GEOL 384.3 and GEOL 334.3

Lab #10: Skin depth

In this lab, you will measure the electromagnetic (EM) wave skin depth in aluminium and also calculate it for Saskatchewan tills. The skin depth is a measure of how the amplitude of an oscillating electromagnetic field decreases with distance into a conducting body. All non-static methods used in geophysics, e. g VLF, MT, Slingram, are subject to this phenomenon.

The skin depth phenomenon is the result of the induction of electric currents in a conducting body. The conductor re-radiates some of the EM wave as reflected or scattered waves and absorbs some of the energy as heat through Ohmic losses. As a result, the EM wave amplitude decreases exponentially with distance into the interior of the conductor:

$$A(z) = A(0)\exp\left(-\frac{z}{\delta}\right), \tag{1}$$

where $A(0)$ is the amplitude at the surface of the conductor, $A(z)$ is the amplitude at distance z into the conductor, and parameter <u>δ is called the skin depth</u>. Thus, the <u>amplitude decreases by a factor of $e^{-1} \approx 0.367$</u> at one skin depth into the conductor, by $e^{-2} \approx 0.135$ at two skin depths, and so on in a geometric progression. The <u>phase</u> of the oscillation is also progressively altered with depth, <u>by about 45° per one skin depth</u> relative to the change in phase that would happen if the conductor were not present.

The skin depth δ is a function of frequency and ground conductivity, and it limits the distance by which a given piece of equipment will be able to 'see' into the Earth at a given frequency. Its relation to physical properties of the material is

$$\delta = \sqrt{\frac{2}{\mu\mu_0\sigma\omega}} = \sqrt{\frac{1}{\pi\mu\mu_0\sigma f}}, \tag{2}$$

where $\mu_0 = 4\pi\cdot10^{-7}$ H/m is the constant needed for the SI units, μ is the magnetic permeability, $\sigma = 1/\rho$ is the conductivity in S/m, ρ is the resistivity in $\Omega\cdot$m , $\omega = 2\pi f$ is the angular frequency, and f is the ordinary frequency in Hz. For non-magnetic materials like aluminium ($\mu \approx 1$), this is approximately

$$\delta = 500\sqrt{\frac{\rho}{f}}\ [\mathrm{m}]. \tag{3}$$

In the above relations, the skin depth is inversely proportional to the square root of frequency, and so the skin depth may be quite large for low frequencies but small for high frequencies. At characteristic frequencies used in geophysical measurements:

- At 1 Hz (frequencies used in 'DC' resistivity measurements), the skin depth in till is more than a kilometer – much thicker than the till.
- At VLF frequencies (about 10 kHz), the skin depth in till might be only ten or fifteen meters – thinner than the till in most places in southern Saskatchewan.

- At Ground-Penetrating Radar (GPR) frequencies (10–100 MHz), the skin depth in Saskatchewan tills is only 0.2–0.7 m.

Therefore, at 10 kHz (VLF), the till behaves like an infinitely thick layer, whereas at 1 Hz (in a resistivity measurement), the till looks thin to an array with large AB distance. For GPR, the till is almost impermeable and presents difficulties for imaging.

Frequency sounding vs. geometric sounding

The relation of frequency to the thickness of the skin layer differentiates between two types of sounding: "geometric" and frequency". Recall that in DC resistivity measurements (low frequency, δ much greater that the target depth), the frequency is insignificant, and we measure conductivity at different depths by varying the AB separation. This approach to controlling the sampling depth is called the geometric sounding. By contrast, at higher frequencies, the sampling depth is determined by the skin depth δ. By varying the frequency, we can vary δ and thereby construct a conductivity vs. depth model. This is called frequency sounding in electromagnetics.

Geometric sounding can also be done at lower frequencies in electromagnetics – by varying the Rx-Tx distance, but usually, frequency sounding is used in EM work.

