Source estimation

This lecture focuses on quantitative characterization of sources of either gravity or magnetic (i.e., potential-field) anomalies measured in the field.

The key points are:

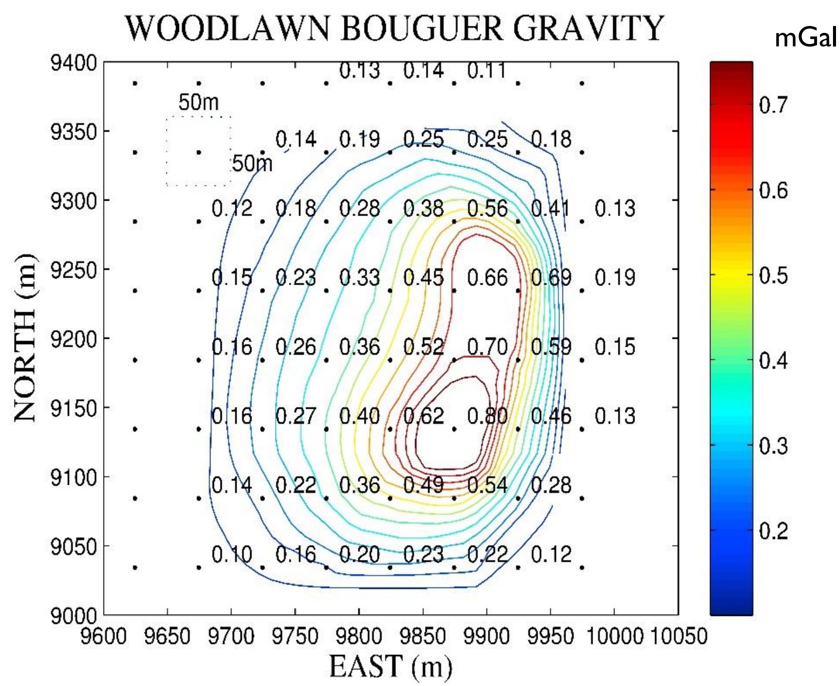
- Source characterization
  - Location
  - Shape and orientation
  - Depth
- Depth estimation
- Excess mass and total mass (for gravity)

- Reading:
  - Dentith and Mudge, Sections 3.10 and 3.11

Source characterization

- Here is an example of a gravity anomaly from your lab #3. The anomaly is produced by a sulphide deposit in Australia.
  - Note that "anomaly" means a localized disturbance of the gravity field which tapers out to zero outside of the anomaly area
  - This means proper corrections should be applied to the data (latitude, free-air, Bouguer, separation of the regional trend, etc.)

- What can we say about the source mass causing this anomaly?
  - Its depth?
  - Its shape?
  - Its size (mass)?
- With respect to these questions, gravity and magnetic anomalies are similar, and we discuss them together
- We will consider these questions in the following slides and lab #3



Source depth ambiguity

- Horizontal positions of gravity and magnetic sources are generally well constrained by the locations and shapes of the anomalies. What about the depths?
  - Generally, the depth is constrained by the lateral (horizontal) dimensions of observed anomalies
- Unfortunately, there exists a fundamental ambiguity of gravity with respect to the depth distribution of mass
  - In principle, for any shape of recorded gravity field $g(x,y)$, masses can be located at any depth below the observation surface and above certain maximum depth
    - In particular, from our discussions of Bouguer slab (thin sheet) in the preceding lecture, the entire mass anomaly can a located immediately below the surface, in a thin layer of surface density

$$\sigma(x,y) = \frac{g(x,y)}{2\pi G}$$

    - This layer is called the "Green's equivalent layer (stratum)". This layer would explain the entire field on and above the survey surface
- Thus, gravity models are inherently nonunique
  - All depth estimates give the maximum possible depth for the source
  - Smooth (regional) variations within gravity or magnetic models can have sources at any depth
    - This is why regional trends need to be removed by corrections
- All inversions for the source look for "the simplest" models which appear to be geologically "likely". These models:
  - Explain the key, localized and most pronounced gravity anomalies at largest depths
  - Try explaining the anomalies by a small number of compact sources, or by higher density/magnetization contrasts

Gravity and magnetic source depth estimation

- Several (broadly related) methods are available
- General rules for method selection:
  1. Try more than one method
  2. Most method give the maximum source depth. Therefore, the shallowest estimate is likely the best
  3. Small isolated features in images are the most reliable. (Overlapping larger features are subject to ambiguity in the preceding slide)

Principles for estimating the depth to the source

- As discussed on the preceding slide, we are looking for localized sources at depth
- Basically, fora localized gravity source, its depth is expressed by the variations of the anomaly (Δg) in the horizontal (X,Y) directions. Thus, we need to identify the characteristic sizes of variations of $\Delta g(x)$ or $\Delta g(y)$.
- There are four key approaches to estimating the depth to the source (H). Note that basically, we can use any measure of horizontal scale of the pattern of Δg:
  1) Depth is proportional to some measure of the lateral extent w of the anomaly: $H = Fw$

     The question is how to measure this w conveniently
  2) Depth is inversely proportional to the derivatives of $\log(\Delta g)$:

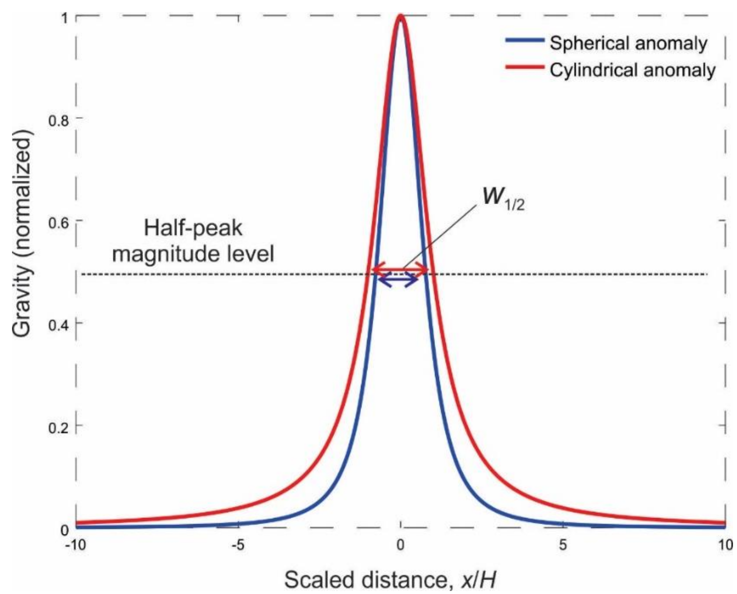
     $$H = F\frac{1}{d(\log \Delta g)/dx} \quad \text{or} \quad H = F\sqrt{\frac{1}{d^2(\log \Delta g)/dx^2}}$$
  3) Depth is inversely proportional to the dominant wavenumber k_D (spatial, or "radial" frequency): $H = F\frac{1}{k_D}$
  4) Depth is proportional to the roll-off of spectral amplitudes: $H = F\frac{d(\log \Delta g)}{dk}$

F is the "form factor" or "shape factor" depending on the shape of the source

- Below, we will only consider approaches 1) and 2) above (commonly used in manual interpretation)
  - Criteria 1) and 2) are often combined in graphical template-based rules

