types of magnetic susceptibility (denoted κ; also often denoted χ):
  - In diamagnetic materials, $\kappa < 0$
    - All electrons in the outer atomic shells of electrons are paired, and so the total spin equals zero. When an external magnetic field $\mathbf{H}$ is applied, the shells are deformed and produce an opposite induced field, which reduces the average field
  - In paramagnetic materials, $\kappa > 0$ and small
    - There exist unpaired electron shells, and therefore the atoms possess magnetic moments $\mathbf{m}$. In an external field $\mathbf{H}$, the moments are turn and tend to be oriented in the direction of $\mathbf{H}$. The field induced by these moments enhances the external field, and hence $\kappa > 0$.
    - However, this paramagnetic polarization effect reduces with absolute temperature as $1/T$
  - In ferromagnetic materials (like iron , nickel, or cobalt), $\kappa > 0$ and large
    - There are many unpaired electrons which interact and attain the same orientation within a whole grain of the material ("magnetic domain"). This creates a very strong enhancement of the external magnetic field
    - This effect disappears above the Curie temperature (and this is how the remanent magnetization gets frozen in when the rock crystallizes and cools down)
    - In ferrimagnetic materials (like magnetite, Fe_3O_4), a part of the magnetic moments gets oriented in the opposite direction. This creates a remanent magnetization even when there is no external magnetic field

Rock magnetization - summary



- In addition, there exist some demagnetization and high-magnetization effects

Magnetic susceptibilities of sedimentary rocks

Rock type	Susceptibility, χ [10^{-6} SI = cgs]
Average	0 to 360
Dolomite (pure)	-12.5 to +44
Dolomite (impure)	20,000
Limestone	10 to 25,000
Sandstone	0 to 21,000
Shales	60 to 19,000

Magnetic susceptibilities of igneous rocks

Rock type	Susceptibility, χ [10^{-6} SI = cgs]
Average for silicic rock	40 to 82,000
Average for mafic rock	500 to 120,000
Granite	10 to 65
Granite with magnetic minerals	20 to 50,000
Rhyolite	250 to 38,000
Pegmatite	3000 to 75,000
Gabbro	800 to 76,000
Basalts	500 to 180,000
Oceanic basalts	300 to 36,000
Peridotite	95,000 to 196,000

Magnetic susceptibilities of minerals

Rock type	Susceptibility, χ [10^{-6} SI = cgs]
Ice	-9
Rocksalt	-10
Gypsum	-13
Quartz	-15
Graphite	-80 to -200
Chalcopyrite	400
Pyrite ore	50 to 5,000
Hematite ore	420 to 38,000
Pyrrhotite	1250 to 6.3×10^{6}
Magnetite	70,000 to 2×10^{7}

Magnetic potential

- In this and the following couple slides, I'll try showing how "magnetic potential" is obtained
- Consider a small shift $\Delta\mathbf{r}$ of the observation point from point $\mathbf{r}$ (figure below)
- If **curl (B)** = 0 (which means that there is no current at the observation point), field **B** can be presented as a gradient of a potential V:

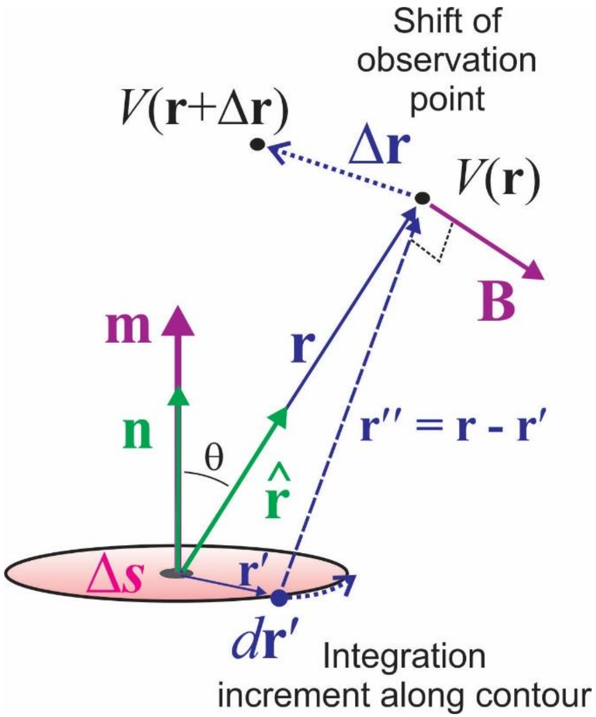
$$\mathbf{B} = -\vec{\nabla} V \qquad \text{or:}$$

$$\Delta V \equiv V(\mathbf{r}+\Delta\mathbf{r}) - V(\mathbf{r}) = -\mathbf{B}\cdot\Delta\mathbf{r}$$

