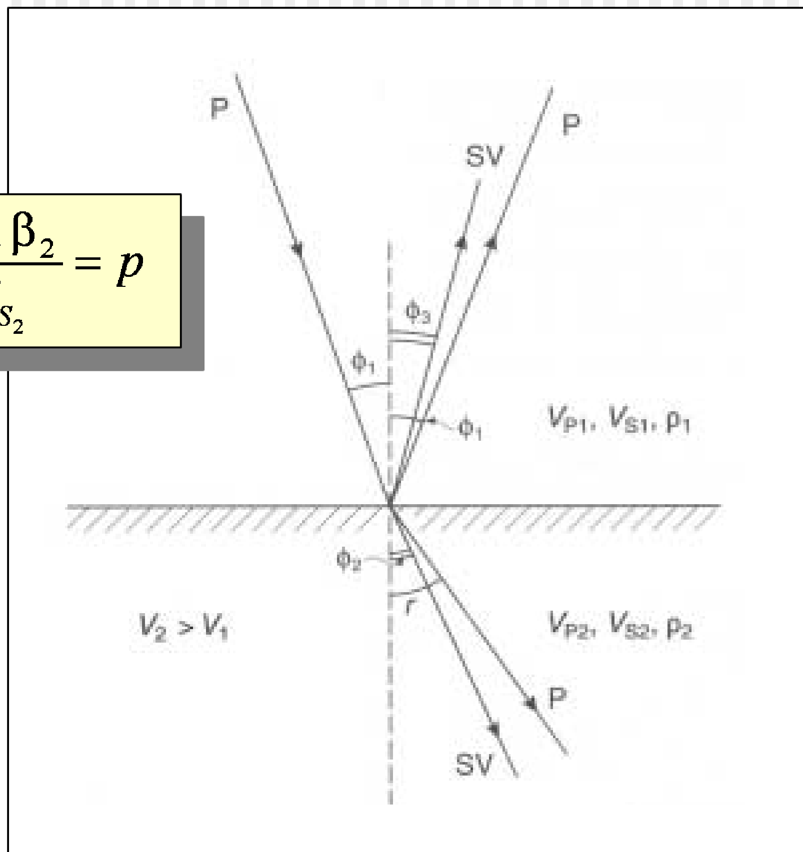
Geometrical Seismics
Refraction

- Refraction paths
  - Head waves
  - Diving waves
- Effects of vertical velocity gradient

- <u>Reading:</u>
  - Sheriff and Geldart, Chapter 4.2 - 4.3.

Snell's Law of Refraction

- When waves (rays) penetrate a medium with different velocity, they *refract*, i.e. bend toward or away from the normal to the velocity boundary.
- The *Snell's Law of refraction* relates the angles of incidence and emergence of waves refracted on a velocity contrast:



$$\frac{\sin i}{V_{P_1}} = \frac{\sin r}{V_{P_2}} = \frac{\sin \beta_1}{V_{S_1}} = \frac{\sin \beta_2}{V_{S_2}} = p$$

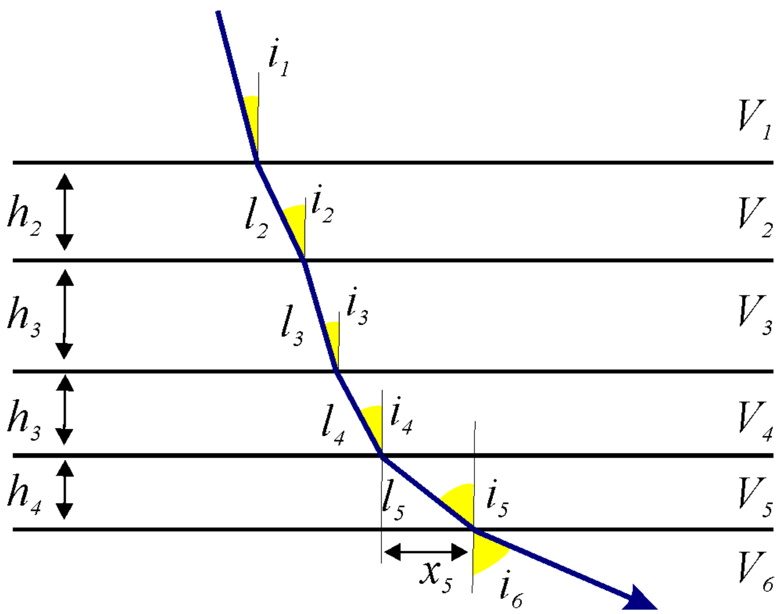
- The constant p is called *ray parameter*
- Note that refraction angles depend on the velocities alone!