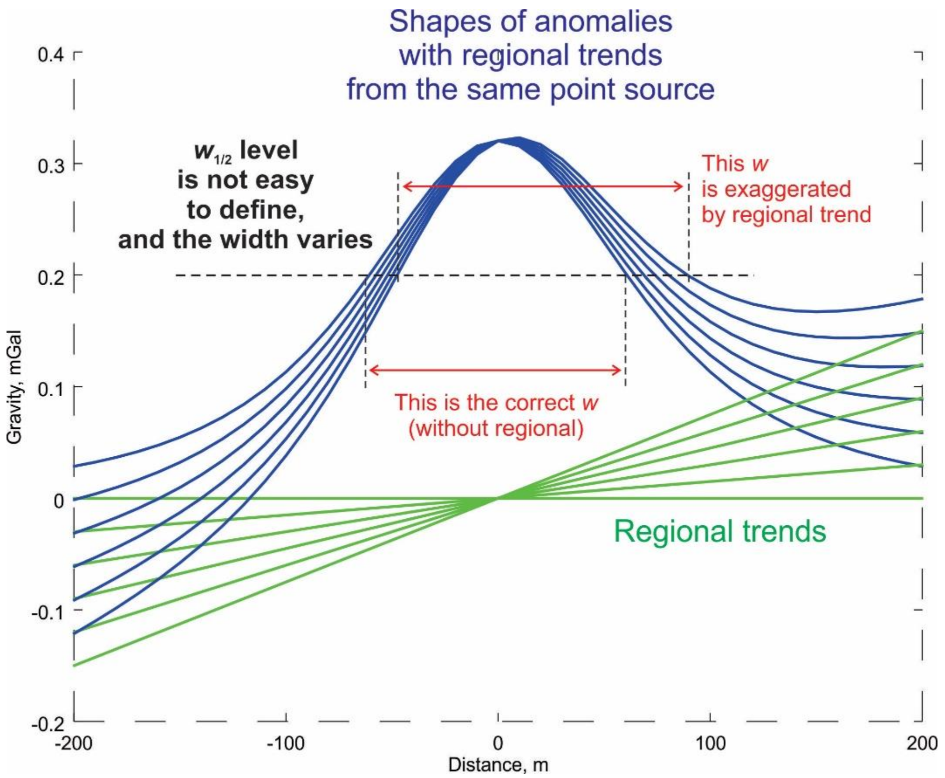
Anomaly-width based method #1: Width at half magnitude

- The depth H to the source of an anomaly is estimated from the width of the anomaly at half-peak magnitude (anomaly height), $w_{1/2}$ (see Figure below):
  - For a spherical source, you can find that: $w_{1/2} = \left(2\sqrt{2^{2/3}-1}\right)H \approx \frac{H}{0.65}$ and therefore $H \approx 0.65 w_{1/2}$.
    - If the line does not pass immediately over the source, *h* would be closer to the total distance to the source
  - For a profile across a cylindrical source: $w_{1/2} = 2H$, and therefore $H \approx 0.5 w_{1/2}$.



The $w_{1/2}$ method and regional trend

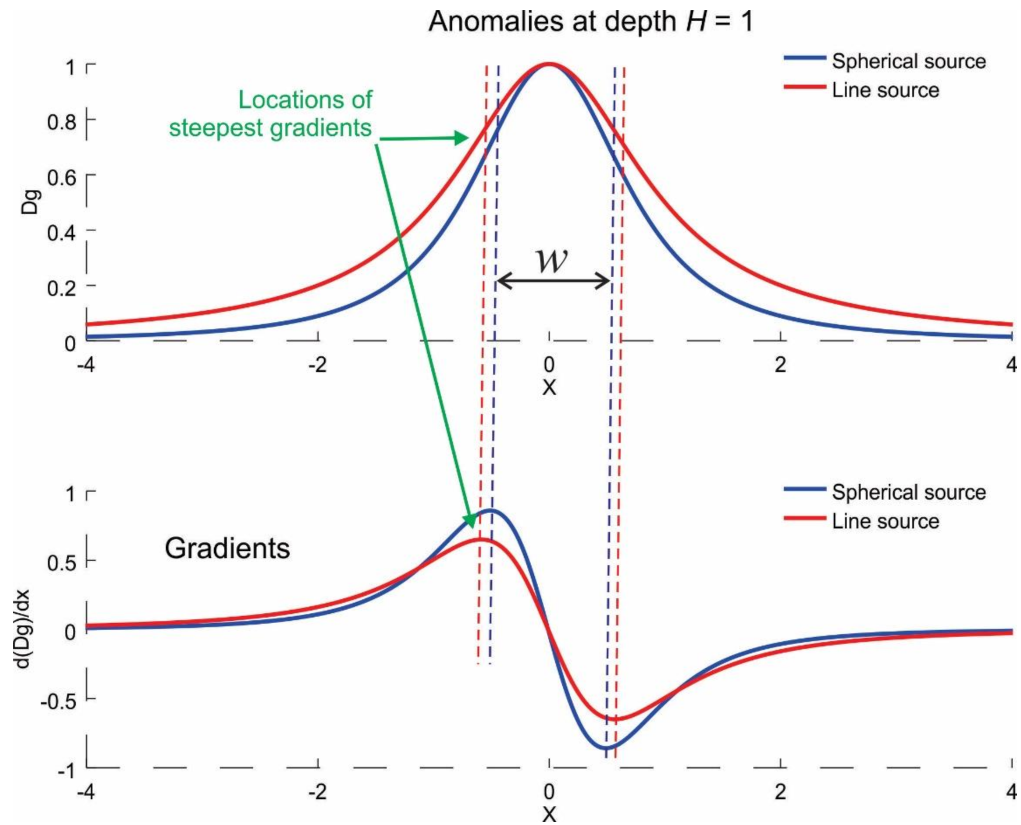
- To use this depth-estimation method, the regional trend should be carefully removed
  - The trend increases the width of the peak and complicates finding the "half-magnitude" level. See this Figure:



Width-based method #2: Width between steepest gradients

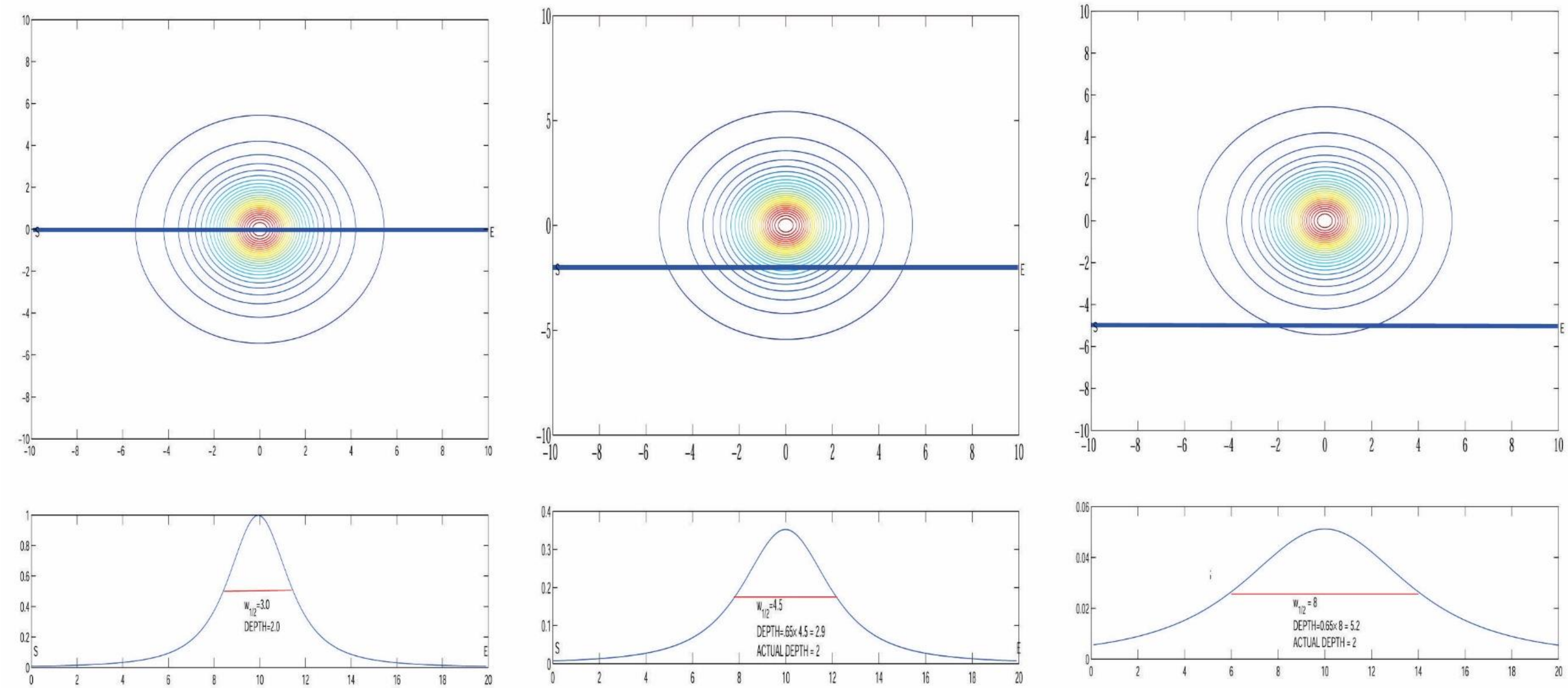
- If w is measured between the points of steepest gradient $d\Delta g/dx$, then for a spherical anomaly: $w = H$ (formfactor $F = 1$) (blue lines in Figure below)
  - For a line source (rod, pipe) across strike: $F = \frac{2}{\sqrt{3}} \approx 1.15$ (red lines)

- Steepest-gradient points are more difficult to eyeball, but this estimate of w is practically unaffected by the regional trend
  - The trend only adds a constant to the gradients



Dependence on placement of gravity profile

- When estimating depth from a 2D profile, make sure it passes over the top of the target mass
  - If the mass is located away from the plane of cross-section, the "depth" will be closer to the total distance to the source (compare these plots)



Profile passes through the peak of a point-source anomaly
Depth estimate based on anomaly half-width is correct

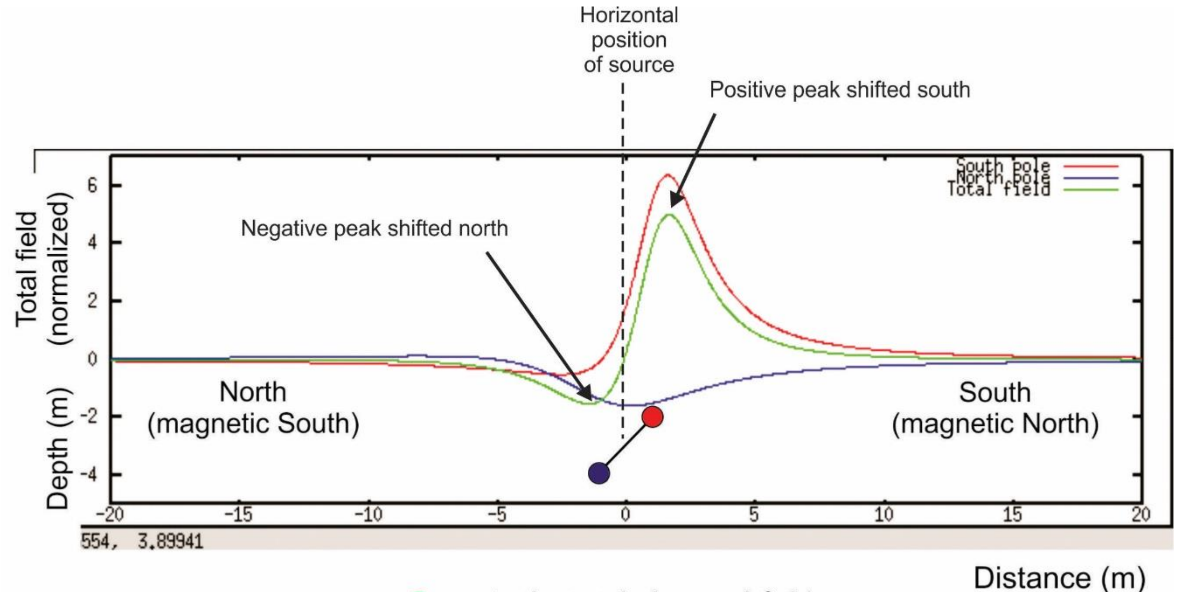
Profile at about half-magnitude of the anomaly
Depth over-estimated by 45%

Profile misses the anomaly
Depth over-estimated by more than a factor of 2

Graphical methods for horizontal position of magnetic source

- For magnetic field, the source is dipole (we will discuss this later)
  - This source can be represented as a combination of the south- (red in the figure) and north-polarity (blue) sources separated laterally and in depth

- The general rule for finding the position of this source dipole is to look for the position of its shallower (south in the northern hemisphere) pole
  - Note that because of the effect of the other pole, the measured positive high (green line in this figure) over this pole is shifted to the south
- The graphical methods shown in the next slides try correcting for this shift



