Resistivity method - Key points of this lecture

- Principles of resistivity measurements
  - Apparent resistivity
  - Geometry factor
- Measurement of resistivity in rock samples
- Resistivity measurements in the field
  - Instrumentation
  - Sounding and profiling
  - Forms of electrode arrays
  - Applied potential
  - Pseudo-sections
- Interpretation
  - Depth and lateral contrasts
  - Examples

- Labs # 4 and 5
- Reading:
  - Dentith and Mudge, Sections 5.4 – 5.6

Principle of resistivity measurement

- According to the name of the method, in "resistivity" measurements, we determine the resistivity ρ or conductivity $\sigma = 1/\rho$ of rock or subsurface layers
- Generally, resistivity within rocks is always measured by (see your labs):
  - Injecting current I into a pair of electrodes connected to the rock sample or some locations on the ground, often denoted A and B (sometimes C_1 and C_2, which mean "current")
  - Measuring voltages (V, difference of electric potentials) between some other points. These points are conventionally denoted M and N (sometimes P_1 and P_2, meaning "potential")
  - Evaluating the resistance ratio $R = V/I$ of the whole circuit
  - Transforming R into apparent resistivity ρ_a (explained on the next slide) by formula:

$$\rho_a = Rk$$

where k is the "geometry factor" depending on the mutual positions of the electrodes

    - Note that since the units for R are Ohm (Ω) and for ρ_a, the units are $\Omega\cdot\text{m}$, then the geometry factor k has units of distance
      - k has the meaning of "A/L", where A is the "characteristic area" crossed by the current, and L is the "characteristic length" of the array
- In modern studies with large numbers of data points, the apparent resistivity step is often bypassed and replaced by direct inversion for the subsurface model (ERT, "Electrical Resistivity Tomography")

Apparent resistivity

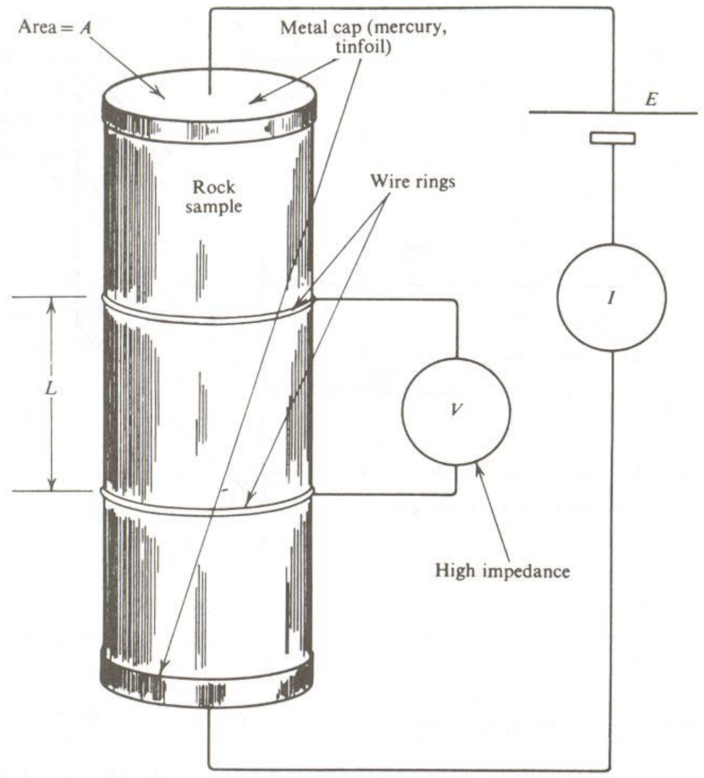
- The apparent resistivity is the key concept used when transforming, presenting, or interpreting resistivity data
  - Usually, “apparent resistivity” ρ_a means the resistivity of a spatially uniform body (rock sample, layer, or the whole subsurface) that would explain the measured resistance R
    - For uniform bodies, ρ_a equals the true resistivity (ρ)
    - For non-uniform bodies, this quantity differs from the true resistivity
  - Apparent resistivity depends on the type and size of the electrode array used and location of measurement

Resistivity of rock samples

- In rock cores, measurement of resistivity is relatively straightforward as shown in this figure
  - Current electrodes are attached to the ends of the core. In a good approximation, the rock is uniform, and the current density is **j** constant throughout the volume:
    - $j = I/A$ (A is the cross-sectional area)
  - Potential electrodes are made of two wire rings (see figure)
    - The voltage measurement circuit has high resistance (impedance), and it does not distort the current
    - Electric field between the rings: $E = V/L$
  - Thus, considering only the portion of the core between the wire rings, the apparent resistivity equals the true one, and the "geometry factor" $k = A/L$:

$$\rho_a = \rho = \frac{E}{j} = \frac{V}{I}\frac{A}{L}$$

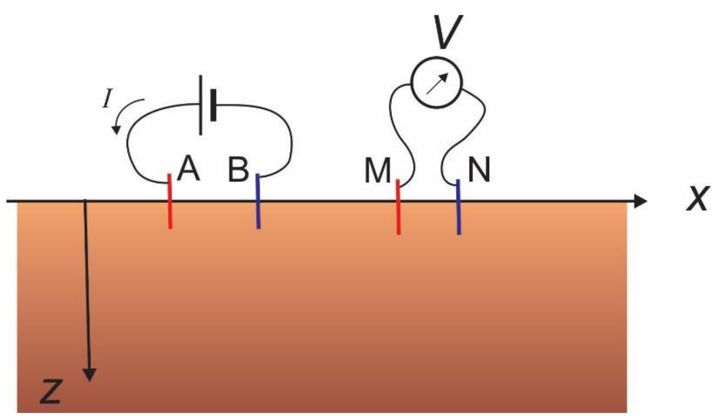
Geometry Factor, k



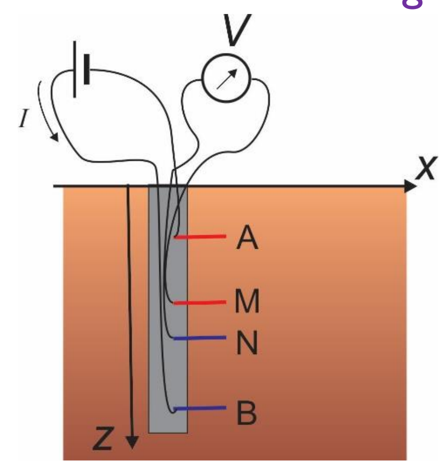
Resistivity measurements in the field

- In the field, the goal of resistivity measurements is to create 1-D ($\rho(z)$), 2-D ($\rho(x,z)$), or 3-D ($\rho(x,y,z)$), or similar conductivity $\sigma = 1/\rho$ images of subsurface layers
  - This is done by using surface or borehole arrays as shown in cartoons below
  - Moving the arrays laterally to cover extended areas is usually called "profiling". Expanding the arrays while remaining at the same place (to vary the depth of coverage) is called "sounding".

