GEOL 384.3 and GEOL 334.3

Lab #11: VLF profile in the Bowl

In this lab, you will acquire short Very Low Frequency (VLF) EM profile across The Bowl at the UofS campus.

Interpretation of VLF basically utilizes the so-called line-of-current model (see next section). An infinite line of current has a magnetic field that is circular and centered on the current. The direction of the magnetic field follows the right-hand rule for a DC current, that is:

- If the thumb of the right-hand points in the direction of the current, the curled fingers point in the circulation sense of the field.

I prefer the "corkscrew rule" for myself:

- If you screw a corkscrew in the direction of the current, the handle turns in the direction of the magnetic field.

For field work, the visualization rule is usually given like this:

- If you are facing a line of current below surface, the magnetic field is directed upward on your right side and downward on your left.

An AC current of course alternates this polarity, and so for display purposes, you need a convention for magnetic field polarity. In VLF, the convention is that the field is up approaching a line current and down moving away regardless of the instantaneous sense of the current.

Brief theory of magnetic field of a line current

When visualizing a line of subsurface current in the field, we usually stand facing the current, take axis Y in the direction of the current, axis Z downward and axis X to the right. The magnetic induction field **B** is then oriented within the XZ plane and directed counter-clockwise. If current I is located at $x = x_0$ at depth z_0 in this plane, the vertical and horizontal components of **B** are

$$B_z = -\mu_0 I \frac{x - x_0}{r^2}, \quad \text{and} \quad B_x = -\mu_0 I \frac{z_0}{r^2}, \tag{1}$$

where $r = \sqrt{\left(x - x_0\right)^2 + \left(z - z_0\right)^2}$ is the distance from observation point (x,z) to the line current, and we disregard the magnetic permeability $\mu \approx 1$. It can be seen from this equation that B_z vanishes right over the current, that is when $x = x_0$. The field is directed upward on the left side ($x < x_0$) and downward on the right side – this means that B_z peaks on the left side of the current (when facing it) and valleys on the right side. At the points where it peaks and valleys, the slope of the $B_z(x)$ curve is zero: $dB_z/dx = 0$. Evaluating this derivative from eq. (1), the result for the location of the peak to valley is:

$$x_{peak} = x_0 - z_0, \quad \text{and} \quad x_{valley} = x_0 + z_0 \quad . \tag{2}$$

This means that the peak and valley occur at distances z_0 from the projection of the current onto the surface, and horizontal distance between the peak and the valley is twice the depth.

Therefore, if we have a plot of the vertical-component magnetic field $B_z(x)$, then we can obtain the depth z_0 as horizontal peak-to-valley distance divided by two:

$$z_0 = \frac{\text{horizontal peak to valley distance in } B_z(x)}{2}. \tag{3}$$

Note that this is independent of the magnitude of the current. The peak-to-valley distance is a function of the depth only.

From the second equation (1), the horizontal component B_x is directed in the negative direction of axis X. Function $B_x(x)$ is symmetric with respect to point $x = x_0$, and peaks right over the current. At this peak, the field strength equals $\mu_0 I/z_0$. To put a scale on its magnitude, a 1A current at 1-m depth produces a B field of $\mu_0 = 4\pi\times10^{-7}$T = 400πnT, or about a thousand nT. From the same eq. (1), a half of this peak level $B_x = \mu_0 I/z_0$ is reached at points $x = x_0 \pm z_0$. The spacing between these points is the half-amplitude width of the anomaly $w_{1/2} = 2z_0$. This means that the rule for estimating depth from $B_x(x)$ is:

If we have a plot of the horizontal-component magnetic field $B_x(x)$, then the target is located beneath its peak and the depth z_0 can be estimated as the half width of the anomaly divided by two:

$$z_0 = \frac{w_{1/2}}{2}. \tag{4}$$

Combining the vertical and horizontal components, the so-called total field (magnitude of vector **B**) equals:

$$T = \sqrt{B_x^2 + B_z^2} = \frac{\mu_0 I}{r}. \tag{5}$$

The total field is the most reliable to measure (because it, for example, equalizes the variations of the components due to misorientations of the instrument or noise). Therefore, we can calculate a depth based on the total field by measuring the horizontal distance $w_{1/2}$ between the two points where $T(x)$ drops to half of its peak value (the width at half-height rule which we used many times for gravity anomalies). For $T(x)$, the peak of the anomaly occurs at $x = x_0$, and its height is $T(x_0)$. Therefore, the two points $x_{1/2}$ at which the anomaly is at half-height can be obtained from (with $z = 0$ on the surface);