Experiment

The experimental setup of this lab is shown in Figure 1. Small transmitter (Tx) and receiver (Rx) coils are placed on two sides of a stack of metal sheets in which the skin depth is being measured. A sinusoidal current is introduced into the Tx coil, which generates an oscillating magnetic field, which is picked up by the receiver coil on the other side of the sheet. The voltages from both Tx and Rx are displayed and recorded on channels A and B of a digital oscilloscope.

To determine δ for a given metal, we will change the number of sheets between the Tx and Rx and try recognizing the exponential decrease of the recorded Rx amplitude (eq. (1)) on the thickness of the stack z.

Once the equipment is set up and both the transmitter and receiver signals are clearly observable on the oscilloscope, we slowly insert one of the Aluminium sheets into the frame. You should see that both the amplitude and the phase of the receiver signal are altered and that the sheet is 'sensed' by the receiver before it comes completely between the transmitter and receiver.

Because the mutual inductance between the Tx and Rx coils is a function of frequency, the signal in the receiver varies with frequency even with no sheets inserted. To see the skin effect, you will need to cancel this variation from your measurements. This will be done by normalizing the measurements at each frequency so that the amplitude at the receiver equals one and the phase is zero when there are no sheets inserted. The normalized receiver voltage is then:

$$\bar{V}_{Rx}(n, f) = \frac{V_{Rx}(n, f)}{V_{Rx}(0, f)}. \tag{4}$$

where $V_{Rx}(0,f)$ is the voltage in the Rx coil when no sheets are present.

The skin effect (eq. 1) says that in an EM wave, the amplitude of the magnetic field should be constant on each side of the conductor and decrease exponentially with distance through it. Therefore, the normalized amplitude V_{Rx} will depend on the thickness of n aluminum sheets, $d = nh$, as

$$\bar{V}_{Rx}(n, f) = \exp\left(-\frac{d}{\delta}\right), \tag{5}$$

where h is the thickness of each sheet (1.59 mm in our case). For $n > 0$, the skin depth δ can then be obtained from this formula as

$$\delta = -\frac{d}{\log\left[\bar{V}_{Rx}(n, f)\right]}. \tag{6}$$

a)

b)

Figure 1. Skin depth experiment: a) operational assembly; b) zoom-in with the metal sheet removed. The Tx and Rx coils wrapped in black electric tape. The Tx coil is placed above the sheet, and Rx is below it.

Assignments

1) Without sheets between the Tx and Rx, set the transmitter to **generate a sinusoidal signal at 500 Hz** and **record the signal strength and phase** in the attached worksheet.

   Try to adjust the transmitted current to the highest amplitude that produces no distortion. With no sheets present, the phase difference should be about 180° at every frequency.

2) **Repeat this measurement at frequencies 1000, 5000, 10000, and 20000 Hz**.

3) Insert sheets of Aluminium in the frame one by one and repeat measurements at each of the five frequencies. If the signal in the receiver becomes unworkably small, you can skip the remaining frequencies. The table suggests where these small amplitudes may occur.

4) In the worksheet, **fill the columns for "Normalized amplitudes" and "Normalized phases"** for each reading. **Use Excel or Matlab or GNU Octave (recommended for GEOL334)**.

   For each frequency, the "Normalized amplitudes" is the dimensionless ratio $\frac{V_{Rx}(n, f)}{V_{Rx}(0, f)}$ of the amplitude recorded with n aluminum sheets to the amplitude with no sheets. Accordingly, the "Normalized phase" is the difference of phases:

$$\Delta\phi(n, f) = \text{phase}\left[V_{Rx}(n, f)\right] - \text{phase}\left[V_{Rx}(0, f)\right].$$

   You should find that all "Normalized amplitudes" equal one and the "Normalized phases" equal zero in all rows with $n = 0$.

5) In 'semilogy' scales, on one plot, **graph the normalized amplitude of the receiver signal vs. frequency** separately for each fixed n: $n = 0$ sheets, $n = 1$ sheet, $n = 2$ sheets, etc.

