Attenuation and dispersion

- Mechanisms:
  - *Absorption* (inelastic);
  - *Scattering* (elastic).
- Mathematical descriptions
- Measurement
- Frequency dependence
- Dispersion, its relation to attenuation

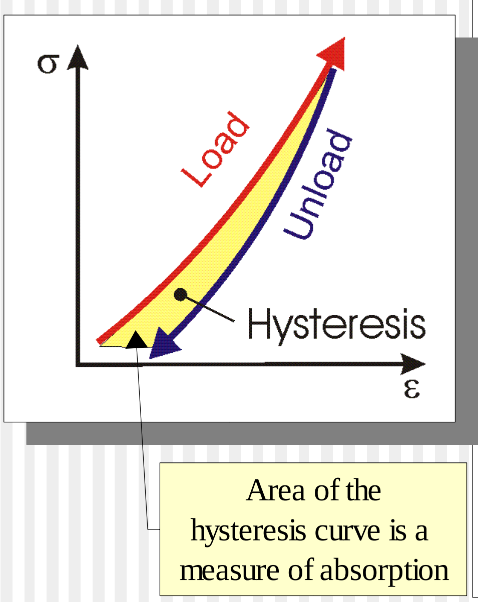
- Reading:
  - Sheriff and Geldart, Sections 2.7; 6.5

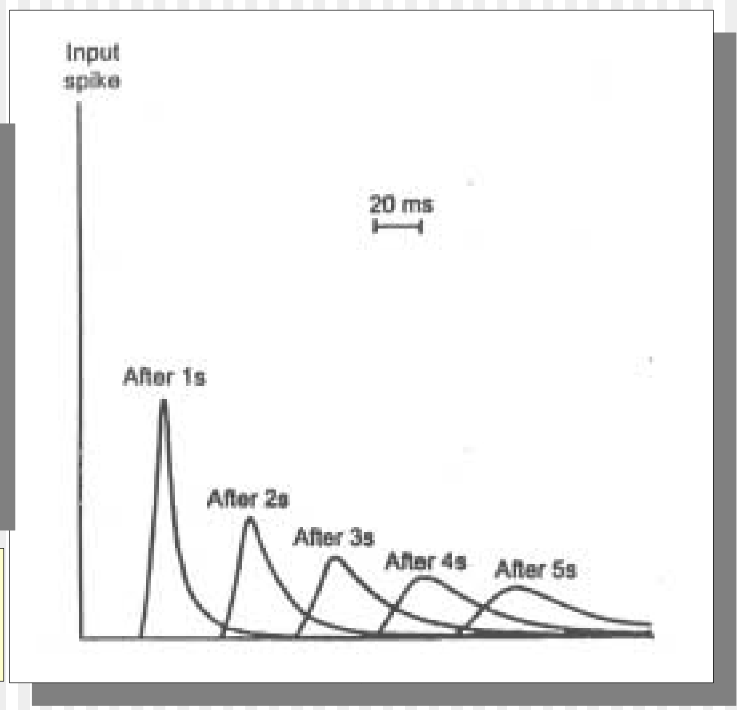
Mechanisms of attenuation

- Three processes lead to reduction of elastic amplitude as the wave propagate away from the source:
  - *Geometrical spreading* - total energy is conserved but distributed over larger wavefronts;
  - *Scattering* (elastic attenuation) – elastic energy is scattered out of the seismic phase of interest;
  - *Inelastic* (intrinsic) attenuation, or *absorption* – elastic energy is converted to heat.

Absorption

- When an elastic wave travels through any medium, its *mechanical* energy is progressively converted to *heat* (through friction and viscosity)
  - On grain boundaries, pores, cracks, water, gas, etc.
  - Loss of elastic energy causes the amplitude to *decrease* and the pulse to *broaden*.