- Field **B** of a loop of current is given by Biot-Savart law:
  - This is the most basic relation in magnetics like Newton's law for gravity (figure on the right)

$$\mathbf{B}(\mathbf{r}) = C_m I \oint \frac{d\mathbf{r}' \times \hat{\mathbf{r}}''}{r''^2}$$

Constant C_m depends on the selected unit system:
In emu system: Dimensionless $C_m = 1$
In SI: $C_m = \mu_0/4\pi = 10^{-7}$ H/m



Magnetic potential

- Therefore, the change in the potential over $\Delta\mathbf{r}$:

$$\Delta V = -C_m I \Delta\mathbf{r}\cdot\oint \frac{d\mathbf{r}'\times(\mathbf{r}-\mathbf{r}')}{r^3} = -C_m I\,\frac{\Delta\mathbf{r}}{r^3}\cdot\oint \mathbf{r}'\times d\mathbf{r}'$$

The first equation here uses: $\hat{\mathbf{r}} \equiv \dfrac{\mathbf{r}}{r}$

The second equation assumes an infinitesimal loop $(r' \ll r)$ and uses $\oint d\mathbf{r}' = 0$

- The contour integral gives twice the area of the loop: $\oint \mathbf{r}'\times d\mathbf{r}' = 2\mathbf{n}\Delta s$
- Hence the change in V over distance $\Delta\mathbf{r}$:

$$\Delta V = -2C_m\frac{I\Delta s\mathbf{n}\cdot\Delta\mathbf{r}}{r^3} = -2C_m\frac{\mathbf{m}\cdot\Delta\mathbf{r}}{r^3} = C_m\Delta\left(\frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2}\right)$$

Magnetic potential and field $\mathbf{B}$

- Therefore, the magnetic potential equals:

$$V(\mathbf{r}) = C_m \frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2} + const$$

The *const* is usually taken zero, so that $V(\infty) = 0$

Note that the potential decreases along the direction of $\mathbf{m}$, and hence:

1. Near the axis of the dipole, $\mathbf{B}$ is generally oriented along $\mathbf{m}$ "in front" of it and opposite to $\mathbf{m}$ "behind" it
2. $\mathbf{B} = 0$ within the plane orthogonal to the dipole

- This is the potential of a dipole source $\mathbf{m}$
- Note two properties different from the monopole potential for gravity:
    1. The potential decreases with distance as $1/r^2$
    2. It also depends on the direction (zero within the plane orthogonal to $\mathbf{m}$ and opposite signs in two directions parallel and opposite to $\mathbf{m}$)

Poisson's relation (relation of magnetics to gravity)

- The dipole field ($V \propto 1/r^2$) equals a derivative of a "monopole field" ($V \propto 1/r$) with respect to the positions of the source ($\mathbf{r}_0$) or observation point ($\mathbf{r}$) :

$$V(\mathbf{r},\mathbf{r}_0) = -\left(\mathbf{n}\cdot\frac{\partial}{\partial\mathbf{r}_0}\right)C_m\frac{m}{|\mathbf{r}-\mathbf{r}_0|} = \left(\mathbf{n}\cdot\frac{\partial}{\partial\mathbf{r}}\right)C_m\frac{m}{|\mathbf{r}-\mathbf{r}_0|}$$

This is simply a spatial derivative in the direction of the magnetic moment, $\mathbf{m}$

- Therefore, the magnetic field of a structure with given direction of magnetization $\mathbf{n}$ can be obtained from the gravity field of the same structure by taking derivative in the direction of magnetization:

$$\mathbf{H}(\mathbf{r}) = \frac{C_m m}{G\rho}\left(\mathbf{n}\cdot\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{g}(\mathbf{r})$$

- This relation between gravity and magnetic fields is called the Poisson's relation