$$T(x_{1/2}) = \frac{\mu_0 I}{r(x_{1/2})} = \frac{\mu_0 I}{\sqrt{(x_{1/2} - x_0)^2 + z_0^2}} = \frac{1}{2}\frac{\mu_0 I}{z_0}, \tag{6}$$

from which $\left(x_{1/2}-x_0\right)^2=3z_0^2$, and the two coordinates at which the anomaly is at half-height are $x_{1/2}=x_0\pm\sqrt{3}z_0$. The difference between these coordinates is the full width at half height of the $T(x)$ curve: $w_{1/2}=2\sqrt{3}z_0$.

> Thus, if measuring the half-amplitude width $w_{1/2}$ from a $T(x)$ plot, we can obtain the depth of the source as:
>
> $$z_0=\frac{w_{1/2}}{2\sqrt{3}}. \tag{7}$$

Note that from eqs. (4) and eq. (1), $\frac{B_x}{T}=\frac{z_0}{r}$ regardless of the current magnitude. This relation can also be used to estimate z_0 by fitting the $B_x(x)/T(x)$ dependence.

Dependence on the orientation of the profile

There is no magnetic field in the direction of the current. Thus, if current is directed along axis Y, then $B_y = 0$. Conversely, if a non-zero B_y component is measured, then the profile is not perpendicular to the current. If the current is directed at angle θ relative to the profile (axis X), then eqs. (1) become:

$$B_z=-\mu_0 I\frac{x-x_0}{r^2}\ \text{(same as in eq. (1))},\qquad B_x=-\mu_0 I\frac{z_0}{r^2}\sin\theta, \tag{8}$$

and the total field also shows the same formula

$$T=\sqrt{B_x^2+B_y^2+B_z^2}=\frac{\mu_0 I}{r}, \tag{9}$$

but distance r is changed is the perpendicular distance from the measurement point to the line current: . $r=\sqrt{\left[\left(x-x_0\right)\sin\theta\right]^2+\left(z-z_0\right)^2}$. Thus, if the profile strikes at less than 90° to the current, then r is smaller, the peak-to-valley distance (as measured along the profile) will be greater and the depth will appear to be greater as well.

> Therefore, in all depth formulas, the peak-to-valley distance and the full width at half height should be multiplied by $\sin\theta$ to obtain the correct depth.

Figure 1 shows how the vertical component B_z would like as the strike of the current with respect to the profile direction changes. The peak amplitude (and valley) do not change in magnitude, and only their positions change. If the relative strike is small, there is hardly any difference, but for relative strikes below 20°, the effect is dramatic. Finally, with a relative strike of 0°, the peak and valley are at infinity, and the vertical component B_z is almost zero along the profile.

Currents that are not infinite lines may appear approximately so if the profile is short compared to the actual length of the current line. Thus, current loops, (square circular, or irregular) may have magnetic fields that are roughly the same as infinite line currents if the profile is short compared to the straight line part of the current, close to its mid-point, and no other parts of the current path are close to the profile.