Scattering

- Wavelength- dependent;
- *Scattering regime* is controlled by the ratio of the *characteristic scale length* of the *heterogeneity* of the medium, *a*, to the wavelength.
- Described in terms of *wavenumber*, $k=2\pi/wavelength$:
  - *ka* << 0.01 (quasi-homogeneous medium) - no significant scattering;
  - *ka* < 0.1 (*Rayleigh scattering*) - produces *apparent Q* and anisotropy;
  - 0.1 < *ka* < 10 (*Mie scattering*) - introduces strong attenuation and discernible scattering noise in the signal.
    - typical for high-resolution seismic studies (boulder clay with 0.5-1 m boulders, $V_P \approx 2000$ m/s, $f \approx 500$ Hz

Quality Factor, *Q*

- Attenuation is measured in terms of *rock quality factor*, *Q:*
  - *Q* is (approximately) frequency-independent

$$A(t)=A(0)\exp^{-\alpha x}=A(0)\exp^{\frac{-\pi ft}{Q}}$$

$x=Vt$

  - Amplitude and energy loss per cycle (wavelength):

$$\ln\left(\frac{A(t+T)}{A(t)}\right)=\frac{-\pi fT}{Q}=\frac{-\pi}{Q}$$

This value, in *dB*, is also often used to characterize attenuation

$$\ln\left(\frac{E(t+T)}{E(t)}\right)=\ln\left(\frac{E(t)-\delta E}{E(t)}\right)=\frac{-\delta E}{E(t)}=\frac{-2\pi}{Q}$$

  - Thus, *Q* measures relative energy loss per cycle:

$$Q=2\pi\frac{E}{\delta E}$$

- Typical values:
  - $Q \approx 30$ for weathered sedimentary rocks;
  - $Q \approx 1000$ for granite.

Typical values of *Q*

Table 6.1 *Absorption constants for rocks*

	Q	δ (dB) = $\eta\lambda$
Sedimentary rocks	20–200	0.16–0.02
Sandstone	70–130	0.04–0.02
Shale	20–70	0.16–0.05
Limestone	50–200	0.06–0.02
Chalk	135	0.02
Dolomite	190	0.02
Rocks with gas in pore space	5–50	0.63–0.06
Metamorphic rocks	200–400	0.02–0.01
Igneous rocks	75–300	0.04–0.01

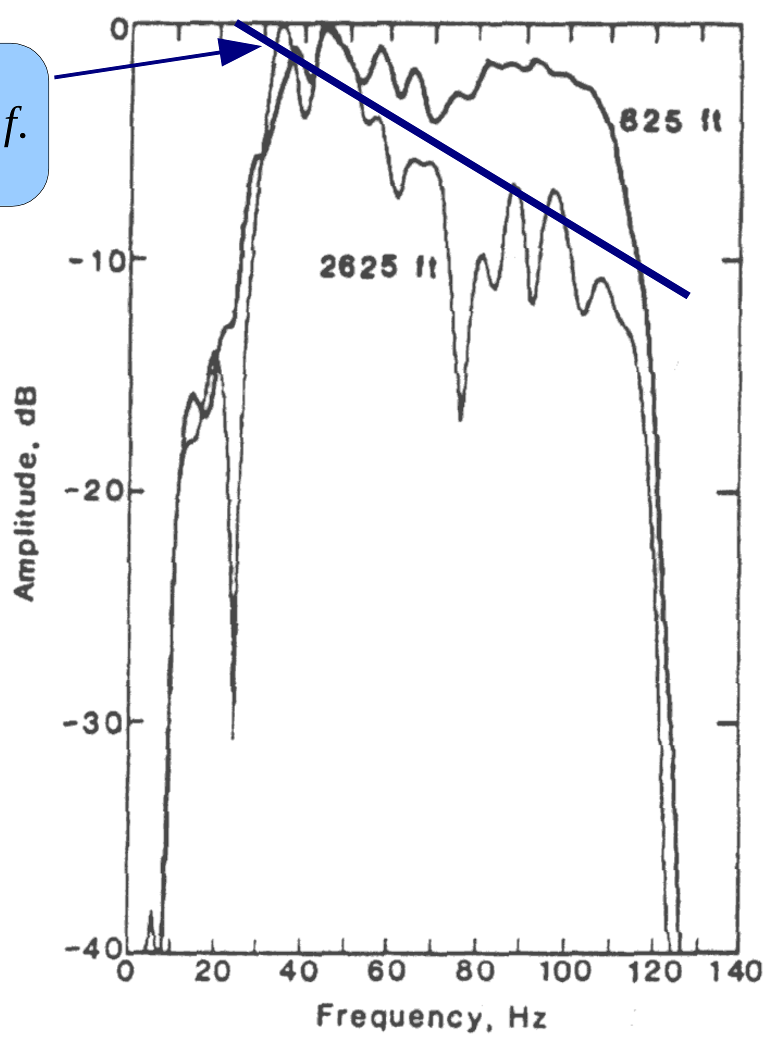
- For sandstones with porosity ϕ% and clay content C %, at 1 MHz and 40 MPa:

$$Q = 179C^{-0.84\phi}$$

Measurement of *Q*

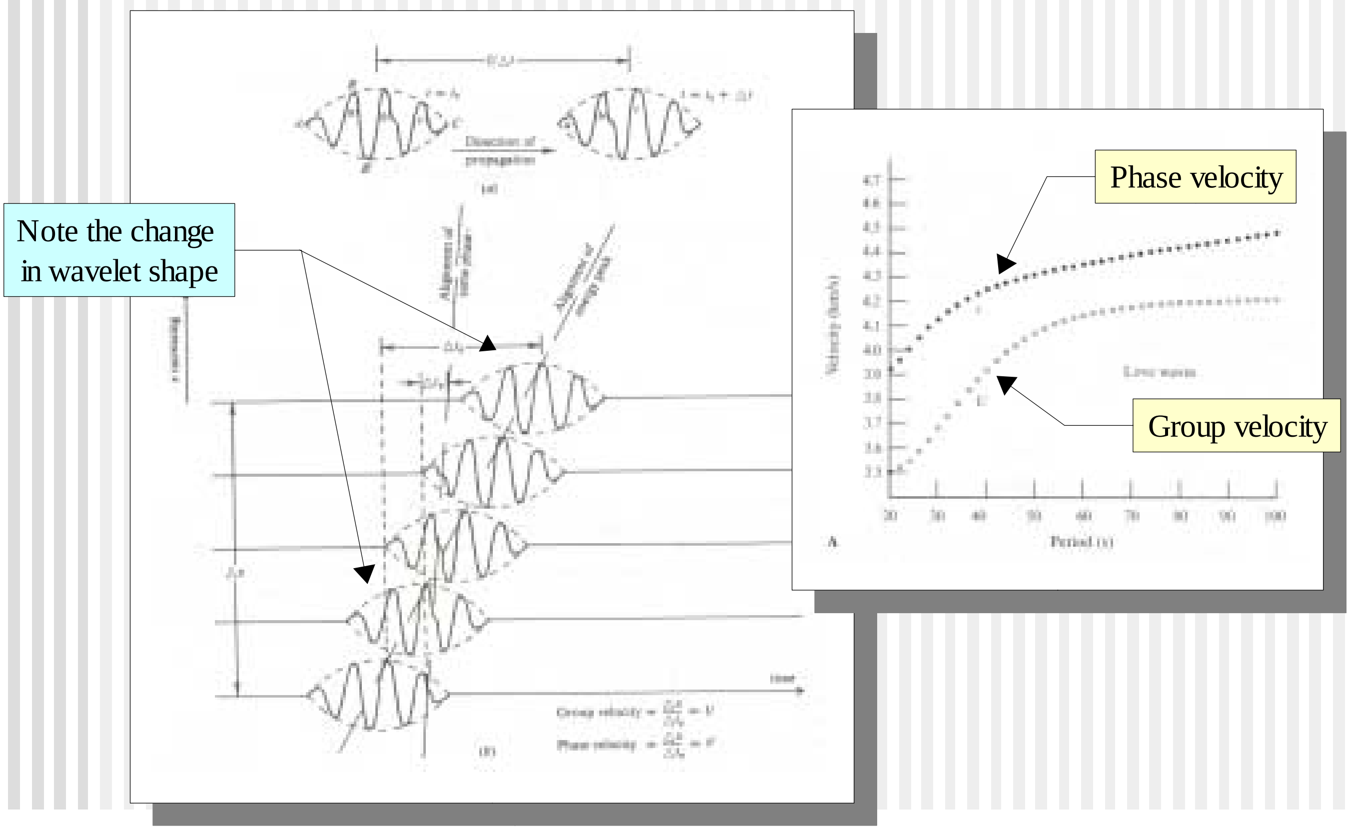
- Take *spectral ratios* of seismic spectra measured at two propagation times
  - The signal in the two windows must be the same in all other respects.