Refraction in a stack of horizontal layers

Ray parameter, *p*, *uniquely specifies* the entire ray.

It does not depend on layer thicknesses or velocities.

Travel times and distances accumulate along the ray to yield the total *T*(*X*)



For any layer: $\sin i_k = pV_k$

$$l_k = \frac{h_k}{\cos i_k} = \frac{h_k}{\sqrt{1-(pV_k)^2}}$$

$$t_k = \frac{l_k}{V_k} = \frac{h_k}{V_k\sqrt{1-(pV_k)^2}}$$

$$x_k = l_k \sin i_k = \frac{h_k(pV_k)}{\sqrt{1-(pV_k)^2}}$$

$$T_n = \sum_{k=1}^{n} t_k \qquad X_n = \sum_{k=1}^{n} x_k$$

Critical Angle of Refraction

- Consider a faster medium overlain with a lower-velocity layer (this is a typical case).
- *Critical angle* of incidence in the slower layer is such that the refracted waves (rays) travel horizontally in the faster layer (sin *r* = 1)
- The critical angles thus are:

$$i_C = \sin^{-1}\frac{V_{P_1}}{V_{P_2}} \quad \textit{for P-waves,}$$

$$i_C = \sin^{-1}\frac{V_{S_1}}{V_{S_2}} \quad \textit{for S-waves.}$$

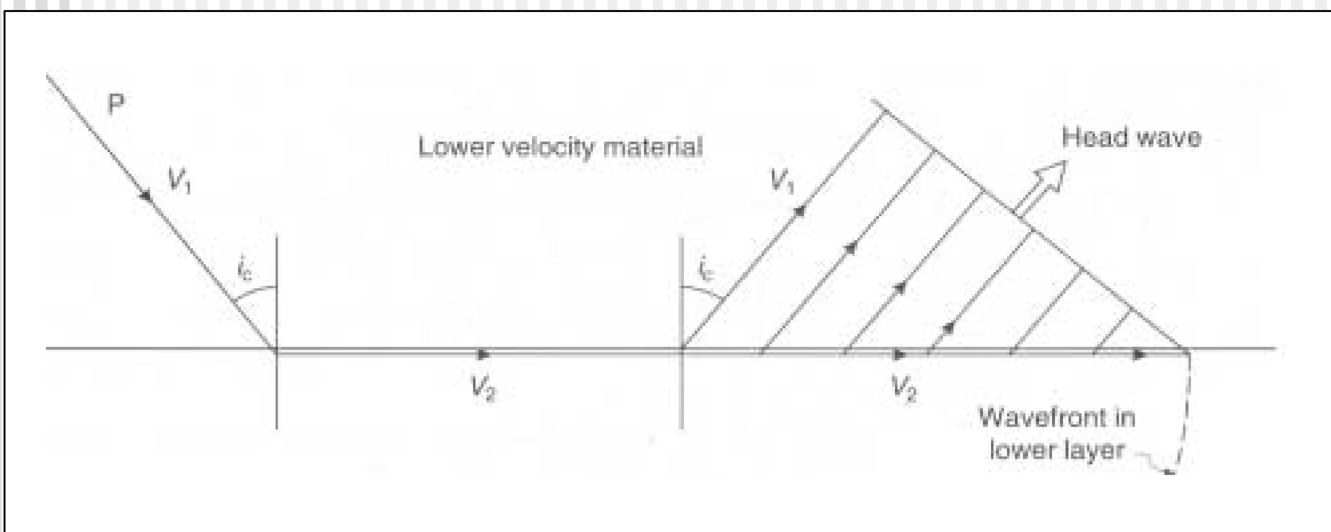
- Critical *ray parameter*: $p^{critical} = \frac{1}{V_{refractor}}$
- If the incident wave strikes the interface at an angle exceeding the critical angle, *no refracted or head wave is generated*.
- Note that i_C should better be viewed as a *property of the interface*, not of a particular ray.