Surface recording



Borehole recording



Field procedures

- There are two general styles of acquisition
  - Field procedures are optimized for safety and minimal movement of long wires and cables:

1. Vertical (depth) sounding
   - Uses a fixed center with expanding spread
   - Measures the vertical variation of resistivity for a given geologic section
   - Frequently done at several locations, even if lateral profiling is the primary objective
     - To establish proper electrode spacing for profiling
     - To improve depth control
2. Lateral profiling (horizontal or downward in a borehole)
   - Current and potential electrodes are shifted over the survey area without altering their relative configuration
   - Focuses on lateral variation of resistivity down to some depth.
   - Best suited for detection of lateral contacts (e.g., steeply dipping dikes, or layers when in borehole).
   - As shown in the following slides, arrays with electrodes at infinity ("pole arrays") or "gradient arrays" are usually used for profiling, because these arrays require fewer wires to be moved

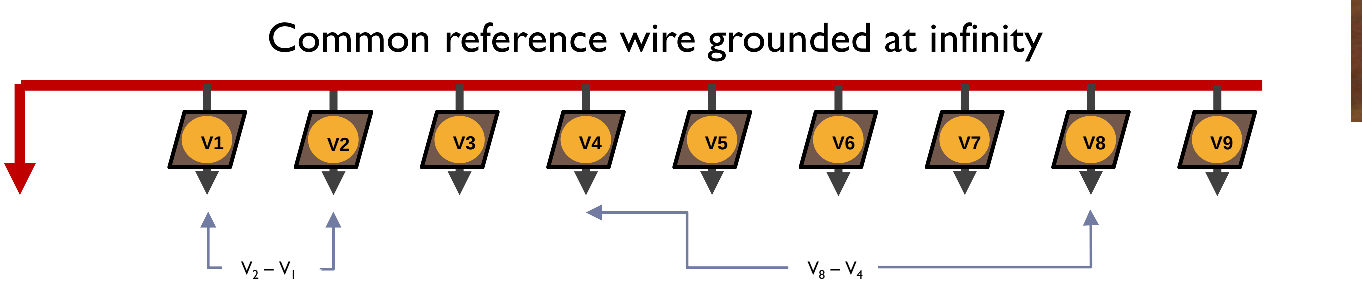
Instrumentation and field gear

- Common resistivity gear looks like this: source/battery, receiver, electrodes, and four wires to locations A, B, M, and N
- For low-voltage measurements (IP, SP, large distances), nonpolarizing electrodes are required

SYSCAL resistivity meter used in our field schools and labs

Large 3-D array recording

- Recent trend in receiver technology uses hundreds of small, independent receivers recording voltages and transmitting data wirelessly
- Only one, common reference wire is needed
- This array provides hundreds of pole-pole recordings of the same source simultaneously
  - “M-N” pairs of these recordings can be used to form multiple arrays
  - Data are inverted by Electric Resistivity Tomography (ERT)



DIAS GEOPHYSICAL

Resistance matrix for an arbitrary electrode array

- Consider the resistance matrix **R** for all locations of electrodes used in the survey
- For any given pairs of current electrodes (A, B) and potential electrodes (M, N), we can extract the resistance matrix $\mathbf{R}_{arr}$ for this position of the 4-electrode array:

$$\mathbf{R} = \begin{bmatrix} R_{11} & \cdots & R_{1A} & \cdots & R_{1B} & \cdots & R_{1L} \\ \vdots & \ddots & \vdots & \cdots & \vdots & \cdots & \vdots \\ R_{M1} & \cdots & R_{MA} & \cdots & R_{MB} & \cdots & R_{ML} \\ \vdots & \cdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ R_{N1} & \cdots & R_{NA} & \cdots & R_{NB} & \cdots & R_{NL} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \ddots & \vdots \\ R_{L1} & \cdots & R_{LA} & \cdots & R_{L3} & \cdots & R_{LL} \end{bmatrix} \Rightarrow \mathbf{R}_{arr} = \begin{bmatrix} R_{MA} & R_{MB} \\ R_{NA} & R_{NB} \end{bmatrix} \Rightarrow \begin{pmatrix} \varphi_M \\ \varphi_N \end{pmatrix} = \mathbf{R}_{arr} \begin{pmatrix} I_A \\ I_B \end{pmatrix}$$

- With this array, opposite currents are sent into A and B:

$$\mathbf{I}_2 = \begin{pmatrix} I \\ -I \end{pmatrix}$$

- …and the difference of potentials at M and N is measured:

$$V = \varphi_M - \varphi_N$$

- …The result is the measured scalar resistance in the array:

$$R = \frac{V}{I} = \left(R_{MA} + R_{NB}\right) - \left(R_{NA} + R_{MB}\right)$$

Geometry factor for an arbitrary surface array

- As explained in the intro to electrical methods lecture, for a homogeneous half-space with resistivity ρ, the potentials produced at points M and N represent sums of contributions from point source at A and a sink (negative source) at B ("basic solution #2" there):

$$V = \varphi_M - \varphi_N = \frac{I\rho}{2\pi r_{\mathrm{AM}}} - \frac{I\rho}{2\pi r_{\mathrm{BM}}} - \left(\frac{I\rho}{2\pi r_{\mathrm{AN}}} - \frac{I\rho}{2\pi r_{\mathrm{BN}}} \right) = \frac{I\rho}{k}$$

$$R_{MA} = \frac{\rho}{2\pi r_{\mathrm{AM}}}, \text{ etc.}$$

in the preceding slide

where $r_{\ldots}$ denote the distances between the corresponding electrodes, and k is the geometry factor for the arbitrary array (the array can even be in 3D):

$$k = 2\pi \left(\frac{1}{r_{\mathrm{AM}}} - \frac{1}{r_{\mathrm{BM}}} - \frac{1}{r_{\mathrm{AN}}} + \frac{1}{r_{\mathrm{BN}}} \right)^{-1}$$

Note that this formula again illustrates the reciprocity - sources AB can be interchanged with receivers MN