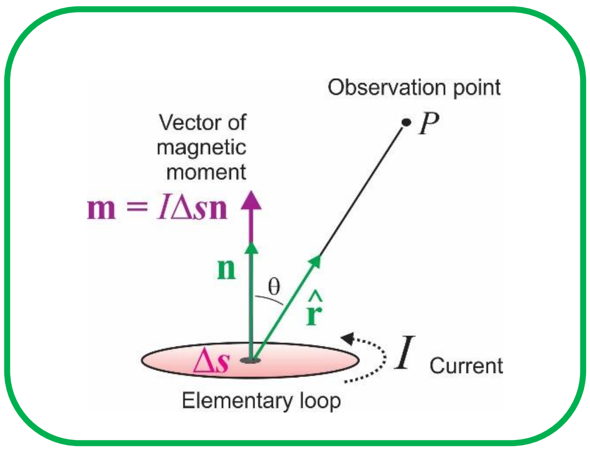
Magnetic monopoles

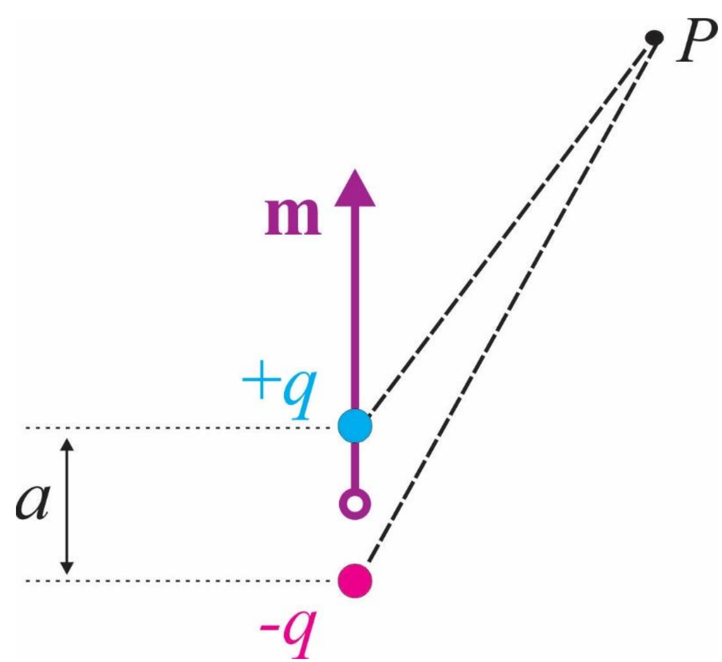
- The dipole field (from which we had started) can be presented as produced by two *monopoles* separated by an arbitrary (small) distance a:

$$V(\mathbf{r}) = V_1\left(q, \mathbf{r} - \frac{a}{2}\mathbf{n}\right) + V_1\left(-q, \mathbf{r} + \frac{a}{2}\mathbf{n}\right)$$

where $V_1(q, \mathbf{r}) = -C_m \dfrac{q}{r}$

and $q = \dfrac{m}{a}$ is called the *pole strength*





Magnetic field of uniform magnetization

- The magnetic potential of a single magnetic moment $\mathbf{m}$:

$$V(\mathbf{r}) = C_m \frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2}$$

- Consider magnetic moment $\mathbf{m}$ uniformly distributed in space with density $\mathbf{M}$:

$$\mathbf{m}_{\text{of a volume}} = \int \mathbf{M}\,dV$$

- Magnetic moment $\mathbf{M}$ within the medium (magnetization) is analogous to polarization $\mathbf{P}$ in electrostatics
  - It can be remanent (frozen in when the rock was formed) or induced by current geomagnetic or man-made fields

- Then, similarly to Gauss's theorem in gravity, factor $1/r^2$ gives a constant factor 4π after integration over all monopole sources within the volume.
- As a result, the potential simply decreases in the direction of $\mathbf{M}$:

$$V(\mathbf{r}) = -4\pi C_m \mathbf{M}\cdot\mathbf{r}$$

and therefore, magnetic field within a uniformly magnetized material is proportional to $\mathbf{M}$:

$$\mathbf{B}(\mathbf{r}) = 4\pi C_m \mathbf{M}$$

Is there an "excess mass" theorem for magnetization?