$$\ln\left(\frac{A(f,t_2)}{A(f,t_1)}\right) = -\frac{\pi f\,(t_2 - t_1)}{Q}\,f.$$



Dispersion

- When phase velocity is dependent on frequency, the wave is called *dispersive:*
  - Wavelet changing shape and spreading out when traveling
  - *Group velocity* (velocity of wave packet, *U*) is different from phase velocity (V):
    - U < V – "*Normal dispersion*";
    - U > V – "*Inverse dispersion*".



Group and phase velocities

- Consider a plane harmonic wave:

$$u(x,t) = Ae^{i\varphi(x,t)} = Ae^{i[k(\omega)x - \omega t]}.$$

  - where $k=\omega/V$ is the *wavenumber*.
  - Note that k is dependent on ω.

- *Phase velocity* is the velocity of propagation of the constant-phase plane ($\varphi(x,t)$=const):

$$V_{phase} = \frac{\omega}{k}.$$

- *Group velocity* is the velocity of propagation of the amplitude peak in the wavelet
  - this is the point where the phase is *stationary* (independent on ω):

$$\frac{d[k(\omega)x - \omega t]}{d\omega} = \frac{dk(\omega)}{d\omega}x - t = 0$$

  - hence: $$U_{group} = \left[\frac{dk}{d\omega}\right]^{-1} = \frac{d\omega}{dk}.$$

Group velocity

- Example: two cosine waves with

  $\omega_1 = \omega_0 - \Delta\omega, k_1 = k_0 - \Delta k$

  $\omega_2 = \omega_0 + \Delta\omega, k_2 = k_0 + \Delta k$

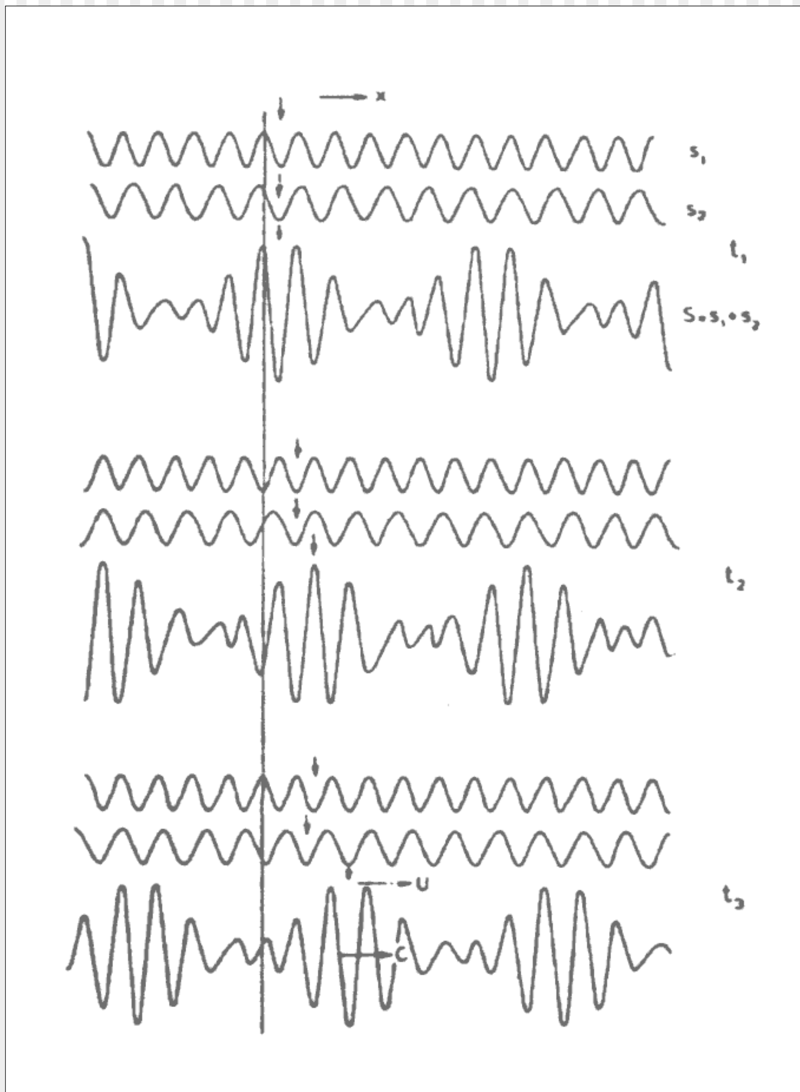
  superimpose to form beats:

  Show that the envelope of these beats travels with group velocity:

  $$U = \frac{\Delta\omega}{\Delta k}.$$

  ...while within the beats, peaks and troughs propagate at approximately:

  $$V = \frac{\omega}{k}.$$



Normal and Inverse dispersion

- When phase velocity is frequency-dependent, group velocity is different from it:

$$U = \frac{d\omega}{dk} = \frac{d(kV)}{dk} = V + k\frac{dV}{dk} = V - \lambda\frac{dV}{d\lambda} \approx V + \omega\frac{dV}{d\omega}.$$

because $k = 2\pi/\lambda = \omega/V$.

- hence:

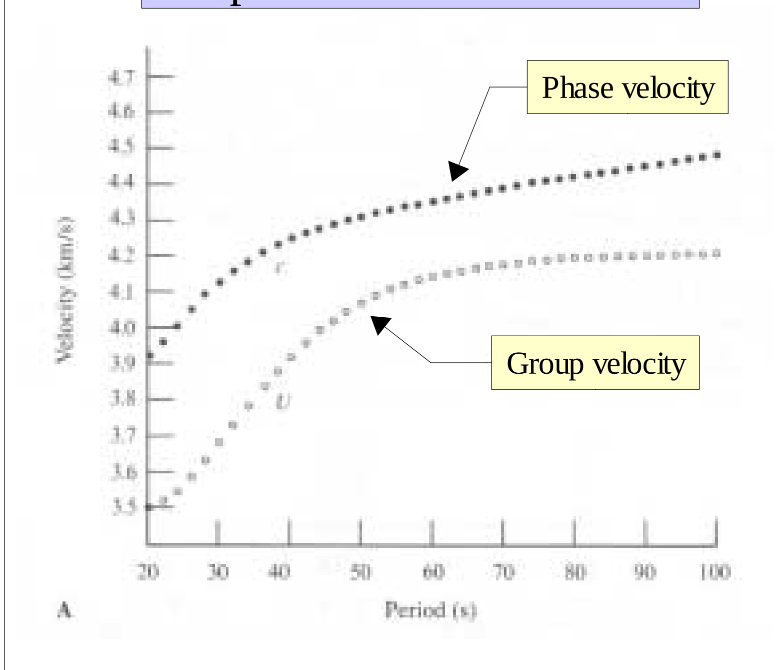
$$\frac{dV}{d\omega} < 0.$$

Normal dispersion
(typically observed in ground roll)

$$\frac{dV}{d\omega} > 0.$$

Inverse dispersion

Example of normal dispersion of Love waves



Attenuation and Dispersion

- Attenuation is *always* associated with dispersion
- Otherwise, attenuating wavelets would spread out symmetrically leading to *noncausuality*
- Thus, attenuating medium is always dispersive.
  - Example: ground roll is quickly attenuated and shows strong normal dispersion.