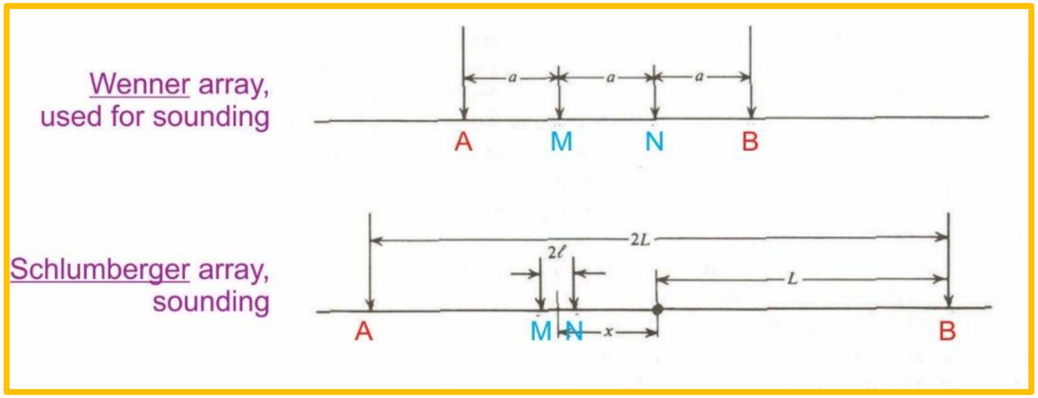
- From the first equation above, the apparent resistivity estimated from resistance $R = V/I$ equals the true one for the uniform half-space:

$$\rho = \frac{V}{I} k$$

Common array configurations

- Different configurations of electrode arrays are used for different targets and goals of experiment (profiling or sounding):

- Wenner array: four electrodes are spaced by a common distance a
  - Single parameter is convenient for calculation, plotting sections, and interpretation
  - However, profiling with this array is difficult, because it requires moving all electrodes and wires every time the array is moved
  - Geometry factor: $k = 2\pi a$



- Schlumberger array: most common for sounding
  - With fixed A and B, several M and N are tried, and then A, B is changed
  - Usually spacing between current electrodes $L >> l$ (spacing between potential electrodes)
  - Geometry factor: $k \approx \frac{\pi}{2l} \frac{\left(L^2 - x^2\right)^2}{L^2 + x^2} = \frac{\pi L^2}{2l}$ (when $x = 0$)

Common array configurations

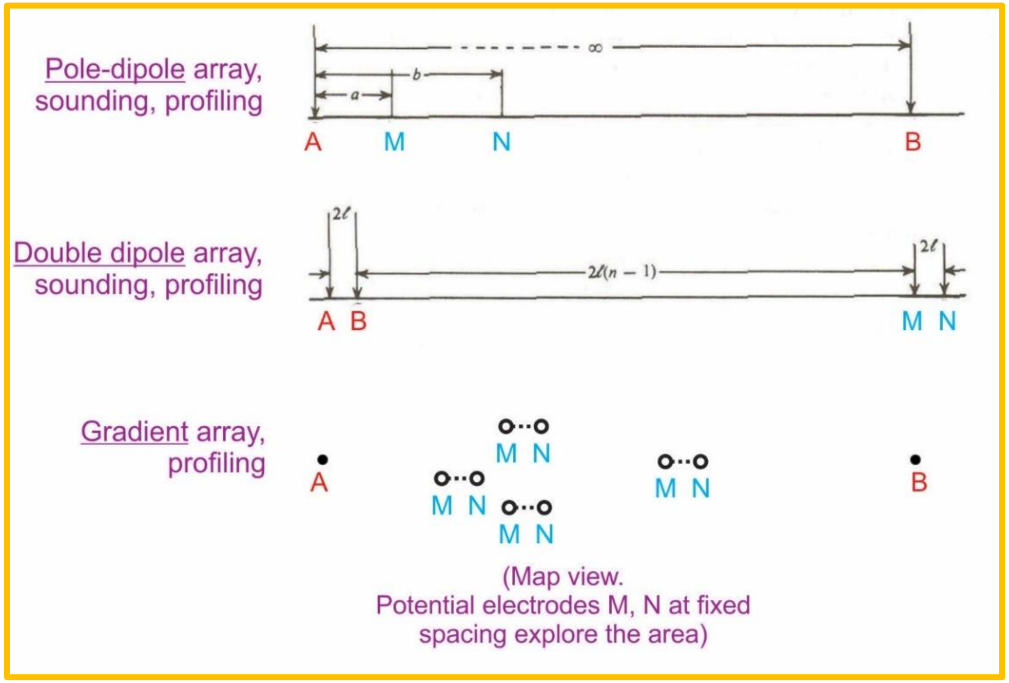
- Pole-dipole array
  - One of current electrodes is at infinity (does not have to be moved), and so profiling is easier

$$k = 2\pi \frac{ab}{b-a}$$

- Double-dipole array
  - Current electrodes are fixed at close spacing, and potential electrodes are moved, also keeping close spacing
  - Dipole-dipole configurations are sensitive to gradients of the field and deeper structures

$$k = 2\pi l\left(n-1\right)n\left(n+1\right)$$

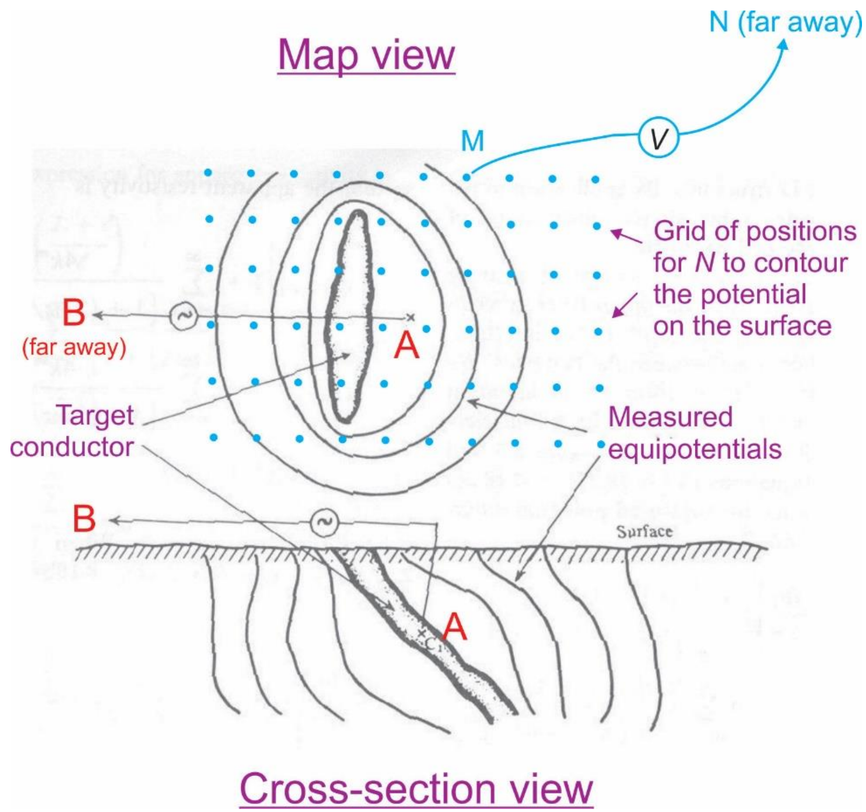
- Gradient array
  - Current electrodes are widely spaced, potential electrodes explore the area between them
  - Also sensitive to field gradients