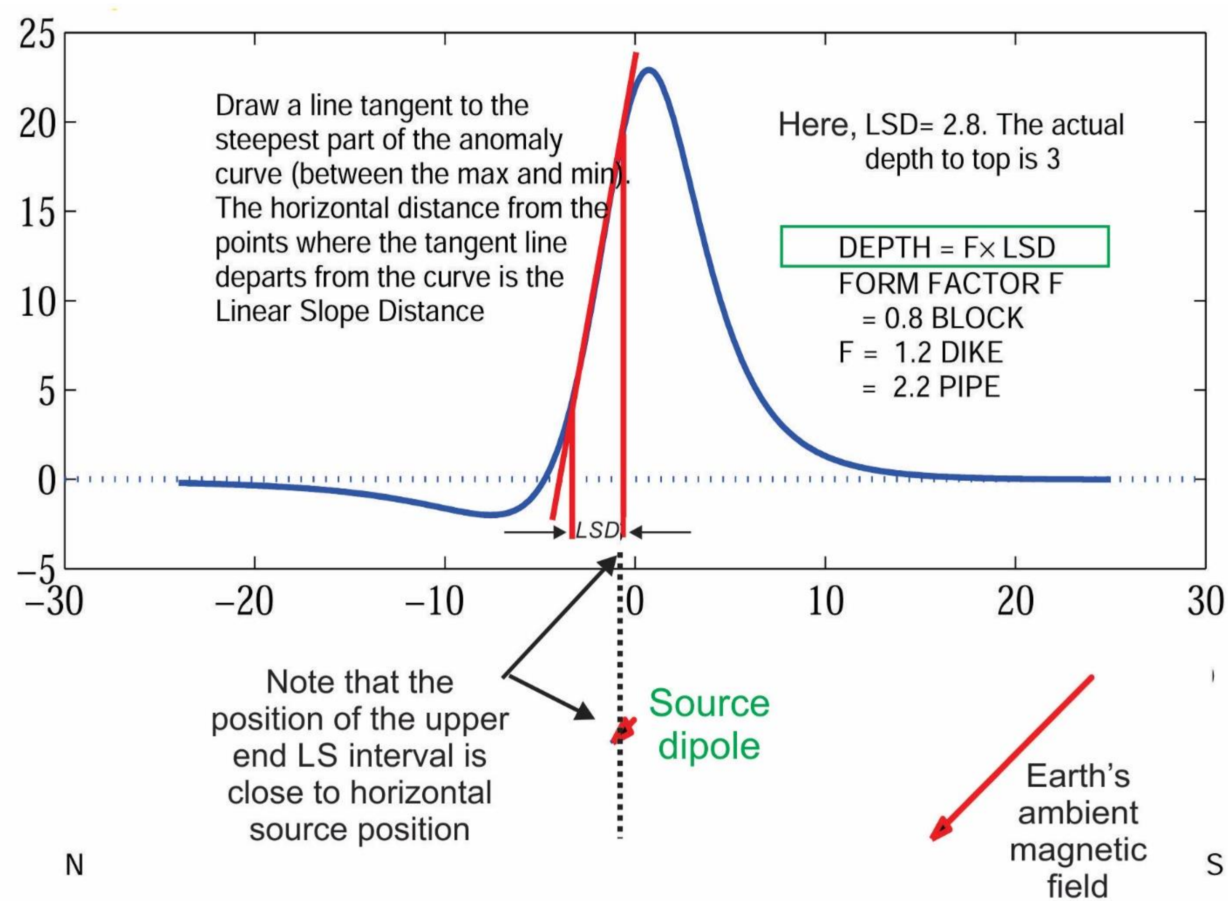
Green is the total observed field,
blue and red are its constituents due to the north and south poles

"Linear slope distance" method for depth and position

- This method utilizes the interval of near-linear and steepest slope between the negative and positive anomaly highs
- The horizontal "linear-slope distance" (LSD) is measured, and depth is estimated as

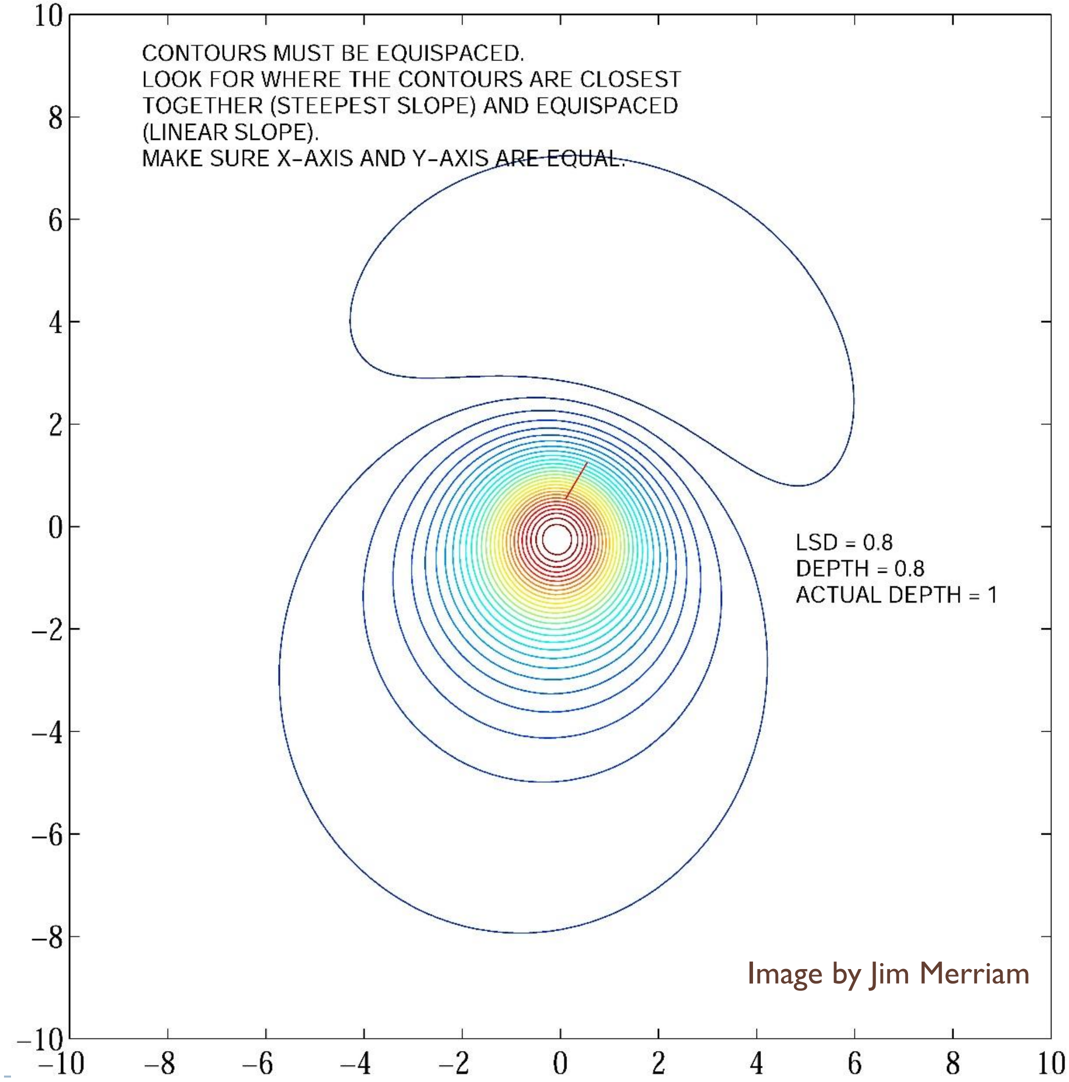
$$H = F \times LSD$$

where the form factor F depends on the shape of the source body (see figure)



Linear slope distance method on contour map

- The LSD method is simple and convenient on equispaced contour maps (see this figure)