- Recall that in gravity, there is an Excess Mass theorem allowing estimation of the mass of an ore body by integrating the gravity measured on the surface:

$$M_e = \frac{1}{2\pi G}\int g_z dA$$

- So maybe a similar theorem can be used for estimating the total magnetization $\mathbf{M}$ of an ore body? The answer to this question is "No".
  - From the preceding slides, the magnetic (flux density) field $\mathbf{B}$ can be presented as "gravity" or "electric" field from magnetic-monopole "charges" $+q$ and $-q$ closely spaced together
  - By Gauss's law, an integral of $\mathbf{B}\cdot\mathbf{n}$ over a closed surface surrounding the target equals $4\pi C_m$ ·(**total charge within the enclosed volume**), but the total charge of all magnetic monopoles equals zero
- Thus, the integral of $\mathbf{B}\cdot\mathbf{n}$ (magnetic flux) over a surface enclosing a magnetic body is always zero
  - The total magnetization does affect the value of the surface integral of a magnetic anomaly, but this value is also influenced by other factors: the shape and depth of the body, relation to the ambient magnetic field, etc.

Magnetic induction

- Magnetic moments within the medium (remanent or induced) cause magnetic field (take SI units now, in which $C_m = \mu_0/4\pi$):

$$\mathbf{H}' = \mu_0 \mathbf{M}$$

- This field adds to the field $\mathbf{H}$ produced by free external currents (such as the Earth's dynamo). The sum is called the magnetic induction field $\mathbf{B}$. This field acts on our magnetometer:

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

- If the magnetization is *induced* by $\mathbf{H}$, then $\mathbf{M}$ is proportional to $\mathbf{H}$: $\mathbf{M} = \kappa \mathbf{H}$

  and then: $\mathbf{B} = \mu_0 \left(1 + \kappa\right) \mathbf{H} = \mu \mu_0 \mathbf{H}$

κ is called magnetic susceptibility

$\mu = 1 + \kappa$ is called (relative) magnetic permeability

$\mathbf{B}$ Is called the magnetic induction vector

Total field

- In field observations, the "total field" is typically measured and reported
- Total field is a scalar quantity – change in the magnitude of the magnetic field vector $\mathbf{B}$:

$$\Delta T = \left|\mathbf{B}_{\text{ambient}} + \Delta\mathbf{B}\right| - \left|\mathbf{B}_{\text{ambient}}\right|$$

- Total field ΔT can be understood as the projection of the anomaly field vector $\Delta\mathbf{B}$ onto the direction of the ambient (geomagnetic) field, $\hat{\mathbf{B}}_{\text{ambient}}$:

$$\Delta T = \sqrt{\left(\mathbf{B}_{\text{ambient}} + \Delta\mathbf{B}\right)^2} - \left|\mathbf{B}_{\text{ambient}}\right| \approx$$

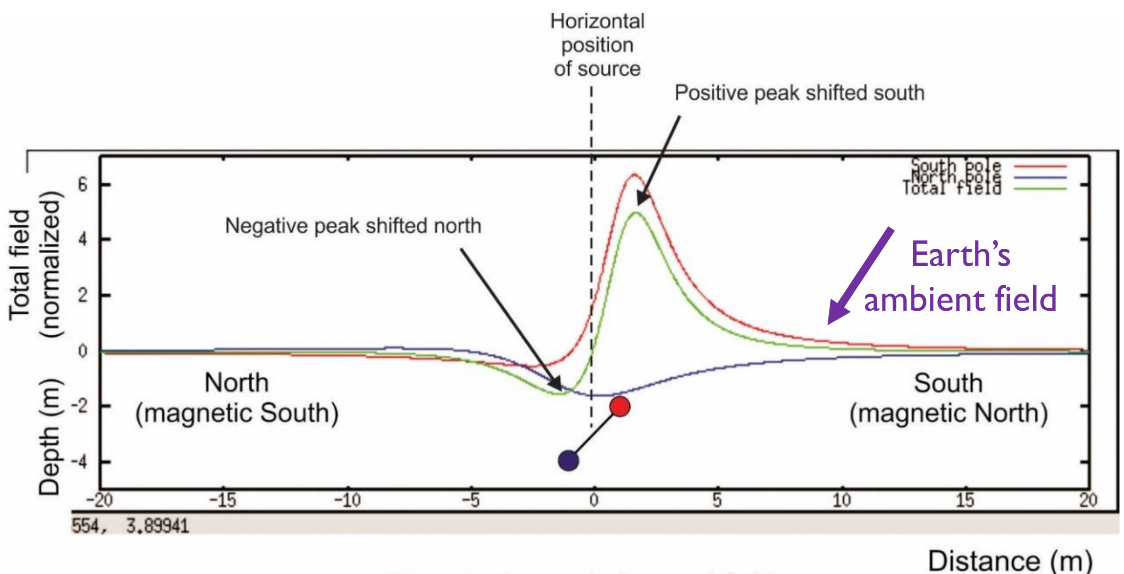
$$\approx \left|\mathbf{B}_{\text{ambient}}\right| \left(\sqrt{1 + \frac{2\mathbf{B}_{\text{ambient}}\Delta\mathbf{B}}{\left|\mathbf{B}_{\text{ambient}}\right|^2}} - 1 \right) \approx \left|\mathbf{B}_{\text{ambient}}\right| \left(1 + \frac{\mathbf{B}_{\text{ambient}}\Delta\mathbf{B}}{\left|\mathbf{B}_{\text{ambient}}\right|^2} - 1 \right) = \frac{\mathbf{B}_{\text{ambient}}}{\left|\mathbf{B}_{\text{ambient}}\right|}\Delta\mathbf{B}$$