Applied potential (Mise-à-la-masse)

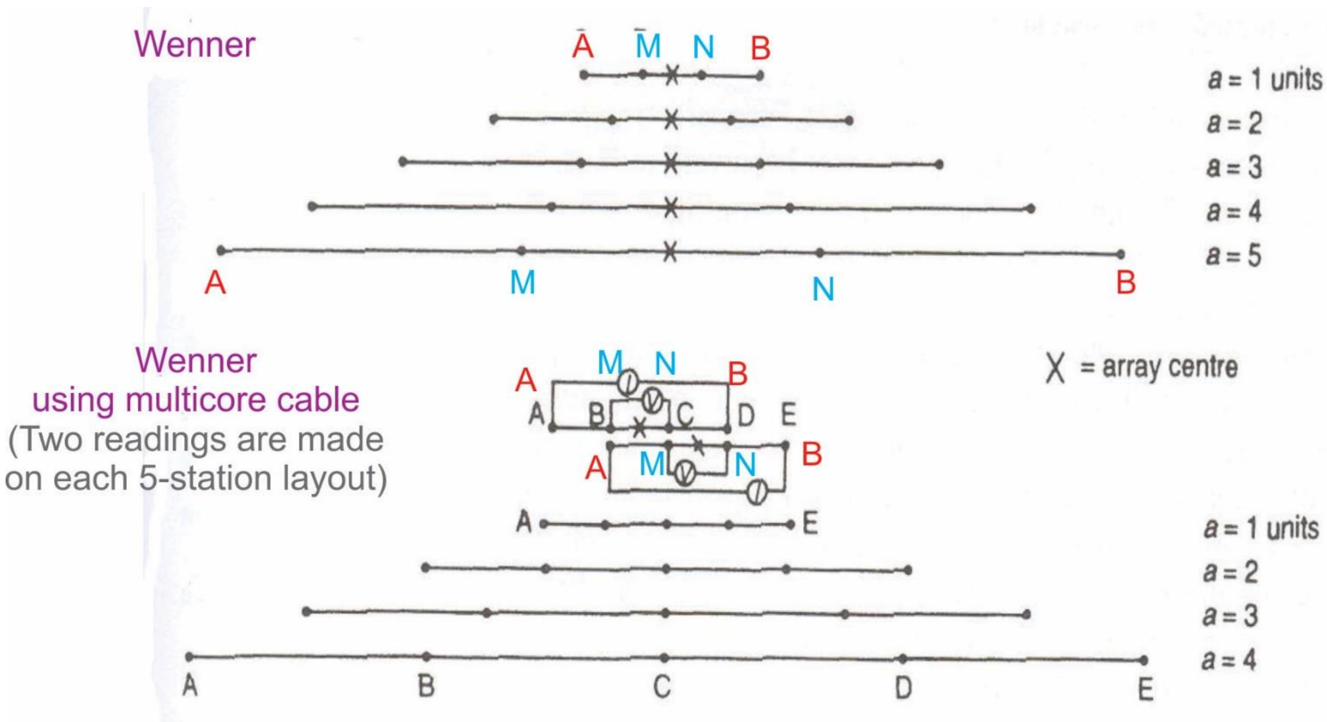
- The Applied potential method uses a pole-dipole array in which the current electrode is embedded into the conductive zone
  - In seismology, there is a similar idea of "salt proximity" surveys (see GEOL335)

- Does not require moving the current electrodes; only one potential electrode M is moved
- Allows mapping the extent, dip, strike, and continuity of the conductive zone better than by usual mapping
  - By using "depth continuation" (numerical solution of the Poisson's equation for the potential), surface map of potential $\varphi(x,y)$ can be transformed into depth image $\varphi(x,y,z)$ (bottom of this figure)