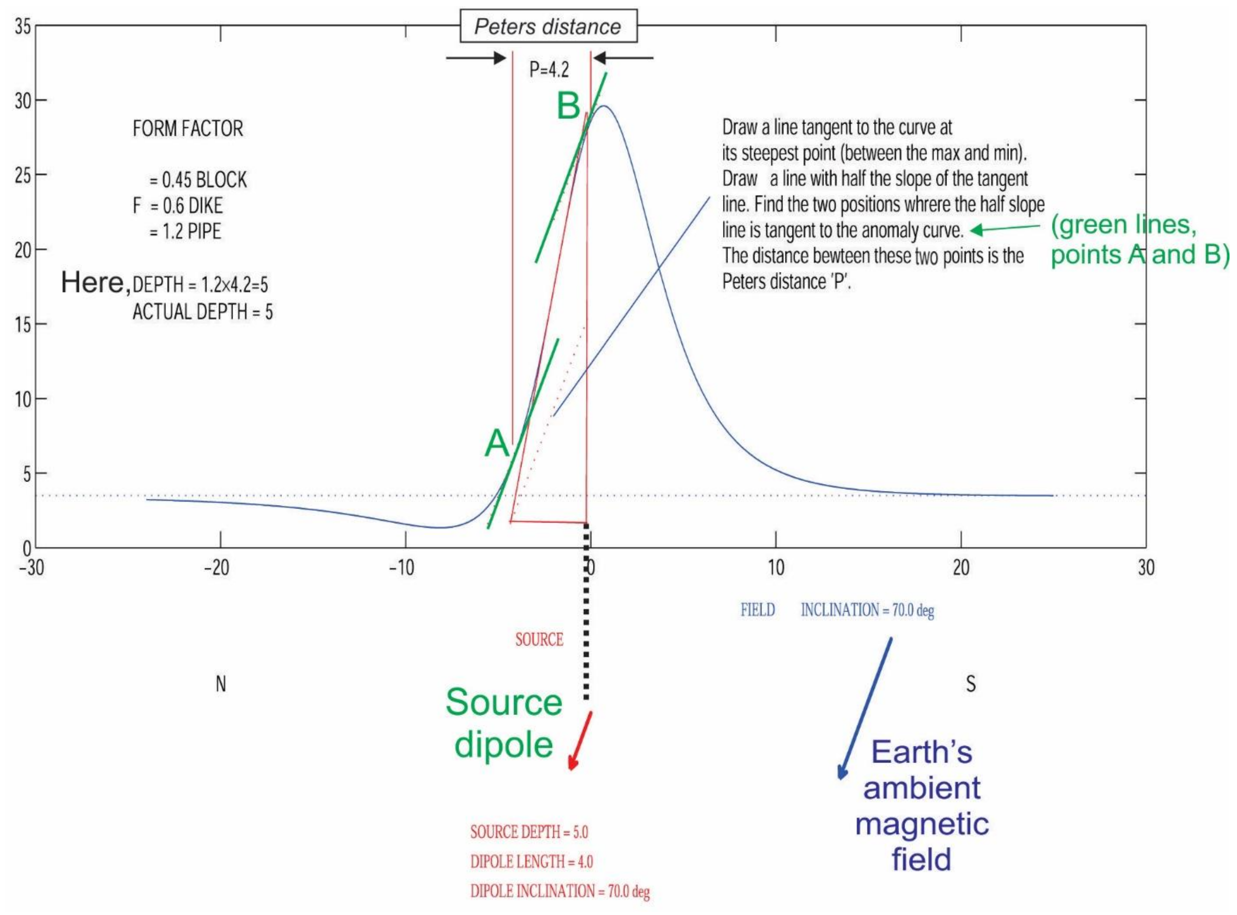


Peters' method for depth and position

- In Peters' method, the characteristic distance P is determined by drawing two parallel tangents at half of the largest slope
- The depth estimate is similar:

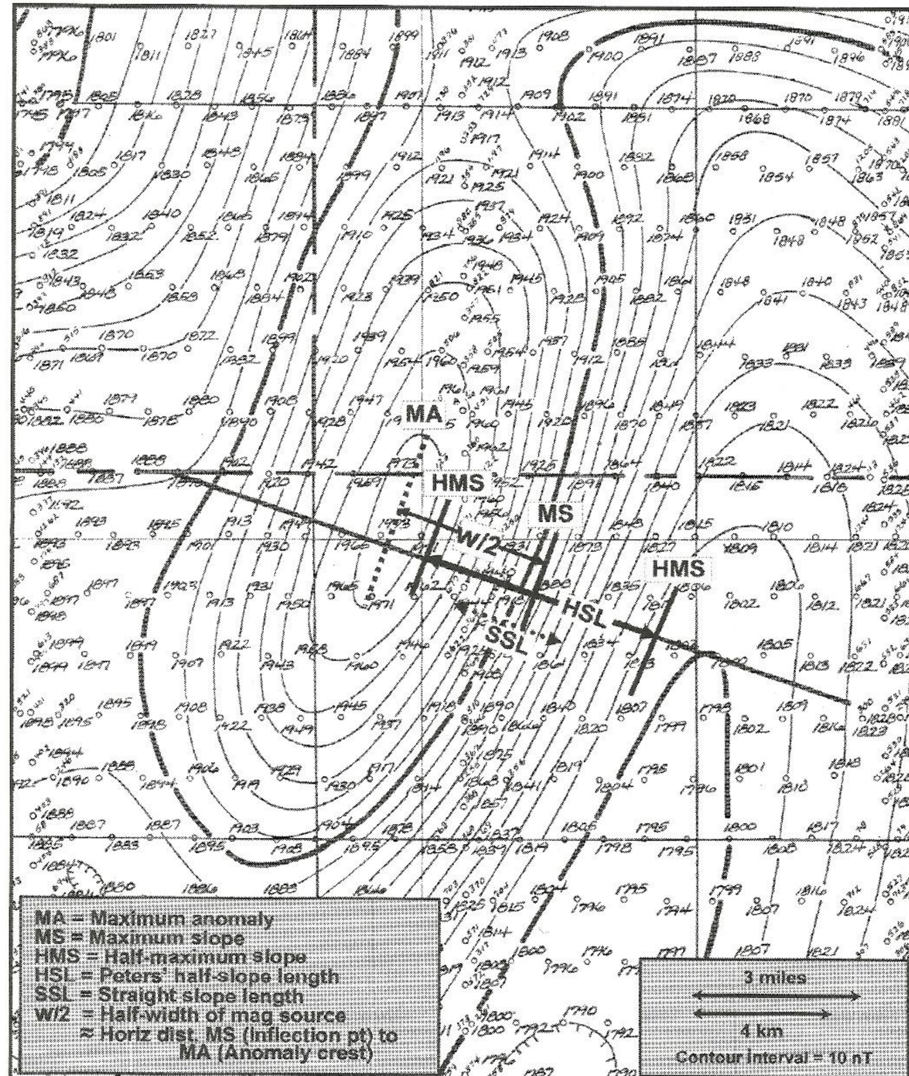
$$H = F \times P$$

with a somewhat different form factor F (see figure)



Peters' method on contour map

- On a contour map, this would mean finding intervals with double contour spacings (HMS in this figure)