Head wave

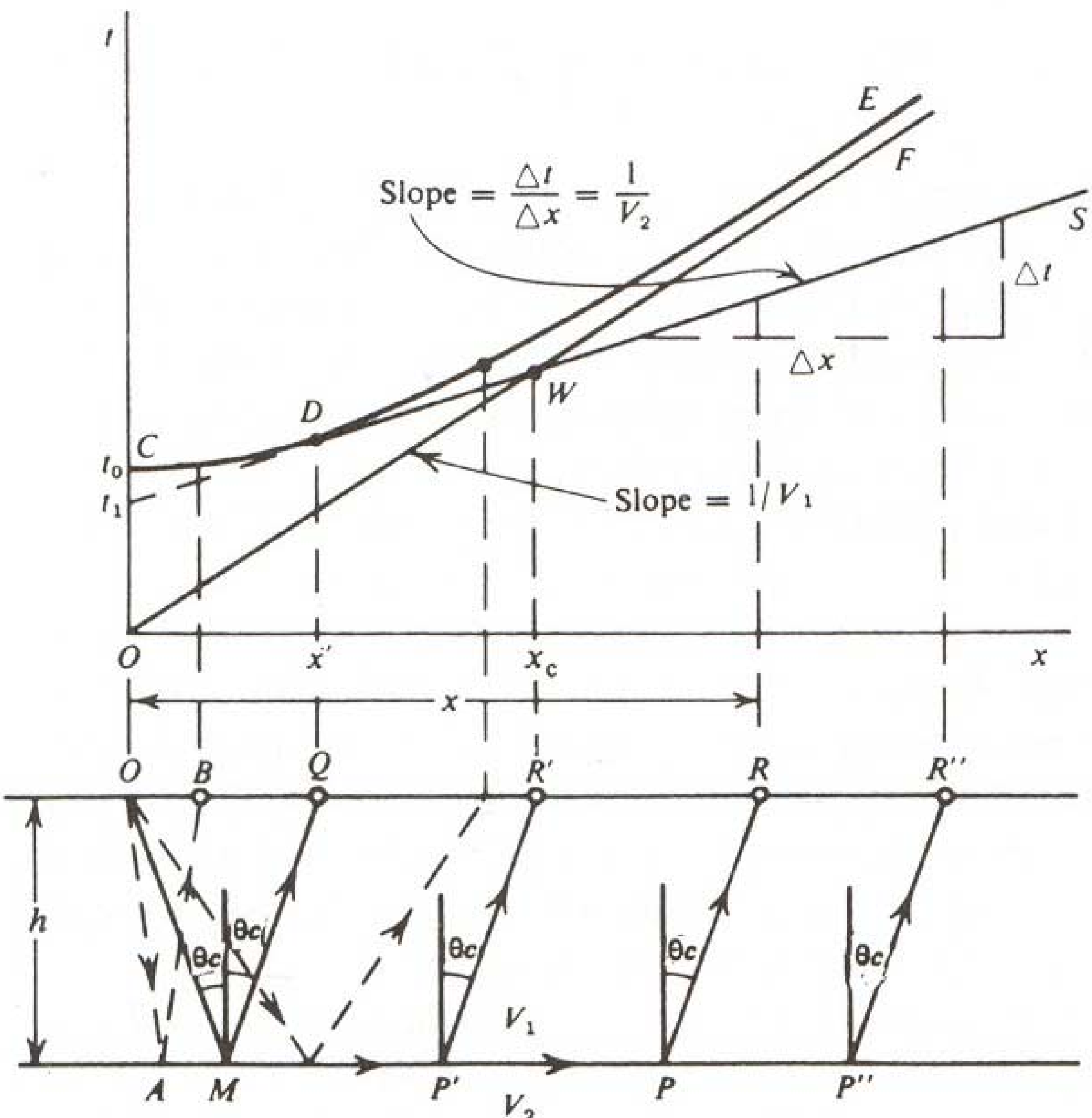
- At critical incidence in the upper medium, a *head wave* is generated in the lower one.
- Although head waves carry very little energy, they are useful approximation for interpreting seismic wave propagation in the presence of strong velocity contrasts.
- Head waves are characterized by planar wavefronts inclined at the critical angle in respect to the velocity boundary. Their travel-time curves are straight lines:

$$t = t_0 + \frac{x}{V_{app}}$$

Here, t_0 is the *intercept time*, and V_{app} is the *apparent velocity*.