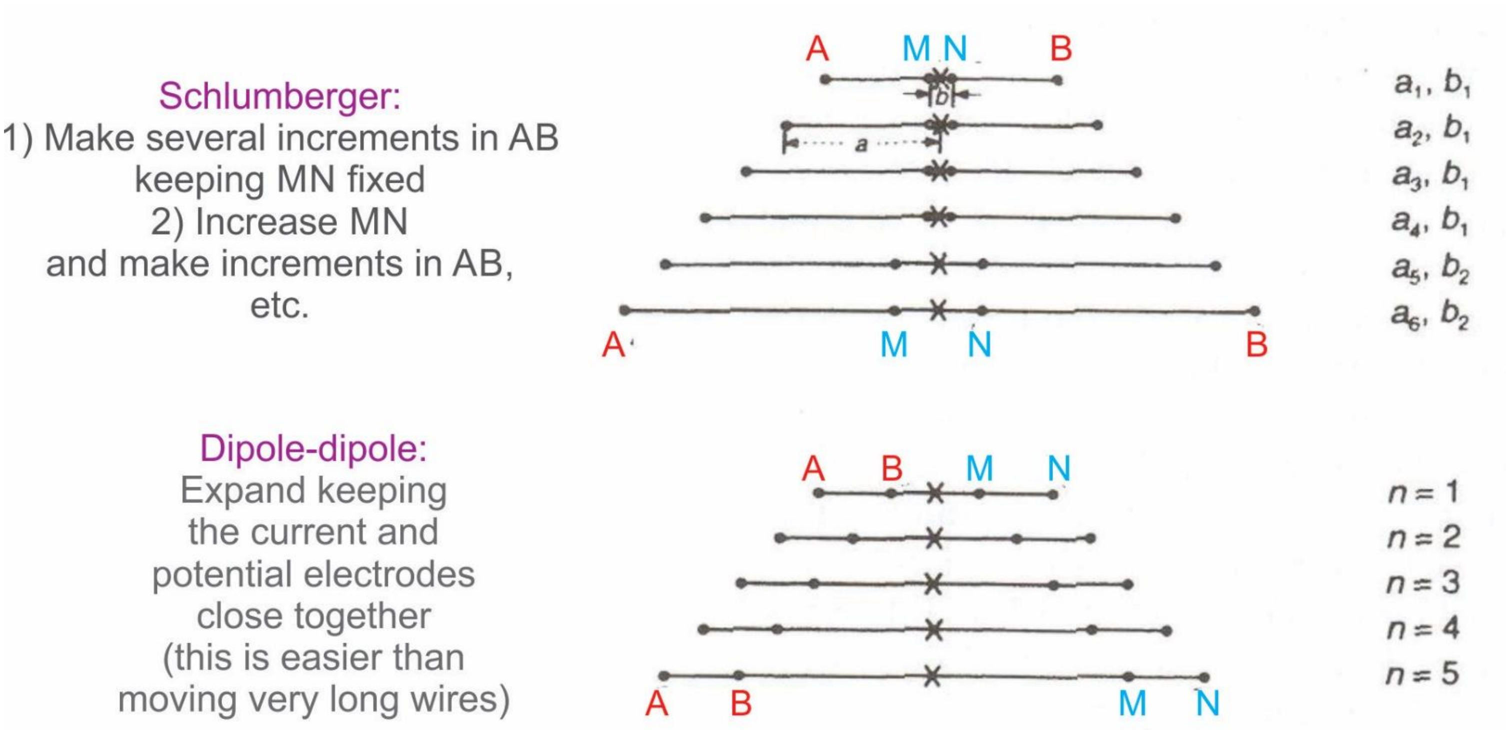
Expanding arrays for depth sounding

- Depth sounding is performed by repeated recording with increasing spacing of the array
- If multicore cable is available, several lateral array positions and spacings a can be recorded in one deployment



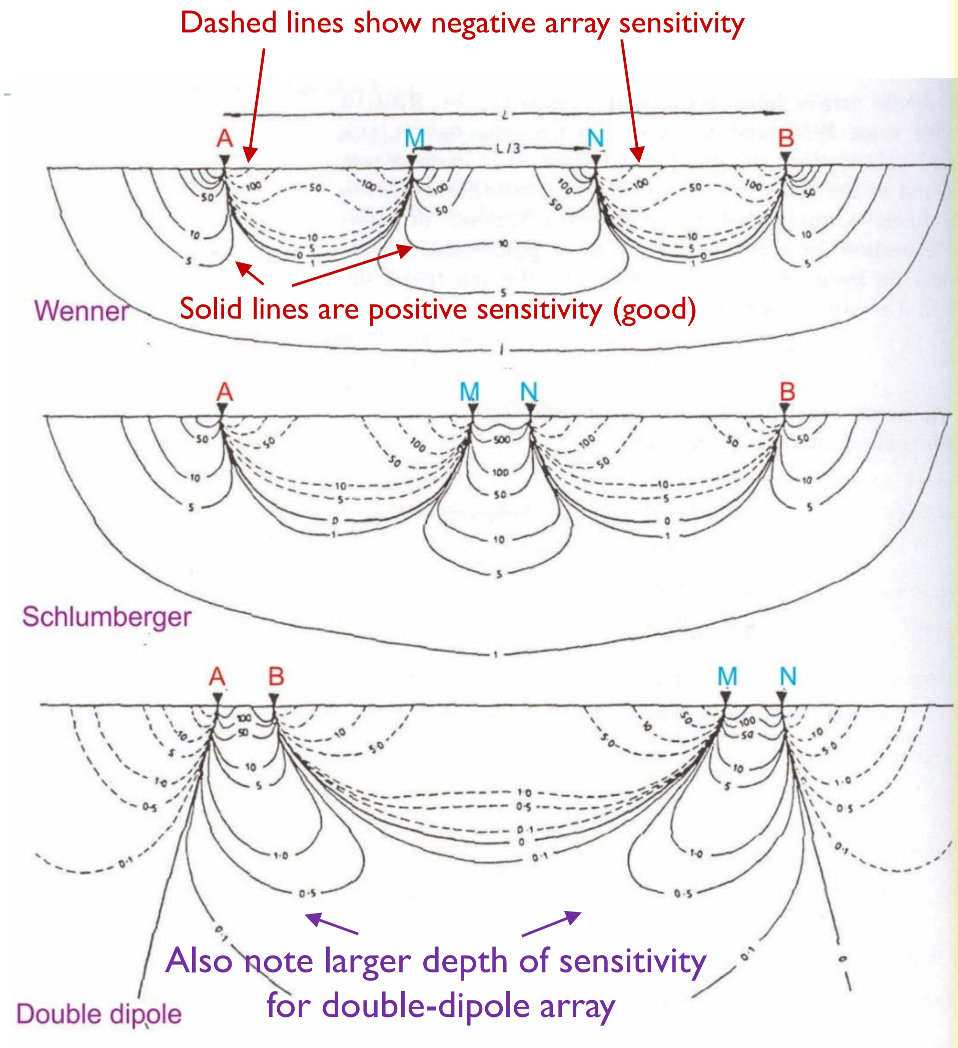
Expanding arrays for depth sounding

- Patterns of expanding spreads for Schlumberger and dipole-dipole arrays
- Again, the idea is to try a range of distances between AB and MN while minimizing moves of long wires