Sokolov's method for depth and position

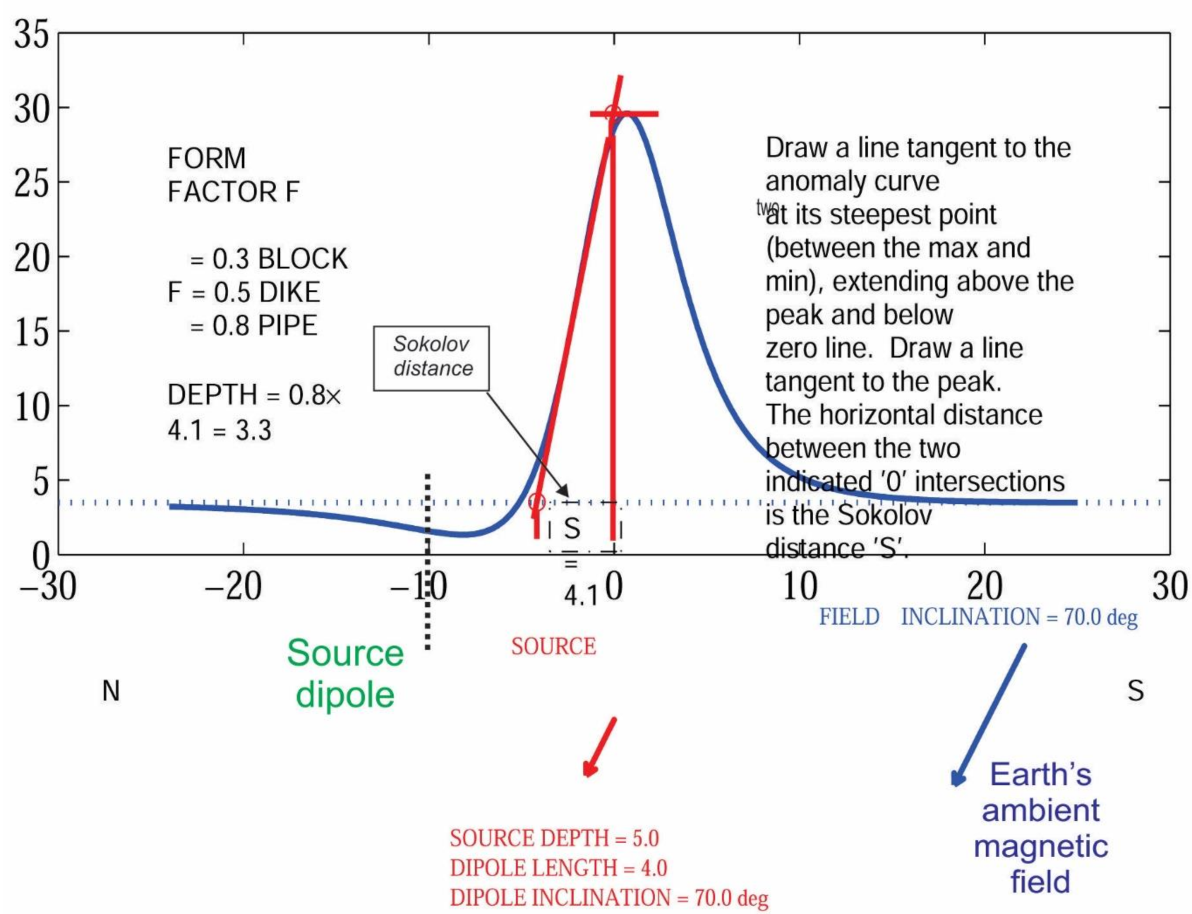
- In Sokolov's method, the characteristic size of the anomaly S (for field denoted $T(x)$) is simply:

$$S = \frac{\max(T)}{\max\left(\frac{dT}{dx}\right)}$$

- Graphically, this is shown in the figure
- The depth estimate is similar:

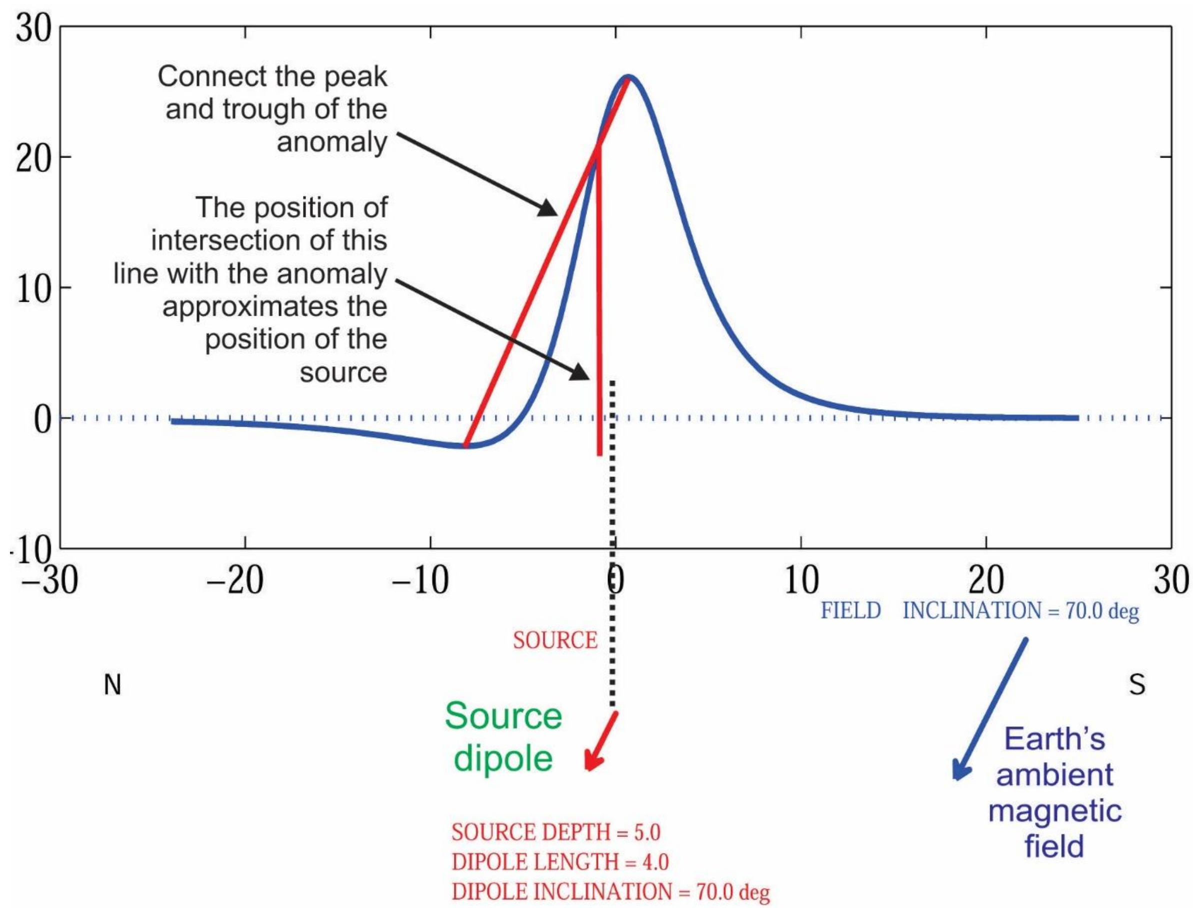
$$H = F \times S$$

with again a somewhat different form factor F (see figure)