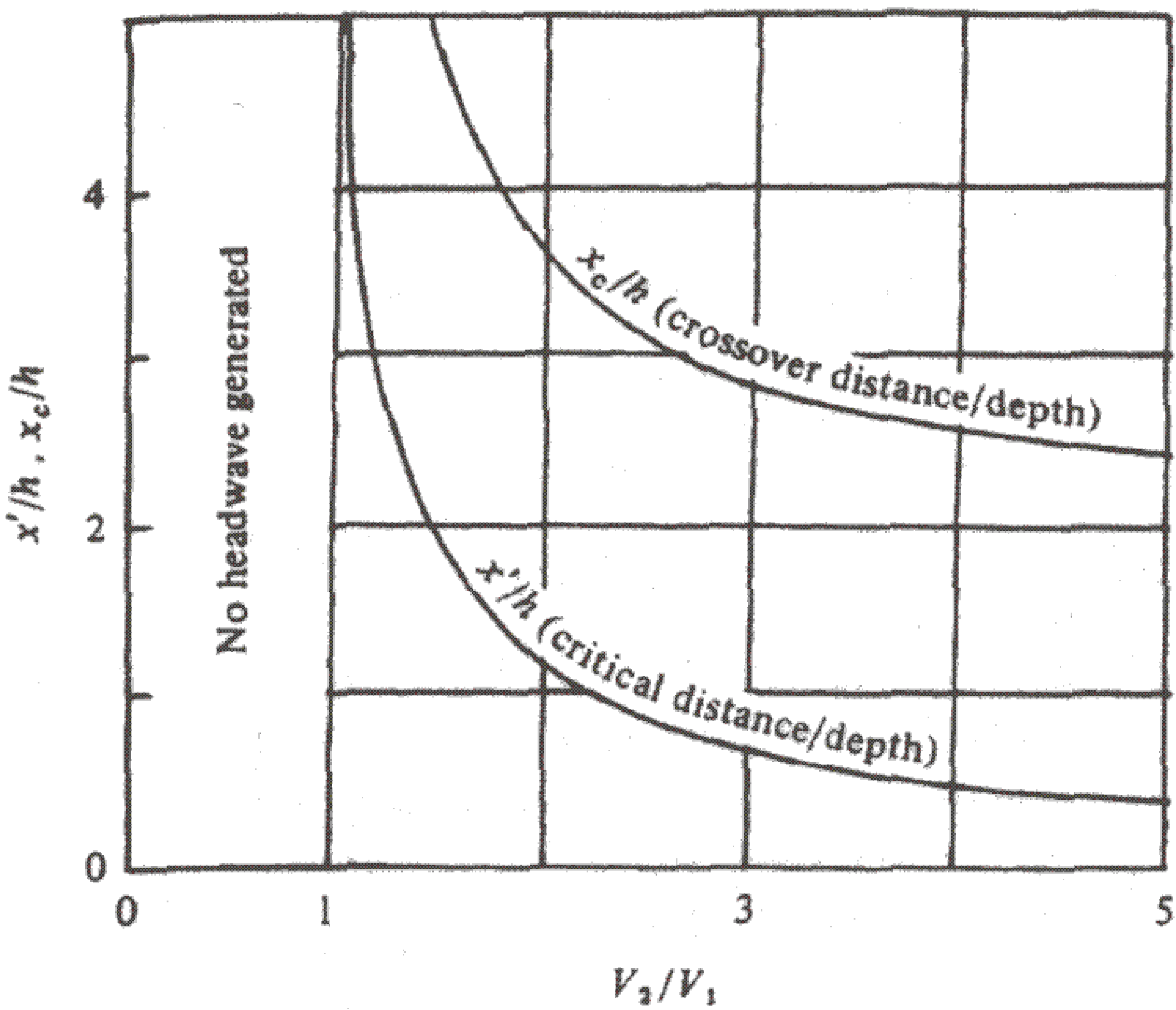


Relation between Reflection and Refraction travel-times



From Sheriff and Geldart, 1995

Critical and Cross-over distances vs. Velocity contrast



- Note that the distances are *proportional* to the depth and *decrease* with increasing velocity contrast across the interface