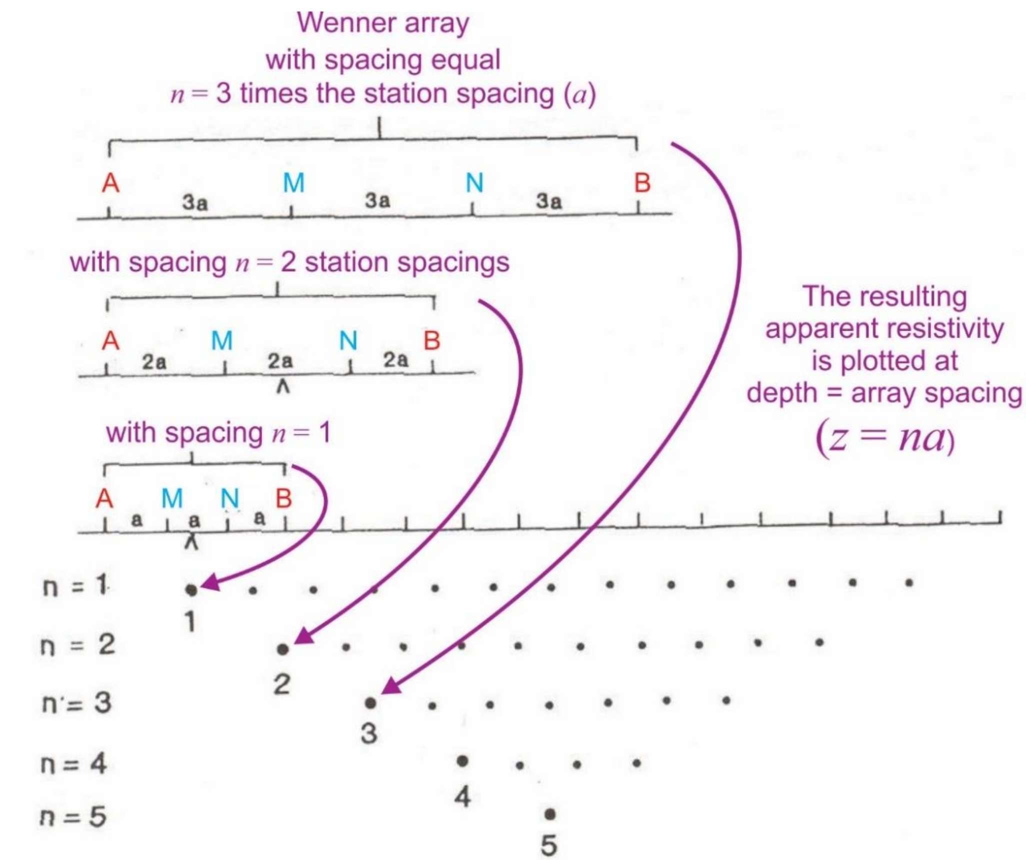
Array sensitivity

- If we insert a resistive body in the ground, will this increase the apparent resistivity measured by an array?
  - The answer to this question is not obvious and not simple
- The answer is given by Array sensitivity model, which is the change of the apparent resistivity (ρ_a) produced by a unit-volume resistive sphere added at point (x,z) in the ground
  - With positive sensitivity, a resistive (or conductive) anomaly in the ground would accordingly increase (decrease) the apparent resistivity. This is how you would intuitively interpret resistivity measurements
- However, note that at shallower depths between the pair of MN and current electrodes, the sensitivity is negative – a resistive/conductive body there would look like an decrease/increase of the apparent resistivity



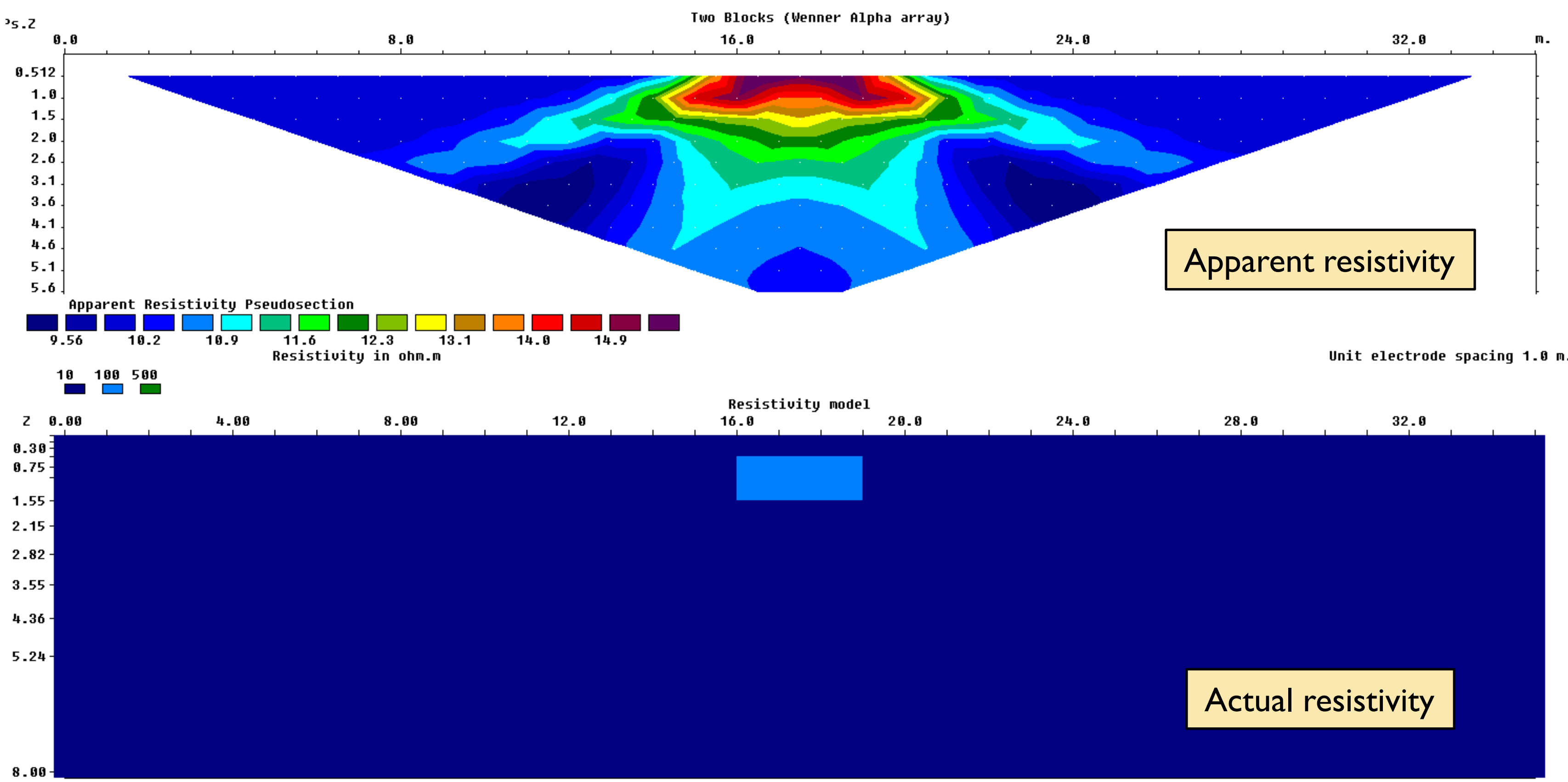
Pseudo-section (pseudo-depth section)

- For profiling with an expanding array (or multicore sounding), the results can be conveniently presented in the form of a "pseudo-depth section"
- For each configuration of the array, the resulting ρ_a is plotted at lateral "pseudo-position" and "pseudo-depth"
  - These coordinates roughly represent the point of maximum sensitivity of ρ_a to true resistivity
  - These coordinates are not accurate, but rather defined by convention
  - They often give a reasonable idea about the distribution of resistivity with depth and laterally
  - Useful for comparing different datasets