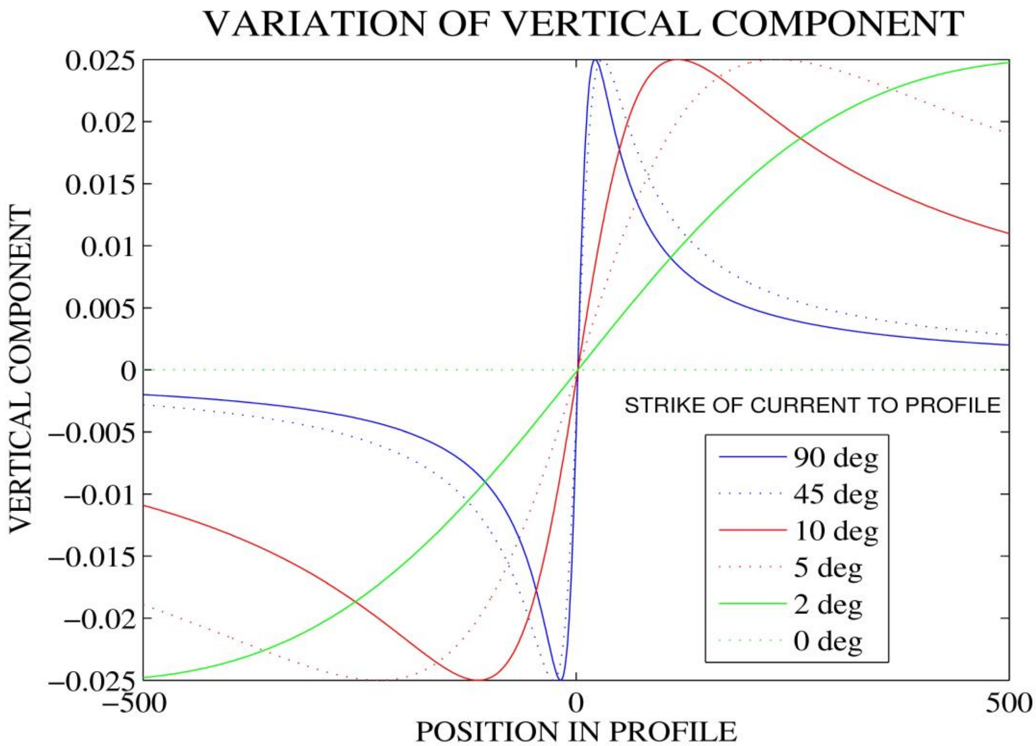

Figure 1. Variation of the vertical component of induction field $B_z(x)$ for several angles θ between the current source and the profile. Each of these profiles can be transformed into the one for q = 0° (blue line) by multiplying coordinates $(x - x_0)$ by $\sin\theta$.

Brief theory of VLF tilt-angle

In VLF, we typically measure the tilt angle. Far from the broadcast antenna (where we are located), if the ground has a uniform or a 1-D (layered) conductivity, the VLF wave has a vertical electric field (so that it does not induce lateral currents) and consequently horizontal magnetic field,. If there is a local variation in conductivity, then there will be a vertical component of the VLF magnetic field, and the net VLF magnetic component will be tilted up or down. This tilt of the magnetic field from horizontal is called the tilt-angle.

The recorded VLF field is a sum of the background field (in the absence of local anomaly; caused by large and deep, uniformly distributed currents) and the anomaly field caused by local currents. Part of the horizontal component is the background and part is from the isolated (line) conductor. All of the vertical component is from the local variations in conductivity, which we are trying to model as a line current.

If the local variation in conductivity has a definite strike, then it will have a response similar to that of a line source of current, and we can use the equations in the preceding section. The tilt-angle is measured by the ratio B_z/B_H, where B_H is the total (called orientation-independent) horizontal magnetic field $B_H = \sqrt{B_x^2 + B_y^2}$. Let us use notation B_h for the horizontal-component magnetic field in the absence of a local anomaly. This regional field will be near-constant on the scale of our target anomaly, which we denote $B_H(x)$.

Because the local anomaly is produced by local current I, we can approximate $B_H(x)$ by the second eq. (1). However, the current I and the background field B_h are unknown, and instead of them, we can measure the magnitude of the local anomaly by its ratio to B_h:

$$\alpha = \frac{\max\left[B_H(x)\right]}{B_h}. \tag{10}$$

In the horizontal-component profile, this α can be determined from the peak value of the anomaly and the "zero-line" field far away from the peak.

With the peak magnitude of $B_H(x)$ determined by α, the total horizontal-component magnetic field will be (from eq. (1))

$$B_H(x) + B_h = B_h\left(1 + \alpha\frac{z_0^2}{r^2}\right). \tag{11}$$

Because the regional background field is produced by currents broadly distributed in the subsurface, it has almost no vertical component, and most of the recorded vertical B_z (field upward and downward directed) is due to the local anomaly. Therefore, the tilt is

$$tilt = tilt_{background}\frac{\frac{x-x_0}{r^2}}{1+\alpha\frac{z_0^2}{r^2}} = tilt_{background}\frac{x-x_0}{r^2+\alpha z_0^2} = tilt_{background}\frac{x-x_0}{(x-x_0)^2+(1+\alpha)z_0^2}, \tag{12}$$

where $tilt_{background} = \frac{\mu_0 I}{B_h}$ is the tilt due to the background filed (far outside of the anomaly). This expression shows the shape of the VLF tilt-angle anomaly you can expect from an induced current source in the subsurface. If α is small, then <u>the tilt-angle anomaly has the same shape as the vertical-component</u> field anomaly, and you can use eq. (3) to estimate the target depth.