Travel times
(Horizontal refractor)

- Direct wave:

$$t(x) = \frac{1}{V_1}.$$

- Head wave:

$$p = \frac{1}{V_2}$$

this is also sin i

$$\sin i = pV_1 \qquad \cos i = \sqrt{1 - (pV_1)^2}$$

$$t = 2\frac{h_1}{V_1 \cos i} + p(x - 2h_1 \tan i) = \frac{2h_1}{V_1 \cos i}(1 - pV_1 \sin i) + px = \frac{2h_1}{V_1}\cos i + px$$

$$t_0 = \frac{2h_1}{V_1}\cos i = \frac{2h_1}{V_1}\sqrt{1 - (pV_1)^2}$$

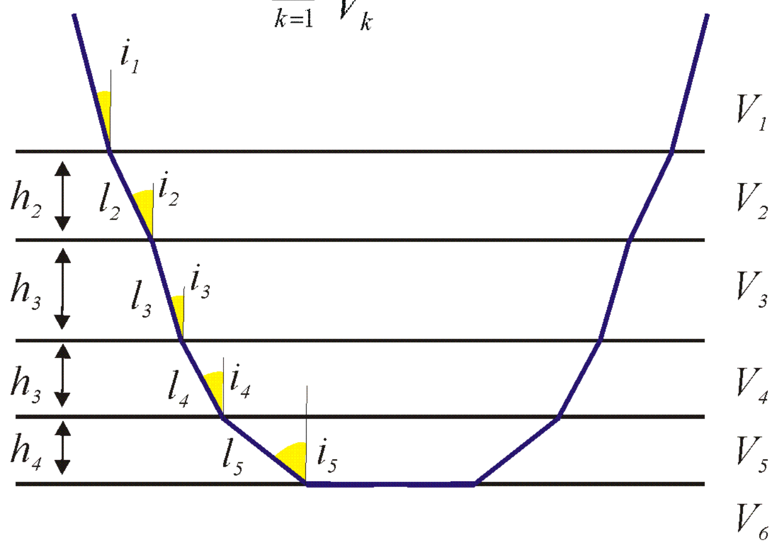
intercept time, t_0

Travel times
(Multiple horizontal layers)

- *p* is the same *critical ray parameter* for the bottom (refracting) interface;
- t_0 is accumulating across the layers:

$$p = \frac{1}{V_{refractor}}$$

$$t = \sum_{k=1}^{n} \frac{2h_k}{V_k} \cos i_k + px$$



For any layer: $\sin i_k = pV_k$

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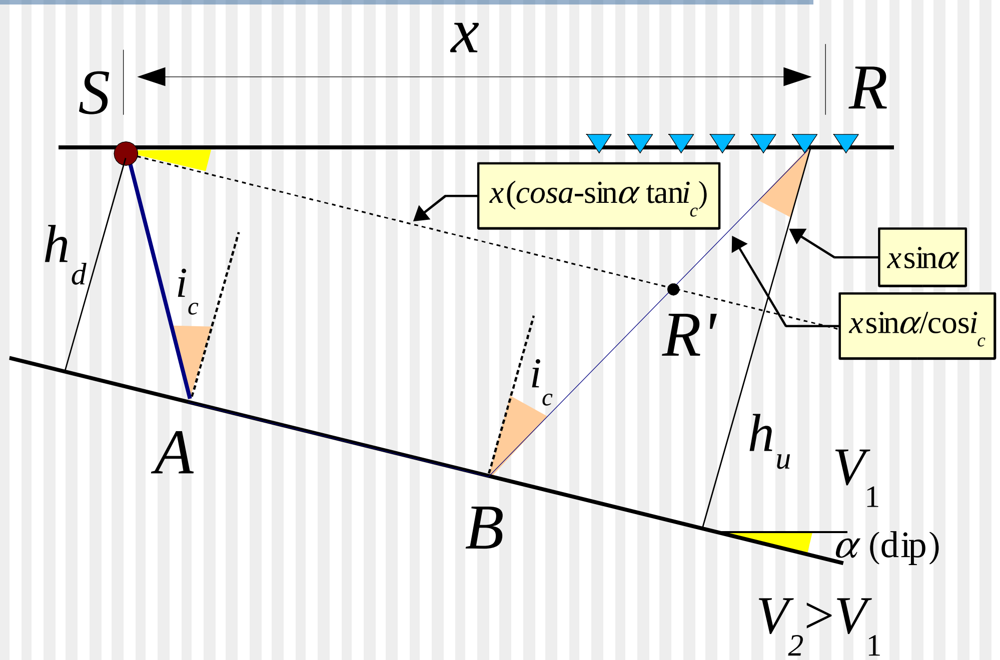
$$x_k = l_k \sin i_k = \frac{h_k (pV_k)}{\sqrt{1-(pV_k)^2}}$$