This ratio is the unit vector $\hat{\mathbf{B}}_{\text{ambient}}$

- It can be shown that as a projection onto a constant direction, ΔT satisfies the Laplace (potential-field, see Introduction) equation
  - This means that ΔT is a potential field, like gravity potential
  - Therefore, it satisfies the Poisson's equation, which gives several ways for useful mathematical transformations of $\Delta T(\mathbf{x})$

Magnetic anomalies

- Compared to gravity, magnetic-field imaging contains an additional difficulty – the magnetization depends on the ambient magnetic field of the Earth, $\mathbf{B}_{\text{ambient}}$. This ambient field varies at different locations (principally with latitude)
  - Here is how an anomaly caused by a magnetized body looks like in northern hemisphere



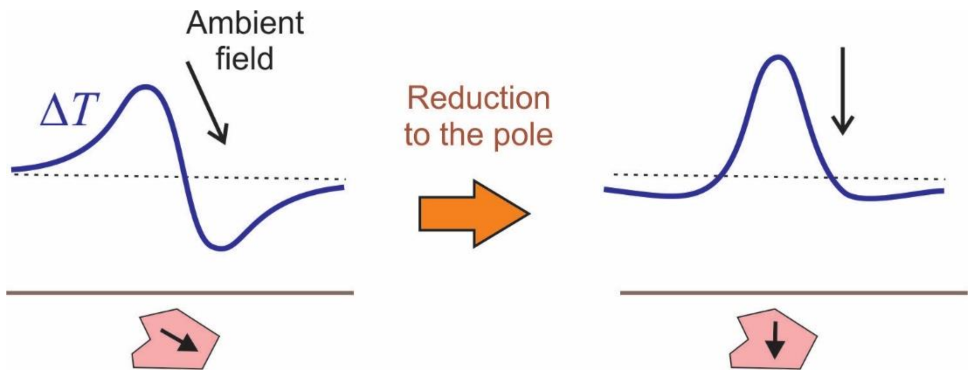
Green is the total observed field,
blue and red are its constituents due to the north and south poles

Note that the Earth's geographic North is also its magnetic South

- Two useful transformations of the magnetic field measured along a profile, $\Delta T(x)$, or a 2-D plane $\Delta T(x,y)$ are shown in the following slides