There is an added complication in VLF caused by the phase shift during induction. The components B_h and B_z from the local line current are in phase with each other, but they may have an unknown phase shift with respect to the background horizontal component. We get around this in VLF by using the total horizontal component as a phase reference and quoting the phase of the vertical component with respect to it as an In-Phase component (the fraction of the vertical component in phase with the horizontal component) and a Quad-Phase component (the fraction of the vertical component 90° out-of-phase with the total horizontal component.

Experiment

A photo from a different survey is shown in Figure 2, and our receiver is shown in Figure 3. In this lab, we will conduct a VLF profile across the Bowl at ½ m spacing. We will start about halfway across the Bowl, move towards the entrance to the Physics building, and try detecting some underground cables. The instrument will record the tilt angle, the In-Phase component, Quad-Phase component, and the Total Field component.

Figure 2. VLF data acquisition looked like this in our 2014 field school (the receiver and antenna are in the cart).

Figure 3. Our OMNI VLF receiver.

With the target oriented strictly orthogonally to the VLF profile, the induced magnetic field **B** will be oriented along the profile, and there will be no transverse field. However, if the conductor deviates from the orthogonal direction by angle θ, then you should see a non-zero horizontal transverse component of **B**. The relative magnitude of the two horizontal components can be used to calculate the strike of the current with respect to the profile:

$$\theta = \arctan\left[\frac{\max(\text{transverse horizontal } B)}{\max(\text{in-profile horizontal } B)}\right]. \tag{13}$$

This value is reported as "Direction" by the VLF receiver.

If θ is nonzero, the apparent source depth for the oblique profile should be greater than it was before by factor $1/\cos(\theta)$. We could still estimate a correct depth by fitting the equation for the vertical component in the oblique case to the observed data.

Assignments

1) Estimate rough directions from the survey area to the three VLF transmitters which we will be using:

   - Near **Cutler, Maine** (24.0 kHz, call sign NAA, located at 44.63°N, 67.27°W);
   - **Jim Creek Naval Radio Station**, near Oso, **Washington** (24.8 kHz, coordinates 48.204°N, 121.917°W).
   - Near **LaMoure, North Dakota** (25.2 kHz, coordinates 46.37°N, 98.34°W).

   VLF recordings will be sensitive to conductors oriented in the direction to the sources, and therefore profiling should be done in an orthogonal direction to the source.

   Plan the direction of the profile and mark it with a measuring rope.

2) **Set up the receiver** for recording from all three of the above transmitters. **Acquire an about 30-m profile at 1-m intervals**.

3) **Transfer the results of the surveys** from the file downloaded from the receiver into the worksheet. Note the names of transmitters added at the beginning of each table downloaded from the receiver.

4) **Plot the tilt angle, the In-Phase component, Quad-Phase component and Total Field component** from each of the three transmitters.

5) **Use the above plots to locate a conductor**. **Estimate its depth from the peak-to valley distance in the tilt-angle curve** (note that the depth is measured from the antenna in the backpack to the conductor). Since the shape of the anomaly is close to $B_z(x)$, use eq. (3).

6) Evaluate which of the three signals has worked best. Could this be related to the frequency of the target or direction to the transmitter?

7) **Answer the following question**: If we had another naval transmitter working at 11 kHz, from which transmitter you would expect a better VLF response for a deeper conductor?

Additional question for GEOL334:

In VLF, we measure a vertical component, a horizontal component in the direction the operator is facing and a horizontal component perpendicular to this direction. If the conductor strikes at right angles to the profile direction, there should be no horizontal component at right angles to the survey direction. However, if there is a horizontal component, it means that the conductor strikes at some angle to the profile, and we can figure out this angle from this information. This component is not displayed directly, but rather as an apparent direction of the magnetic horizontal field (column "Direction" in the work sheet). If this

"Direction" is constant along the profile, then there is no secondary magnetic component orthogonal to the survey direction.

8) **Plot column "Direction" and see if the conductor strikes at 90° to the profile**.

Hand in:

Brief answers to the questions highlighted in **bold** above with figures embedded in a Word or PowerPoint document by email.