Travel times
(Dipping refractor)



$$t = \frac{2h_d}{V_1}\cos i_c + \frac{1}{V_2}x(\cos\alpha - \sin\alpha\,\tan i_c) + \frac{1}{V_1}\frac{x\sin\alpha}{\cos i_c}$$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1\cos i_c}\left[\overbrace{\frac{V_1}{V_2}}^{\sin i_c}(\cos\alpha\,\cos i_c - \sin\alpha\,\sin i_c) + \sin\alpha\right]$$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1\cos i_c}\left[\cos\alpha\,\cos i_c\,\sin i_c + \sin\alpha\,\cos^2 i_c\right]$$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\left[\cos\alpha\,\sin i_c + \sin\alpha\,\cos i_c\right]$$

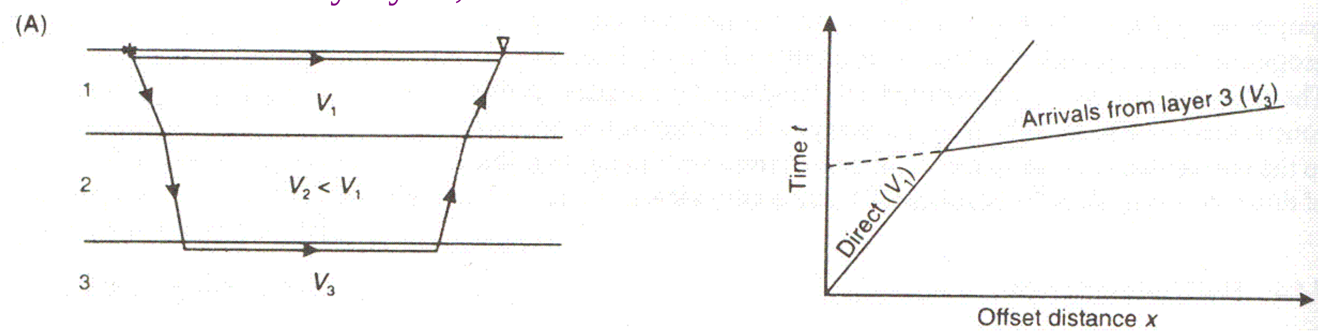
$$\boxed{= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\sin(i_c + \alpha)}$$

would change to '-' for up-dip shooting

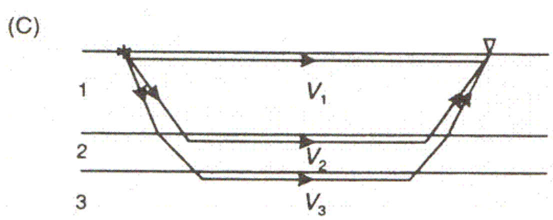
Hidden-Layer Problem

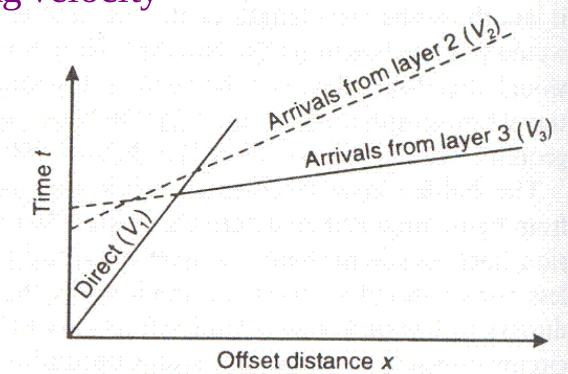
- Velocity contrasts *may not manifest themselves* in refraction (first-arrival) travel times. Three typical cases:

Low-velocity layers;

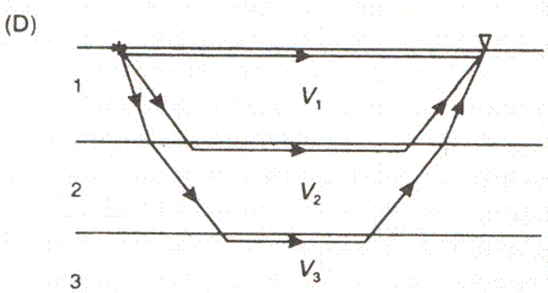


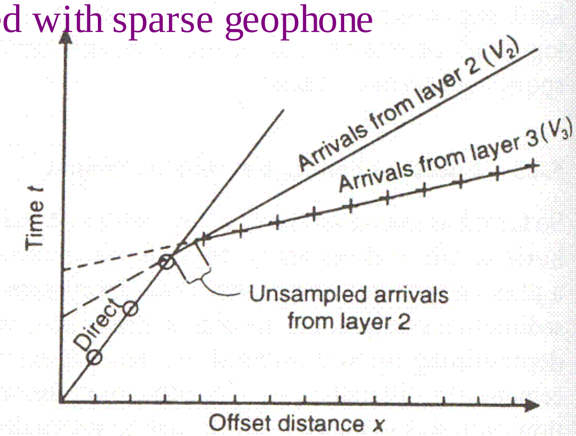
Relatively thin layers on top of a strong velocity contrast;





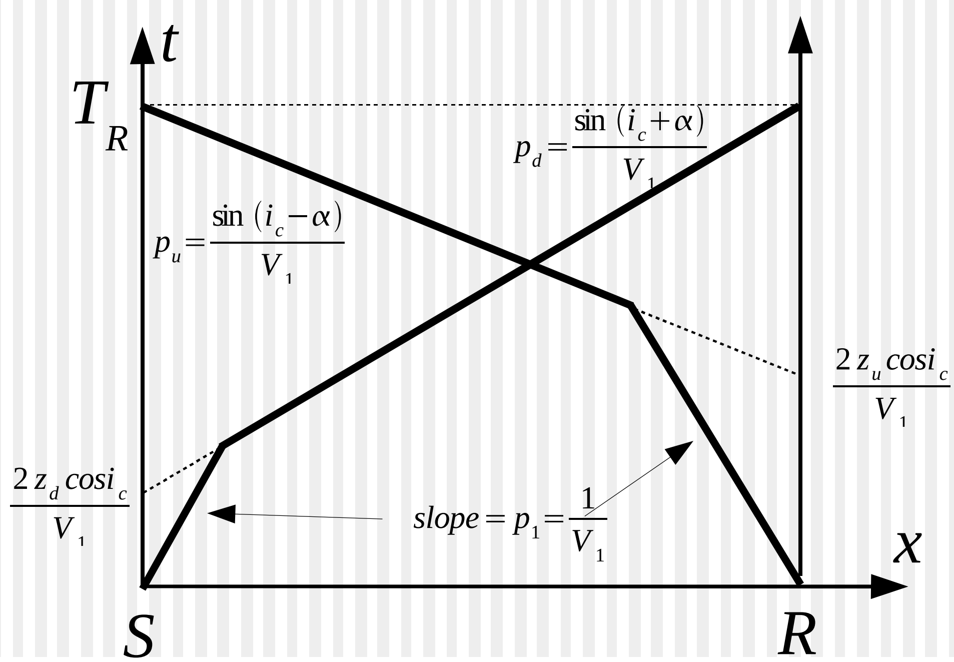
Short travel-time branch may be missed with sparse geophone coverage.





