Attenuation and dispersion

- **Mechanisms:**
  - *Absorption* (inelastic);
  - *Scattering* (elastic).

- Mathematical descriptions
- Measurement
- Frequency dependence
- Dispersion, its relation to attenuation

**Reading:**
- Sheriff and Geldart, Sections 2.7; 6.5
Mechanisms of attenuation

- Three processes lead to reduction of elastic amplitude as the wave propagate away from the source:
  - *Geometrical spreading* - total energy is conserved but distributed over larger wavefronts;
  - *Scattering* (elastic attenuation) – elastic energy is scattered out of the seismic phase of interest;
  - *Inelastic* (intrinsic) attenuation, or *absorption* – elastic energy is converted to heat.
Absorption

- When an elastic wave travels through any medium, its *mechanical* energy is progressively converted to *heat* (through friction and viscosity)
  - On grain boundaries, pores, cracks, water, gas, etc.
  - Loss of elastic energy causes the amplitude to *decrease* and the pulse to *broaden*.

Area of the hysteresis curve is a measure of absorption
Scattering

- Wavelength-dependent;
- **Scattering regime** is controlled by the ratio of the characteristic scale length of the heterogeneity of the medium, $a$, to the wavelength.
- Described in terms of wavenumber, $k = \frac{2\pi}{\text{wavelength}}$:
  - $ka << 0.01$ (quasi-homogeneous medium) - no significant scattering;
  - $ka < 0.1$ (Rayleigh scattering) - produces apparent $Q$ and anisotropy;
  - $0.1 < ka < 10$ (Mie scattering) - introduces strong attenuation and discernible scattering noise in the signal.
  - typical for high-resolution seismic studies (boulder clay with 0.5-1 m boulders, $V_p \approx 2000$ m/s, $f \approx 500$ Hz)
Quality Factor, $Q$

- Attenuation is measured in terms of *rock quality factor*, $Q$:
  - $Q$ is (approximately) frequency-independent
    \[ A(t) = A(0) \exp^{-\alpha x} = A(0) \exp^{-\frac{-\pi f t}{Q}} \]
    \[ x = V t \]
  - Amplitude and energy loss per cycle (wavelength):
    \[ \ln \left( \frac{A(t+T)}{A(t)} \right) = -\pi f T \frac{T}{Q} = -\frac{\pi}{Q} \]
    \[ \ln \left( \frac{E(t+T)}{E(t)} \right) = \ln \left( \frac{E(t) - \delta E}{E(t)} \right) = -\frac{\delta E}{E(t)} = -2\pi \frac{E}{Q} \]
  - Thus, $Q$ measures relative energy loss per cycle:
    \[ Q = 2\pi \frac{E}{\delta E} \]

- Typical values:
  - $Q \approx 30$ for weathered sedimentary rocks;
  - $Q \approx 1000$ for granite.
**Typical values of \( Q \)**

**Table 6.1 Absorption constants for rocks**

<table>
<thead>
<tr>
<th>Category</th>
<th>( Q )</th>
<th>( \delta ) (dB) = ( \eta \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedimentary rocks</td>
<td>20–200</td>
<td>0.16–0.02</td>
</tr>
<tr>
<td>Sandstone</td>
<td>70–130</td>
<td>0.04–0.02</td>
</tr>
<tr>
<td>Shale</td>
<td>20–70</td>
<td>0.16–0.05</td>
</tr>
<tr>
<td>Limestone</td>
<td>50–200</td>
<td>0.06–0.02</td>
</tr>
<tr>
<td>Chalk</td>
<td>135</td>
<td>0.02</td>
</tr>
<tr>
<td>Dolomite</td>
<td>190</td>
<td>0.02</td>
</tr>
<tr>
<td>Rocks with gas in pore space</td>
<td>5–50</td>
<td>0.63–0.06</td>
</tr>
<tr>
<td>Metamorphic rocks</td>
<td>200–400</td>
<td>0.02–0.01</td>
</tr>
<tr>
<td>Igneous rocks</td>
<td>75–300</td>
<td>0.04–0.01</td>
</tr>
</tbody>
</table>

- For sandstones with porosity \( \phi \)% and clay content \( C \)% at 1 MHz and 40 MPa:
  \[
  Q = 179C^{-0.84\phi}
  \]
Measurement of $Q$

- Take *spectral ratios* of seismic spectra measured at two propagation times
  - The signal in the two windows must be the same in all other respects.

\[
\ln \left( \frac{A(f, t_2)}{A(f, t_1)} \right) = -\frac{\pi f(t_2 - t_1)}{Q} f.
\]
Dispersion

- When phase velocity is dependent on frequency, the wave is called **dispersive:**
  - Wavelet changing shape and spreading out when traveling
  - **Group velocity** (velocity of wave packet, $U$) is different from phase velocity ($V$):
    - $U < V$ – “Normal dispersion”;
    - $U > V$ – “Inverse dispersion”.

![Diagram showing wave dispersion](image)
Group and phase velocities

Consider a plane harmonic wave:

\[ u(x, t) = Ae^{i\varphi(x, t)} = Ae^{i[k(\omega)x - \omega t]} \]

where \( k = \frac{\omega}{V} \) is the wavenumber.

Note that \( k \) is dependent on \( \omega \).

**Phase velocity** is the velocity of propagation of the constant-phase plane \((\varphi(x, t) = \text{const})\):

\[ V_{\text{phase}} = \frac{\omega}{k}. \]

**Group velocity** is the velocity of propagation of the amplitude peak in the wavelet

- this is the point where the phase is stationary (independent on \( \omega \)):

\[
\frac{d}{d\omega} \left[ k(\omega)x - \omega t \right] = \frac{dk(\omega)}{d\omega} x - t = 0
\]

- hence:

\[ U_{\text{group}} = \left[ \frac{dk}{d\omega} \right]^{-1} = \frac{d\omega}{dk}. \]
Group velocity

Example: two cosine waves with

\[ \omega_1 = \omega_0 - \Delta \omega, \quad k_1 = k_0 - \Delta k \]
\[ \omega_2 = \omega_0 + \Delta \omega, \quad k_2 = k_0 + \Delta k \]

superimpose to form beats:

Show that the envelope of these beats travels with group velocity:

\[ U = \frac{\Delta \omega}{\Delta k} \]

...while within the beats, peaks and troughs propagate at approximately:

\[ V = \frac{\omega}{k} \]
Normal and Inverse dispersion

- When phase velocity is frequency-dependent, group velocity is different from it:
  \[ U = \frac{d\omega}{dk} = \frac{d(kV)}{dk} = V + k \frac{dV}{dk} = V - \frac{\lambda}{d\lambda} \frac{dV}{d\omega} \approx V + \omega \frac{dV}{d\omega}. \]

  because \( k = \frac{2\pi}{\lambda} = \frac{\omega}{V}. \)

- hence:

  \[ \frac{dV}{d\omega} < 0. \]
  **Normal dispersion**
  (typically observed in ground roll)

  \[ \frac{dV}{d\omega} > 0. \]
  **Inverse dispersion**

Example of normal dispersion of Love waves
Attenuation and Dispersion

- Attenuation is *always* associated with dispersion.
- Otherwise, attenuating wavelets would spread out symmetrically leading to *noncausuality*.
- Thus, attenuating medium is always dispersive.
  - Example: ground roll is quickly attenuated and shows strong normal dispersion.