Comparison of true and apparent resistivities

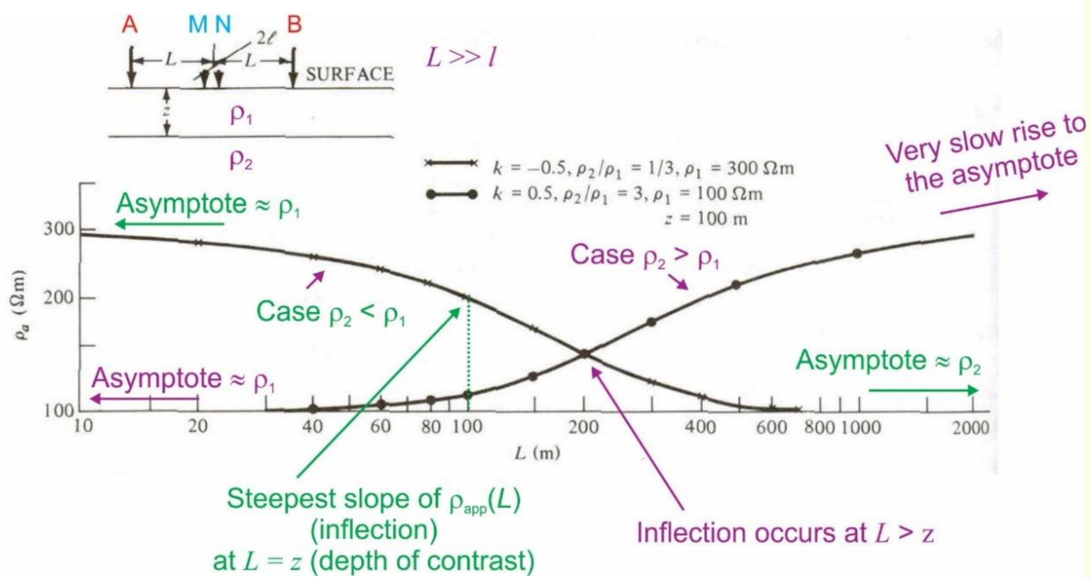
- Resistivity model with a resistive block and pseudo-section measured using Wenner array
- Calculated using program RES2DMOD (free, by M.H. Loke)



Interpretation of depth variations of resistivity – basic idea

- For a resistive layer overlaying a more conductive ground ($\rho_2 < \rho_1$ and green in the figure below), the resistivities can be seen from the asymptotes of ρ_{app} in depth sounding (e.g., by Wenner or Schlumberger arrays)
  - For $L << z$, the current flows mostly through the upper layer, and $\rho_a \approx \rho_1$
  - For, $L >> z$, the current flows mostly through the lower layer, and $\rho_a \approx \rho_2$
  - The inflection in the $\rho_{app}(L)$ curve occurs near $L = z$ (depth of the contrast)

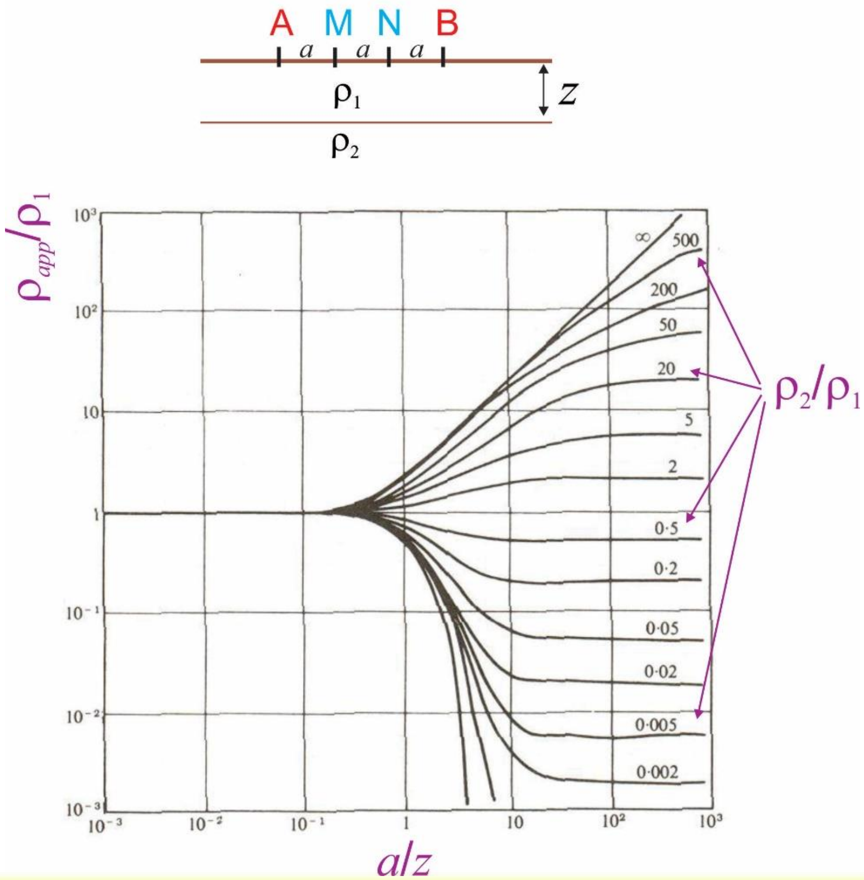
- For $\rho_2 > \rho_1$ (resistive deep part of the model), the situation is not so easy (purple):
  - The asymptote at $L << z$ is still correct because the current flows through the upper layer
  - However, the asymptote at $L >> z$ is practically not achieved
    - This is because the current is concentrated within low-resistivity layers and tends to avoid resistive ones



Interpretation – two-layer models

- Quantitative fitting of two-layer resistivity models can be done by plotting the observed and $\rho_a(array\_spacing)$ dependencies in scaled (unitless) axes and matching them against modeled master curves
  - For Wenner array, only one set of master curves is needed for variable resistivity contrast between layers

Master curves for Wenner array over two layers



Interpretation – "complete curve matching" method

- That was for Wenner array, and here is how you can find ρ_1, ρ_2, and z for any array:
- From the "basic case #4" in the preceding lecture, recall that for a two-layer resistivity, the potential φ at distance r from a current source or sink can be modeled as

$$\varphi(r) = \frac{I\rho_1}{2\pi}\left(\frac{1}{r} + \sum_{i=1}^{\infty}\frac{2k^i}{r_i}\right)$$

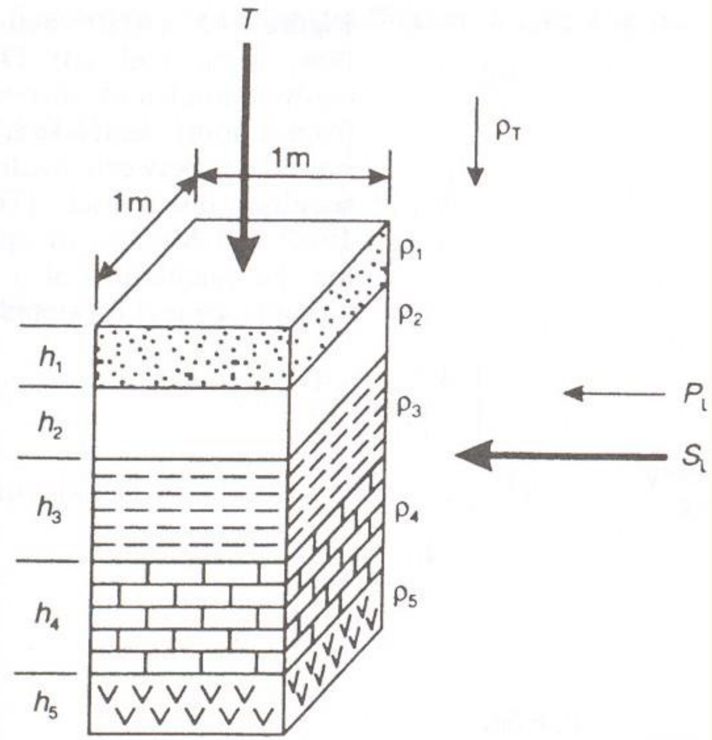
were k is the "reflection coefficient" for resistivity: $k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{\rho_2/\rho_1 - 1}{\rho_2/\rho_1 + 1}$