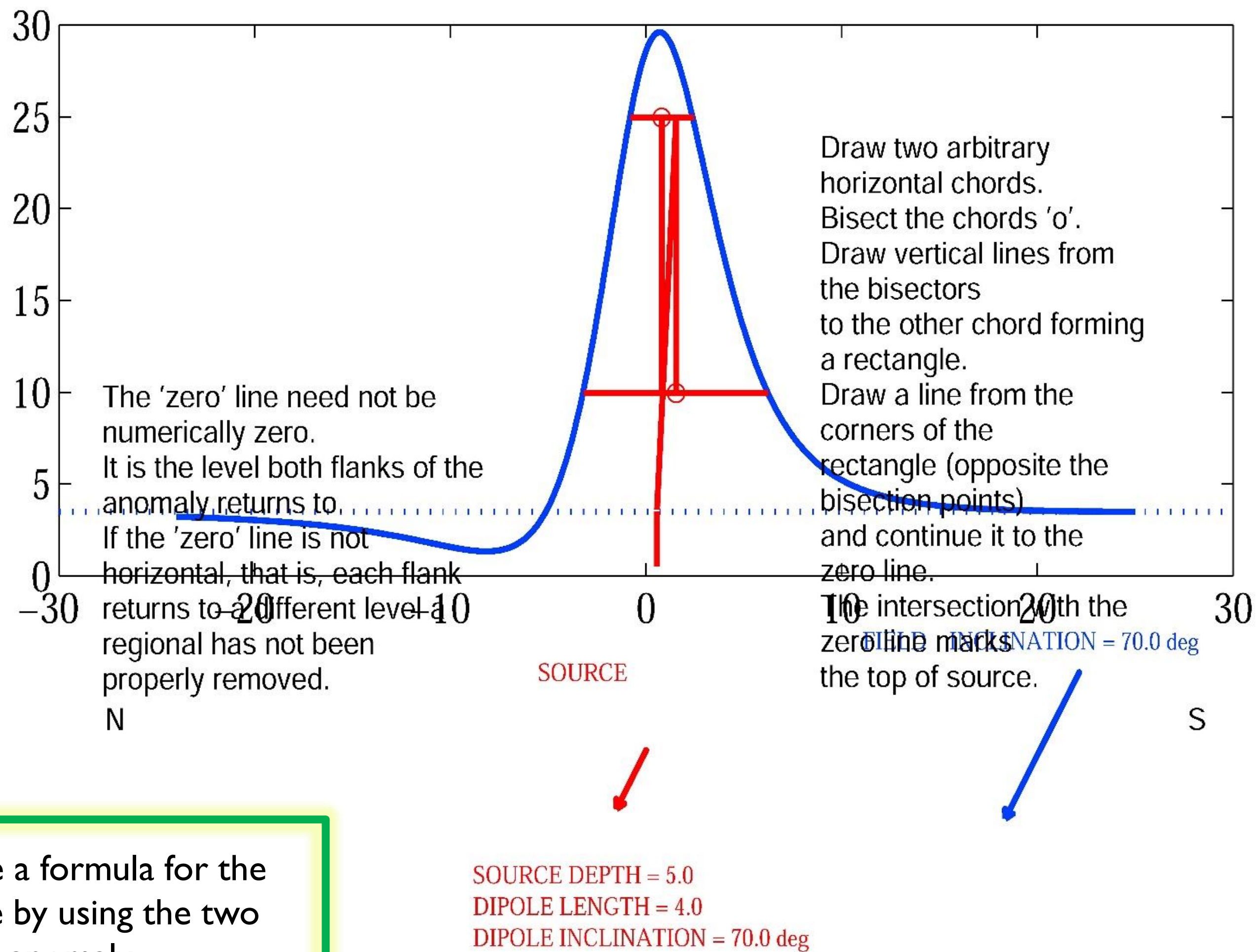
E-line method for position of the top of the source

- In the following methods, only the horizontal position of the top of the source (the shallower pole) is located
- In "E-line" method, the source is located as shown in the figure



Werner's first method for the top of the source

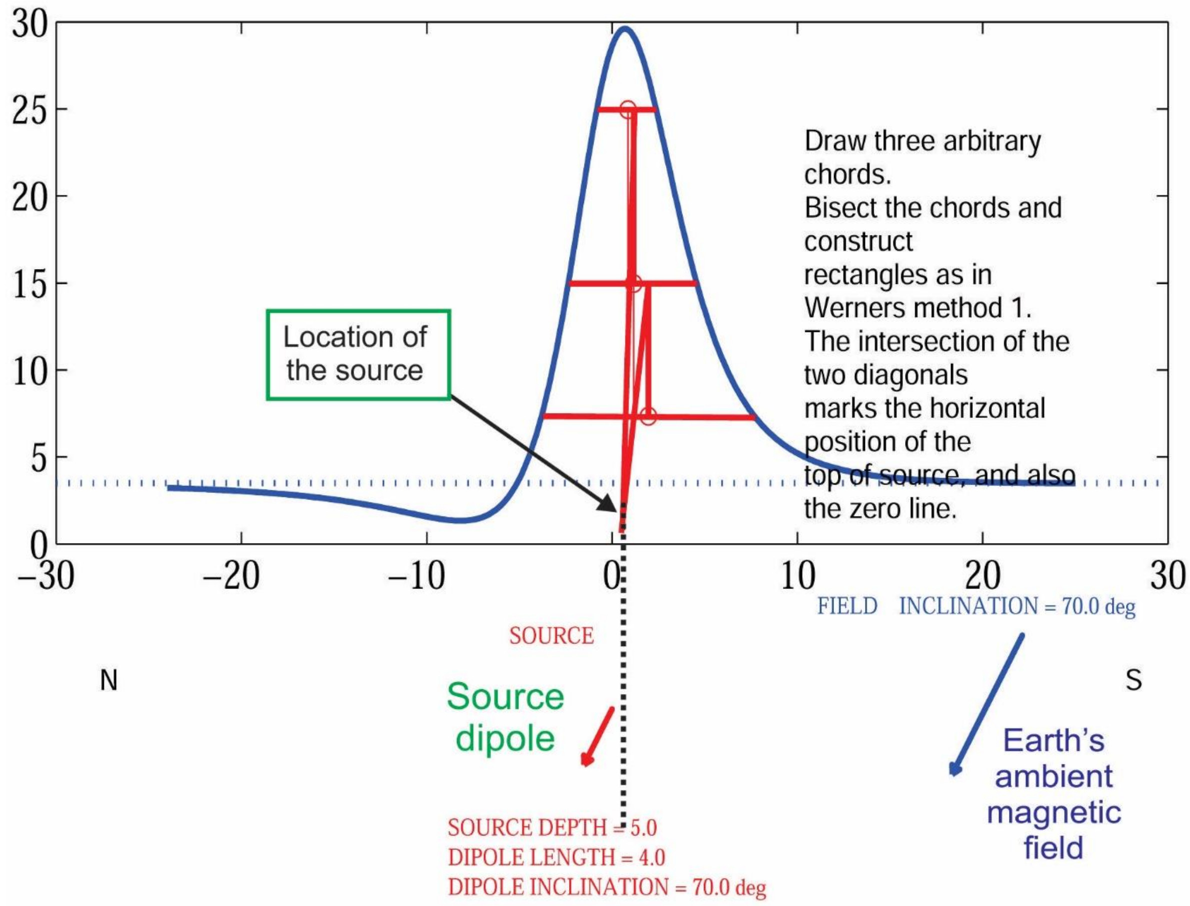
- In the first method by Werner, the source is located by bisecting the intervals of two arbitrary levels of the anomaly (figure)
- This method relies on the "center line" (level of the field in the absence of the anomaly)



Exercise for GEOL334: write a formula for the predicted position of the source by using the two levels and ranges of the anomaly