Reduction to the pole

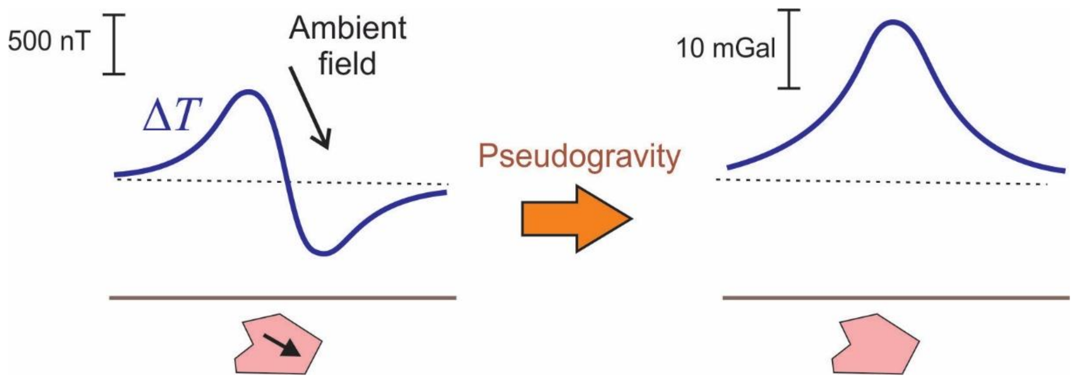
- Total-field magnetic anomalies are usually shifted in space because of the inclination of the ambient field
- The Reduction to the Pole operation transforms the total magnetic field anomaly measured in an inclined field, ΔT, into an anomaly that would be produced by the same structure with vertical magnetization and in a vertical ambient field.
  - As if recording at the geographic North pole
  - Details will be given in Geol480



- Benefits:
  - The reduced anomaly is symmetric and centered over the source
  - The width of the anomaly corresponds to the source depth more directly
  - Simpler interpretation process

Pseudo-gravity

- Pseudogravity transforms the total magnetic field anomaly, ΔT, into a gravity anomaly that would be produced by the same structure
  - Also assuming uniform density and magnetization



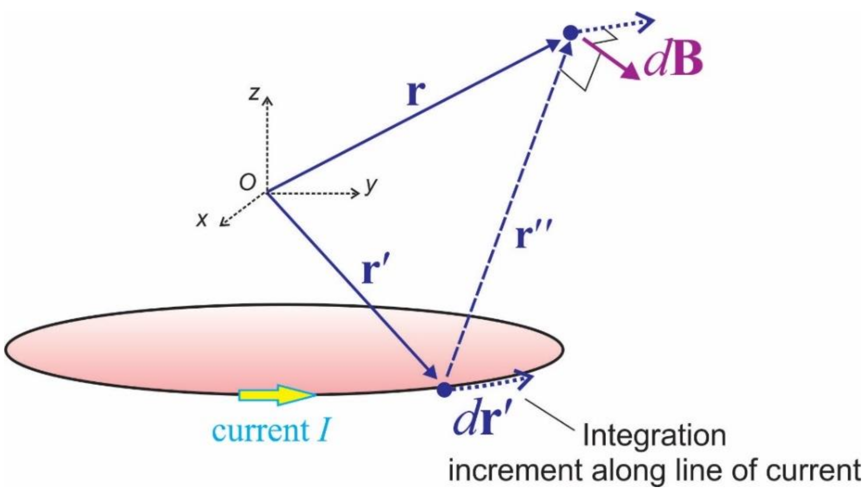
- Benefits:
  - Gravity anomalies are often easier to interpret and to estimate depth
  - Gravity anomalies are more symmetric, peaks centered over the source
  - Some structures (e.g., mafic plutons) produce both gravity and magnetic anomalies, and so they can be compared directly
  - Tubular structures can be identified by maximum horizontal gradients

Magnetic field of a large-scale current

- Up to this point, we were talking about magnetic field measured at some distance from a small loop of current (magnetic dipole, magnetized body)
- Now, assume that the current occupies a large space, and we are measuring $\mathbf{B}$ at a proximity to the loop (maybe inside the loop)
  - Then the dipole representation is not so convenient
- If the current can be broken down to line sources (like current in wires), then the magnetic field is given by the Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \mu\mu_0 I \oint \frac{d\mathbf{r}' \times \hat{\mathbf{r}}''}{r''^2}$$

  - See figure showing elementary field $d\mathbf{B}$ produced by a segment of current $d\mathbf{r}'$:
- Orientation of field $d\mathbf{B}$ (orthogonal to both current and $\mathbf{r}''$) is often memorized by mnemonic rules
  - Called "Right hand", "thumb", "corkscrew", etc.
  - See Lab #11



Circuital relations for current

- The magnetic field in a vicinity of a current is easy to obtain from circuital relations
  - This is analogous to the Gauss theorem in gravity and electrostatics
- Consider a closed circuit C bounding some surface S. Then:
  - For **B** field and total current crossing surface S (Ampere's law)

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \iint_S \mathbf{j}_{\text{total}} \cdot d\mathbf{s} = \mu_0 I_{\text{total enclosed by C}}$$