Reversed travel times

- One needs *reversed* recording (in opposite directions) for resolution of dips.
- The *reciprocal times*, T_R, must be the the same for reversed shots.
- Dipping refractor is indicated by:
  - Different *apparent velocities* (=1/p, TTC slopes) in the two directions;
    - determine V_2 and α (refractor velocity and dip).
  - Different *intercept times*.
    - determine h_d and h_u (interface depths).



Determination of refractor velocity and dip

- *Apparent velocity* is $V_{app} = 1/p$, where p is the *ray parameter (*i.e., slope of the travel-time curve).
  - Apparent velocities are measured directly from the observed TTCs;
  - $V_{app} = V_{refractor}$ only in the case of a horizontal layering.
  - For a dipping refractor:
    - Down dip: $V_d = \frac{V_1}{\sin(i_c + \alpha)}$ (*slower than* V_1);
    - Up-dip: $V_u = \frac{V_1}{\sin(i_c - \alpha)}$ (*faster*).
- From the two reversed apparent velocities, i_c and α are determined:

$$i_c + \alpha = \sin^{-1}\frac{V_1}{V_d}, \quad i_c - \alpha = \sin^{-1}\frac{V_1}{V_u} \mapsto \boxed{\begin{aligned} i_c &= \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} + \sin^{-1}\frac{V_1}{V_u}\right), \\ \alpha &= \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} - \sin^{-1}\frac{V_1}{V_u}\right). \end{aligned}}$$

- From i_c, the refractor velocity is: $\boxed{V_2 = \frac{V_1}{\sin i_c}.}$

Approximation of small refractor dip

- If refractor dip is small:

$$\frac{V_1}{V_d} = \sin(i_c + \alpha) \approx \sin i_c + \alpha \cos i_c,$$

$$\frac{V_1}{V_u} = \sin(i_c - \alpha) \approx \sin i_c - \alpha \cos i_c,$$

  and therefore:

$$\sin i_c \approx \frac{V_1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right).$$

- and:

$$\frac{1}{V_2} \approx \frac{1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right).$$

- Thus, the *slowness of the refractor* is approximately the mean of the up-dip and down-dip *apparent slownesses*.

Diving waves

- Consider velocity gradually increasing with depth: $V(z)$.
- Rays will bend upward at any point and eventually will return to the surface
  - Such waves are called *diving waves*.
- An *implicit* solution for the travel-time curve (x,t) can be obtained from the multiple-layer refraction formulas:

$$x(p) = 2\int_0^{h_{max}} \frac{pV(z)\,dz}{\sqrt{1-(pV(z))^2}},$$

$$t(p) = 2\int_0^{h_{max}} \frac{dz}{V(z)\sqrt{1-(pV(z))^2}},$$

where h_m is the depth at which $pV(h_m)=1$.

Diving waves

Linear increase of velocity with depth

- Consider: $V(z) = V_0 + az$.

  a is generally between 0.3-1.3 1/s.

- Hence, denoting $u=pV=\sin i$:

Parametric representation of the (x,z,t) through u

$$x(u)=\int_{z_0}^{z}\frac{pV\,dz}{\sqrt{1-(pV)^2}}=\frac{1}{pa}\int_{u_0}^{u}\frac{u\,du}{\sqrt{1-u^2}}=$$

$$=\frac{1}{pa}\left(\sqrt{1-u^2}-\sqrt{1-u_0^2}\right)\equiv\frac{1}{pa}\sqrt{1-u^2}+x_c$$

$$z(u)=\frac{1}{pa}(u-u_0)=\frac{1}{pa}u+z_c$$

Denote the constants (center of the circular ray path)

- The raypath is a *circle*:

$$(x-x_c)^2+(z-z_c)^2=\left(\frac{1}{pa}\right)^2.$$

- and time: $$t(p)=\int_{z_0}^{z}\frac{dz}{V\sqrt{1-(pV)^2}}=\frac{1}{a}\int_{0}^{h_{max}}\frac{du}{u\sqrt{1-u^2}}=$$

$$=\frac{1}{a}\ln\left[\frac{u}{1-\sqrt{1-u^2}}\right].$$

Diving waves

Layers with low velocities and high velocity gradients create complex travel-time curves