   You should get a horizontal line for the no-sheets case. For $n > 0$, you should see lines with downward slopes increasing with n. **Is this so?**

6) In the rows with $n > 0$ in the worksheet (or Matlab matrix), **calculate the column for "Skin depth"** by using eq. (6) and the column of normalized amplitudes you calculated in step 4).

7) You should expect seeing near constant values in the "Skin depth" column. **Is this so?** For each frequency, **plot the skin depths vs. *n* and identify the range of values independent of *n***. For larger n, the signal may become too weak and the estimates unreliable.

8) **Average the values of skin depths δ** within each of these near-constant ranges and place them in one of the fields of column "Final skin depth" (one value for each frequency). These values will be your final estimates of skin depth, and the variations from the averaged values will give estimates of their uncertainties. **Plot the final δ values vs. frequency** on a semi-logarithmic ('semilogx') scale.

9) **For bonus points:**

   There are other ways for estimating δ from the table in the worksheet. For example, eq. (4) can be written without normalization for all values of n including $n = 0$ as:

$$\log V_{Rx}(n, f) = \log V_{Rx0}(f) - \frac{d}{\delta}, \tag{6}$$

where $V_{Rx0}(f)$ is the "true" voltage when $d = 0$ (this is different from what we measured with no sheets, because all measurements involve experimental errors). Equation (6) can be treated as an equation of linear regression for $ax_n + b = y_n$, where $x_n = d = nh$ and $y_n = \log V_{Rx}(n, f)$ for an n-sheet measurement, $a = -1/\delta$, and $b = \log V_{Rx0}(f)$.

For each frequency, evaluate the coefficients *a* and *b* of this regression, and the slope will give you $\delta = -1/a$. Compare this value to those found above.

10) **In column "Theoretical skin depth" in the worksheet, calculate the skin depth for aluminum** by eq. (3) and compare it to the results of the measurements. The resistivity of aluminium is $\rho = 2.5 \cdot 10^{-8}\ \Omega \cdot$m.

11) Finally, **for each frequency, plot the phase as a function of the total thickness of aluminium**.

    Note that the phase of a sinusoidal signal is always determined with an uncertainty of 360m degrees, where m is an arbitrary integer. Therefore, when you plot the phases, **look whether you can improve the continuity of phase variations** by adding multiples of 360° to the recorded phase values.

    Depending on the polarity of connecting the wires to the oscilloscope, the phase of the received signal should have been close to 0° or 180° when no sheets were in place. That is, the receiver voltage is 0° or 180° out of phase with the transmitter signal. However, each time you add a sheet, the receiver signal should lag progressively further behind the transmitter signal. **Do you see this from your plot?**

Additional tasks for GEOL334:

12) In linear scales, **plot the original amplitudes vs. total thickness of sheets** (d). Do the plots look like the linear dependencies on d in eq. (6)? Next, **redo this plot using units of skin depth (ratio d/δ) instead of meters**. In this plot, you should see that the amplitude falls off by $1/e \approx 0.37$ per skin depth regardless of frequency.

13) **Calculate the skin depth in till at 1 Hz, 10 Hz, 100 Hz, 1 kHz, and 10 kHz.** The resistivity of till is about 20 Ohm·m.

    These values will show about how deep you will be able to investigate till with these frequencies, and these frequencies are about the range you have available in geophysics.

14) **Estimate the resistivity of aluminium by using your measurements of skin depth** and eq. (3). A good way to do this is to linearize the equation. Square the obtained skin depth values and plot δ^2 vs. $500^2/f$. <u>The slope of this line is the resistivity</u> ρ. Check this slope against the known resistivity of aluminium ($2.5 \cdot 10^{-8}\ \Omega \cdot$m).

15) **Plot the phase again, only this time, instead of having the thickness of aluminium measured in meters as the *x*-axis, use the thickness measured in skin depths** (ratio d/δ). This curve should be fairly straight over a reasonably long portion and close for all frequencies. **By how much does the phase change <u>across one skin depth</u>?**

Hand in:

Brief answers to the questions highlighted in **bold** above with figures embedded in a Word or PowerPoint document by email.