  - For **H** field and the free current

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \mu_0 \iint_S \mathbf{j}_{\text{free}} \cdot d\mathbf{s} = I_{\text{free enclosed by C}}$$

Magnetic field of a line source of current

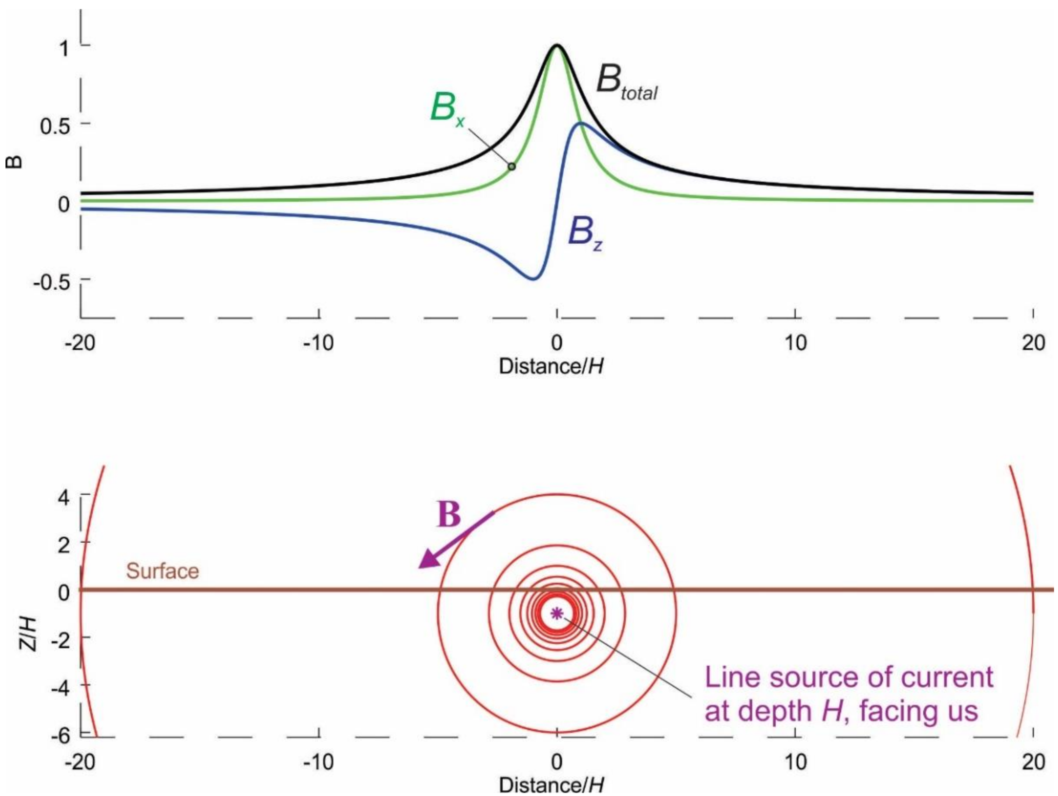
- A simple and useful model of large-scale current is a straight line of current. In this case, the Biot-Savart integral gives field $\mathbf{B}$, which is:
  - Orthogonal to both the line of current and radius $\mathbf{r}$ connecting the line to the observation point
  - Directed clockwise if viewing in the direction of the current
  - With magnitude proportional to current I and inversely proportional to distance r from the line of current:

$$\left|\mathbf{B}(r)\right| = \frac{\mu\mu_0 I}{2\pi r}$$

- This line-source model is often used to interpret VLF profiles (see Lab #11)

Magnetic anomalies of a line source of current

- A line source underground (and similarly above ground) creates characteristic shapes of anomalies:
  - Symmetric for B_x
  - Antisymmetric for B_z
  - Symmetric and wider for total magnitude of vector $\mathbf{B}$ field



See Lab #11 for more detail

Field surveying - airborne

- Large areas covered with aircraft-based surveying (airplanes, helicopters, drones)
- Using multiple magnetometers, magnetic gradiometry (measuring gradients of the magnetic field) is performed

Sander Geophysics' Magnetic Gradiometer System

Sander Cessna Caravan system

Field surveying - aeromagnetic

Leveling aeromagnetic surveys

60 surveys in Alaska:

- unlevelled

- after local levelling corrections

- after final levelling corrections, including long-wavelength corrections (IGRF)