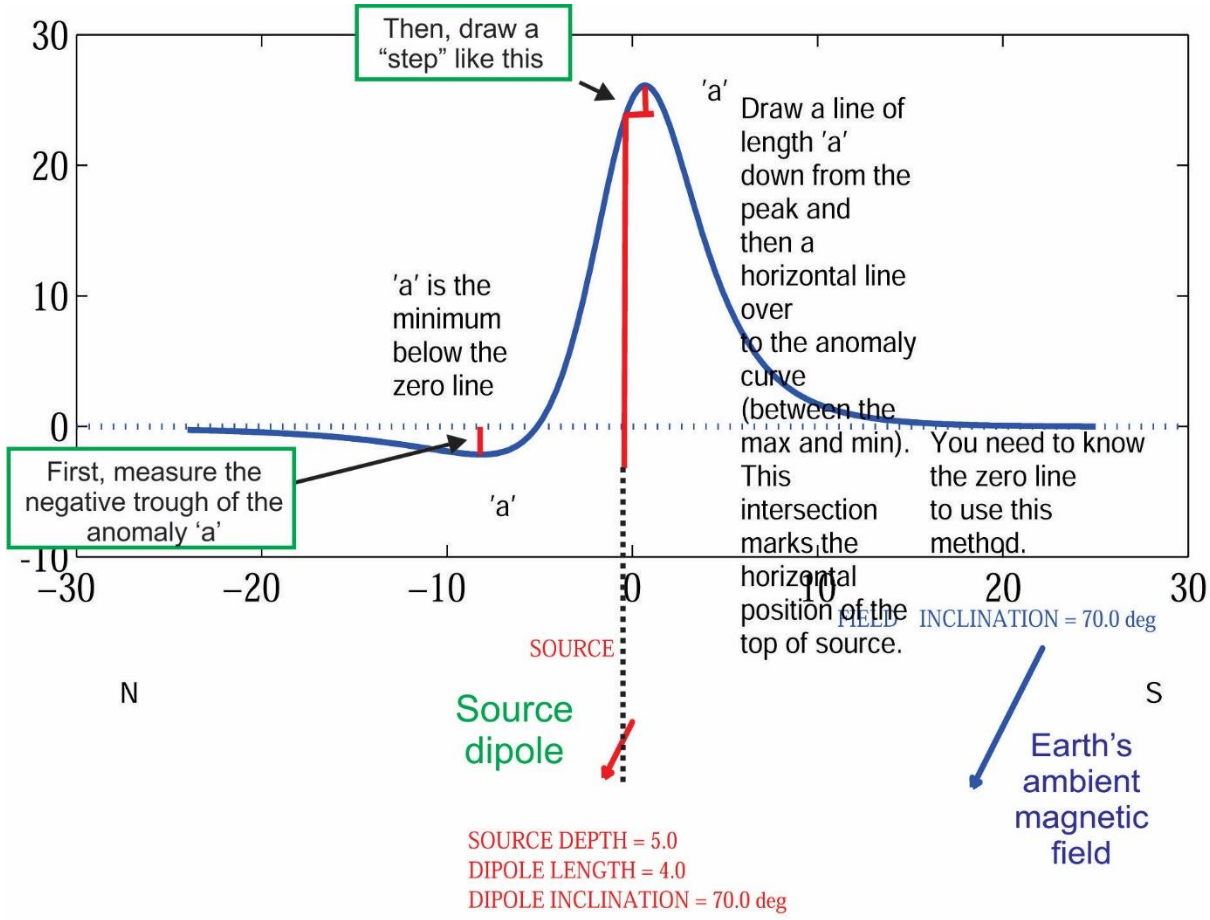
Werner's second method for the top of the source

- The second Werner's method works without the center line (it estimates the center line by using TWO pairs of chords)



Logachev's method for the top of the source

- Logachev's method also needs knowledge of the center line
- If the center line is set at contour level $T = 0$, the position of the source is found by stepping from the positive peak by the contour interval equal the level of the negative peak

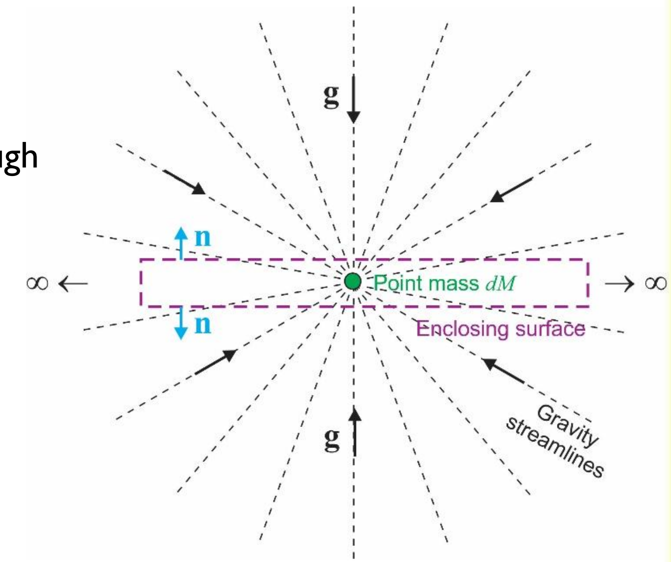


Excess mass

- After (and if) all corrections are performed satisfactorily, we obtain a localized "gravity anomaly" which is due to some excess mass M_e located within the subsurface. M_e measures the density in excess of the uniform background used as Bouguer density
  - If the anomaly $\Delta g > 0$, M_e is positive (a denser body compared to host rock background)
  - If $\Delta g < 0$, M_e is negative (a less dense body or cavity within host rock)
- Similar to most other key relations, the excess mass follows from Gauss's law. Its relation is similar to Bouguer gravity formula: $g = 2\pi G\sigma$, where σ is the mass per unit area below the surface.
- If we consider a point mass dM enclosed between two infinite planes (figure on the right), Gauss's law tells us that the total flux of **g** through the two planes equals

$$\int \mathbf{gn}dA = -4\pi G dM$$

where **n** is the normal unit vector to the surfaces, and dA is an element of the area



- Since a half of the streamlines crosses each of the two surfaces, the flux of **g** through each of them equals

$$\int \mathbf{gn}dA = -2\pi G dM$$

- Therefore, for the observation surface (the upper one): $\int \mathbf{gn}dA = -2\pi G dM$

Excess mass and Total mass

- Thus, the integral of the gravity g_z over the entire observation surface is proportional to the mass below the surface:

$$\int \mathbf{gn}dA = -\int g_z dA = -2\pi G dM$$

- For a distributed mass, $M_e = \int dM$, and the relation to the surface integral of measured gravity is the same:

$$\int g_z dA = 2\pi G M_e$$

- Therefore, M_e can be determined by integrating the gravity anomaly (see lab #3)

$$M_e = \frac{1}{2\pi G}\int g_z dA$$

- In these relations, M_e is the "excess mass" (difference from the host-rock background): $M_e = \Delta\rho V$. The "total mass" (what you would dig out of the ground) is $M_t = (\rho + \Delta\rho)V$

  where ρ is the density of host rock, $\Delta\rho$ is the density contrast, and V is the volume

- Therefore, the total mass of the deposit is estimated from measured (and Bouguer and terrain-corrected) gravity as:

$$M_t = M_e \frac{(\rho + \Delta\rho)}{\Delta\rho} = \frac{1}{2\pi G}\left(1 + \frac{\rho}{\Delta\rho}\right)\int g_z dA$$