Unfortunately, this is also denoted "k", but this is not geometry factor

- From this $\varphi(r)$, the $\rho_a(L)$ can be expressed for any array by using geometry factors (see this lecture)
- Therefore, the complete $\rho_a(L)$ curve can be easily modeled for parameters z and k. The complete-curve matching method is then:
  1. From the asymptote at $L \to 0$, estimate $\rho_1 = \rho_a(L \to 0)$
  2. Divide your data $\rho_a(L)$ by ρ_1, model it for a range of z and k, and find the best-fit pair (z,k)
  3. From the value of k, determine ρ_2 (equation above): $\rho_2 = \rho_1 \frac{1+k}{1-k}$

Layer equivalence

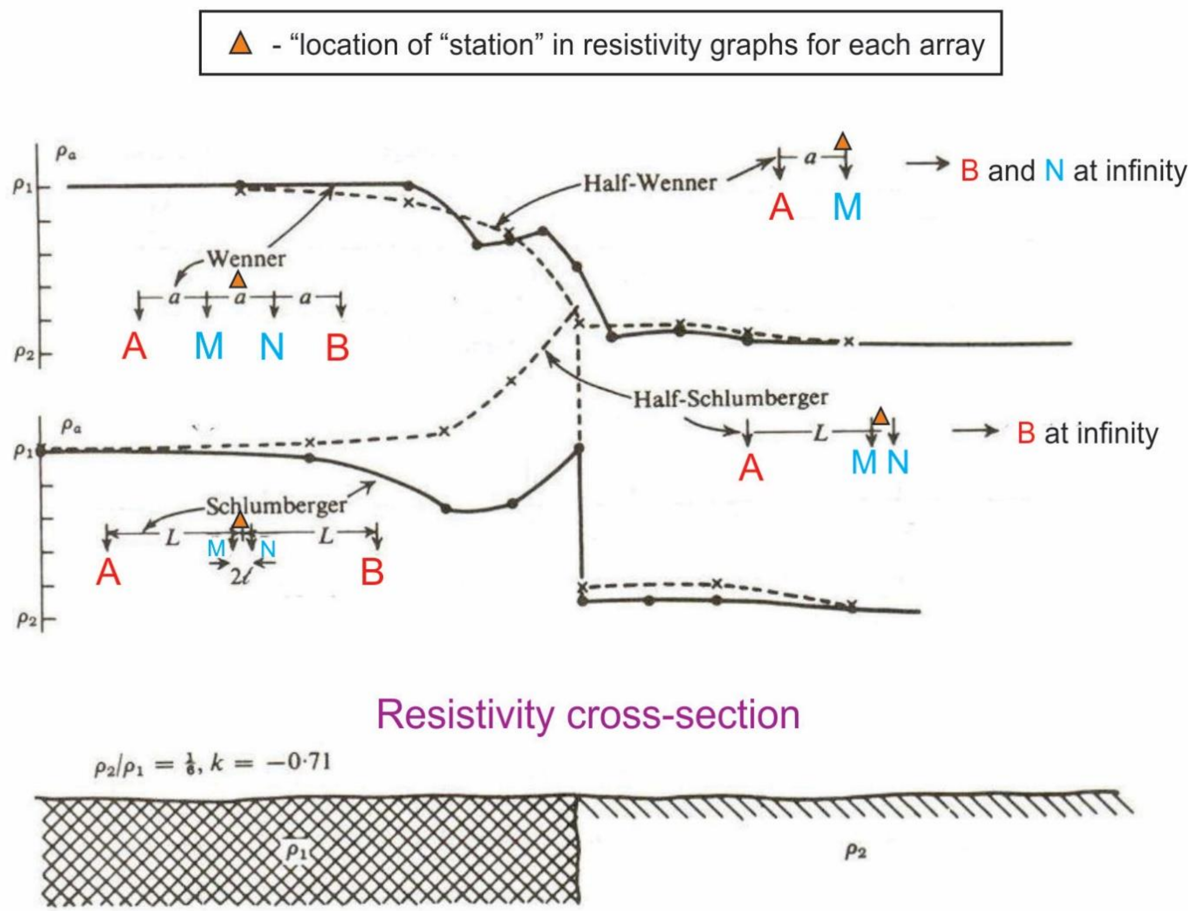
- Similar to gravity, there exists a significant uncertainty in resistivity interpretation. This uncertainty should be realized to avoid pitfalls.
  - The problem is that resistivity is likely anisotropic, i.e. different in vertical and longitudinal (horizontal) directions
- Here is a list of all (in principle) measurable parameters for a stack of layers ("dar Zarrouk parameters"):
  - Longitudinal (horizontal) conductance $S = h/\rho_L = h\sigma_L$
  - Longitudinal resistivity $\rho_L = h/S$
  - Transverse resistance $T = h\rho_T$
  - Transverse resistivity $\rho_T = T/h$
  - Anisotropy $A = \rho_T/\rho_L$
- If considering anisotropy, it is impossible to uniquely determine both resistivity and thickness of any layer!



Profiling example: lateral resistivity contrast

- Note that due to the extent of array, $\rho_a(x)$ shows a complex patterns when the array passes a vertical contact
  - Note the rising $\rho_a(x)$ when approaching the low-resistivity zone (ρ_2). This is because of the negative array sensitivity discussed above
- These lateral patterns should be removed by rigorous inversion

Apparent resistivity profiles over a vertical resistivity contrast for four arrays



Profiling example: resistive dike

- Example of a narrow dike of thickness b equal half of the spacing of the electrode array (a or L) shows the shape of ρ_a response in detail
  - Symmetric/asymmetric for symmetric/asymmetric arrays
  - Response is usually much smaller than true resistivity but close for pole-dipole array
  - The response is inverted for the pole-pole array (because of its negative sensitivity at this point)
  - When using the double dipole array, TWO images of the dike are obtained when either the current or potential electrodes pass the target (because of reciprocity)