Surface waves

- Rayleigh and Love waves
- Particle motion
- Phase and group velocity
- Dispersion of surface waves

- Reading:
  - Sheriff and Geldart, 2.5
  - Telford et al., 4.2.4, 4.2.6

Mechanism

- Surface waves are always associated with a boundary.
- The (e.g., horizontal) boundary disrupts vertical wave propagation *but* provides for special wave modes propagating along it.
- Because there are 2 or 4 boundary conditions to satisfy (e.g., displacement and stress continuity), surface waves are always build up of 2 or 4 interacting wave modes:
  - *P* and *SV* wave modes (*Rayleigh* or *Stoneley* waves);
  - Two *SH* modes (*Love* waves).
- Such waves are “tied” to the surface and exponentially diminish away from it.

Surface wave potentials

- General wave equations for potentials:

$$\nabla^2 \phi = \frac{1}{V_P^2} \frac{\partial^2 \phi}{\partial t^2}, \qquad \textit{P}\text{-wave}$$

$$\nabla^2 \psi_V = \frac{1}{V_S^2} \frac{\partial^2 \psi_V}{\partial t^2}, \qquad \textit{SV}\text{-wave}$$

$$\nabla^2 \psi_H = \frac{1}{V_S^2} \frac{\partial^2 \psi_H}{\partial t^2}. \qquad \textit{SH}\text{-wave}$$

- Surface waves are combinations of solutions with *complex* (e.g., pure imaginary) wavenumbers along *z.*
  - e.g., for Rayleigh wave:

$$\phi = A e^{-mz} e^{i(kx - \omega t)},$$

$$\psi_V = B e^{-nz} e^{i(kx - \omega t)}.$$

  - Question: why are not such solutions allowed without a boundary?

Depth dependence

- To satisfy the wave equations for any *k* and ω, *m* and *n* must equal (*show this*):

$$m^2 = \sqrt{k^2 - \frac{\omega^2}{V_P^2}},$$

P-wave component in Rayleigh wave

$$n^2 = \sqrt{k^2 - \frac{\omega^2}{V_S^2}}.$$

SV-wave component

  - note that therefore, for <u>*any*</u> surface wave:

$$k > \frac{\omega}{V_S} > \frac{\omega}{V_P},$$

    and so $V_{Surface} = \frac{\omega}{k} < V_S$.

- To further describe the solution, we need to:
  1) consider ω and *A* as free variables;
  2) determine B and $k(\omega)$ from the <u>*boundary conditions*</u>.

"Dispersion relation"

Rayleigh waves
("ground roll")

- *Rayleigh waves* propagate along the free surface
- The displacements are as usual:

$$\vec{u}_P(\vec{x},\vec{z}) = \left(\frac{\partial \phi}{\partial x}, 0, \frac{\partial \phi}{\partial z}\right),$$

P-wave

$$\vec{u}_S(\vec{x},\vec{z}) = \left(\frac{-\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x}\right),$$

SV-wave

- and traction:

$$\vec{F}_P(\vec{x},\vec{z}) = \left(2\mu \frac{\partial^2 \phi}{\partial x \partial z}, 0, \lambda \nabla^2 \phi + 2\mu \frac{\partial^2 \phi}{\partial z^2}\right),$$

$$\vec{F}_S(\vec{x},\vec{z}) = \left(\mu\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}\right), 0, \lambda \nabla^2 \phi + 2\mu \frac{\partial^2 \psi}{\partial x \partial z}\right),$$

- For a free surface, the boundary conditions read: $\sigma_{xz} = \sigma_{zz} = 0$,
- Solution:

$$\phi = e^{-mz} e^{i(kx - \omega t)},$$

$$\psi_V = B e^{-nz} e^{i(kx - \omega t)}.$$

we can set A=1 and seek B and $k(\omega)$

Rayleigh waves
("ground roll")

- Result (e.g., for σ=0.25), relative P- and S-wave amplitudes:

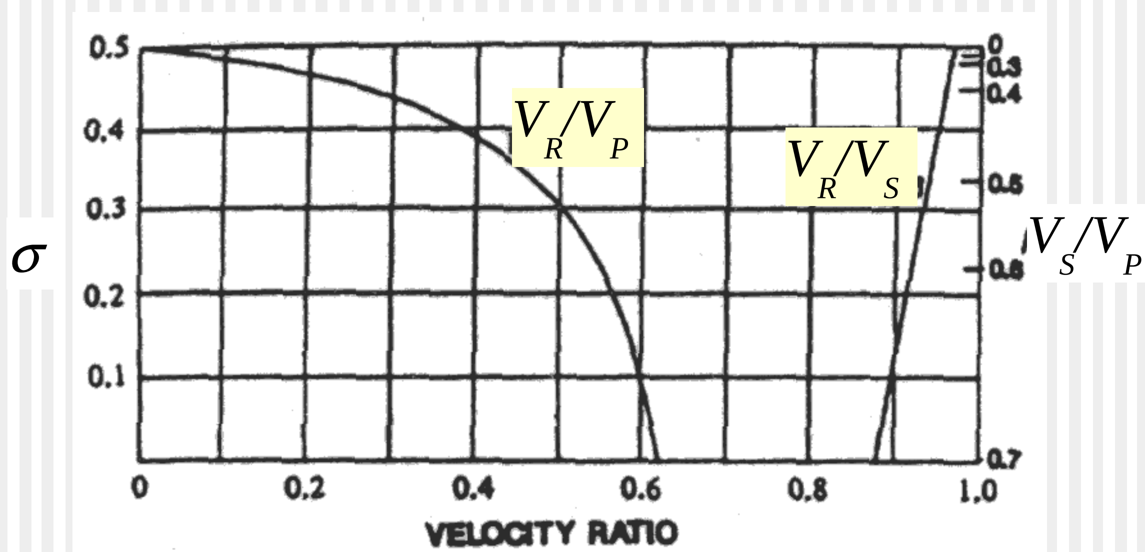
$$\phi = e^{-0.848kz} e^{i(kx-\omega t)}, \quad \textit{P}\text{-wave}$$

$$\psi_V = 1.468 i e^{-0.393z} e^{i(kx-\omega t)}. \quad \textit{SV}\text{-wave}$$

- ...and dispersion relation:

$$k = V_R \omega.$$ **(This means *no* dispersion!)**

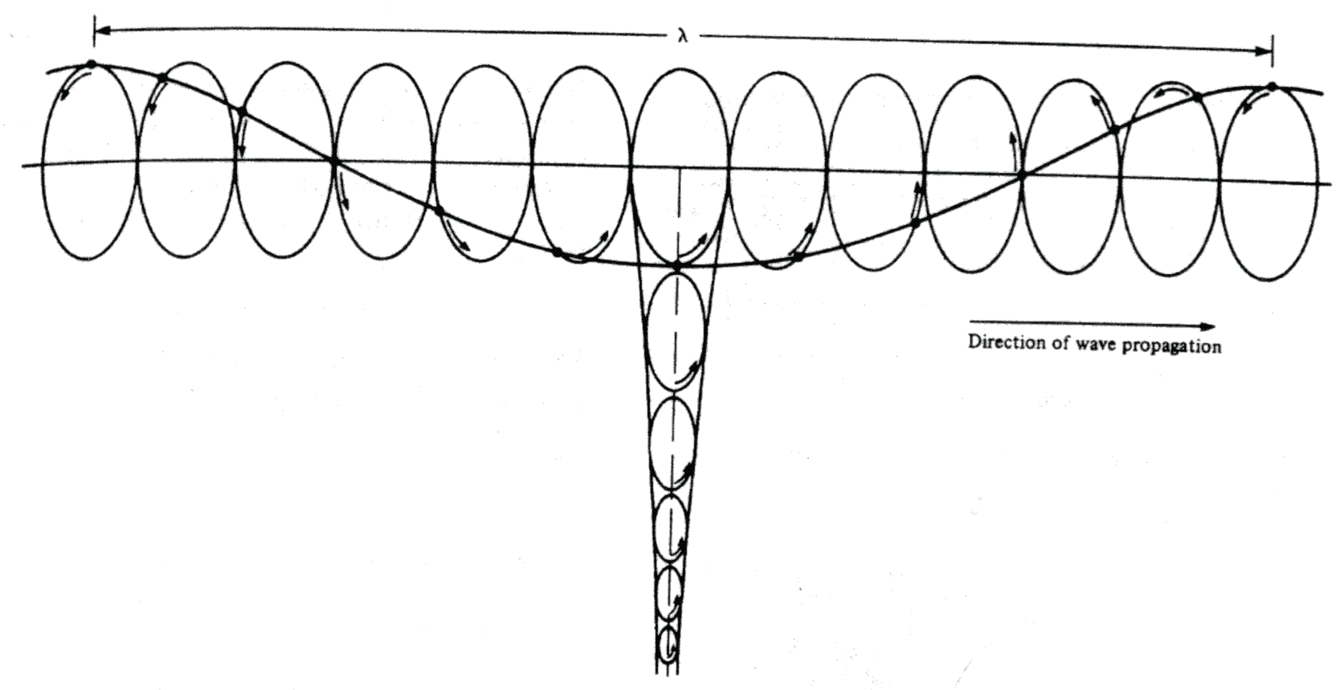
- Rayleigh wave velocity
- $V_R = 0.919 V_S$
- For varying σ:



Rayleigh waves
("ground roll")

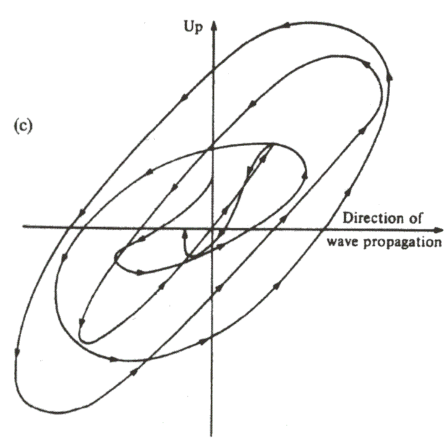
How does it follow from the equations for potentials and displacements?

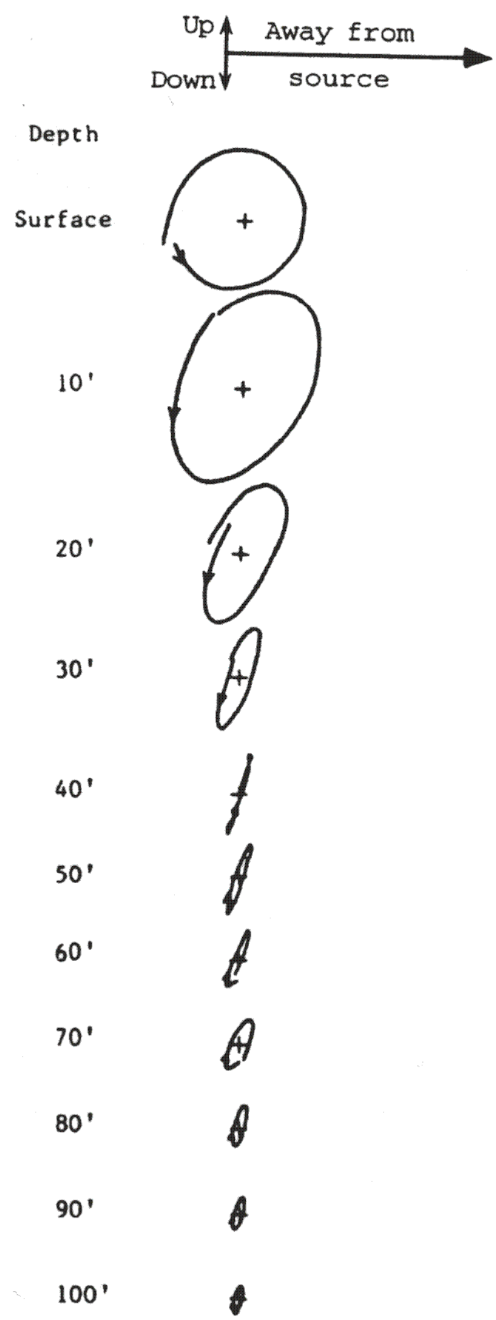
- Particle motion is elliptical and *retrograde* (counter-clockwise when the wave is moving left to right):



Rayleigh waves
("ground roll")

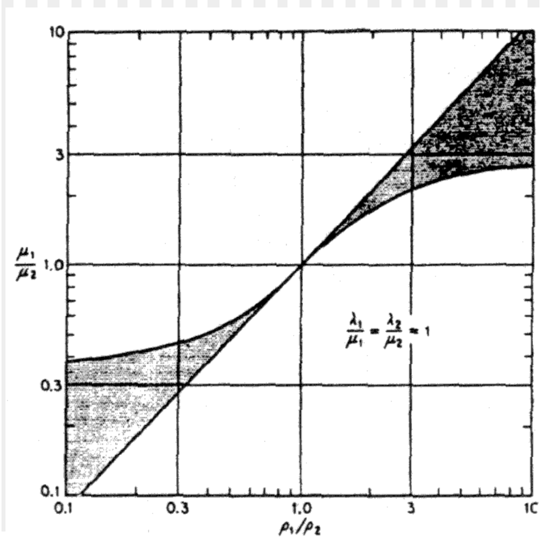
- Real Earth is never a *uniform* half-space, and thus in Rayleigh waves:
  - Particle motion paths are tilted and complex;
  - Retrograde motion may change into prograde at depth;
  - Normal dispersion *is present*.





Stoneley waves

- These waves propagate along the contact of *two* semi-infinite media
  - They are *P*/*SV* in nature, like Rayleigh waves;
  - They always exist when one of the media is a fluid;
    - An important example is *tube wave* propagating along a fluid-filled borehole.
  - If both media are solids, Stoneley waves exist only when $V_{S1} \approx V_{S2}$ and ρ and μ lie within narrow limits:



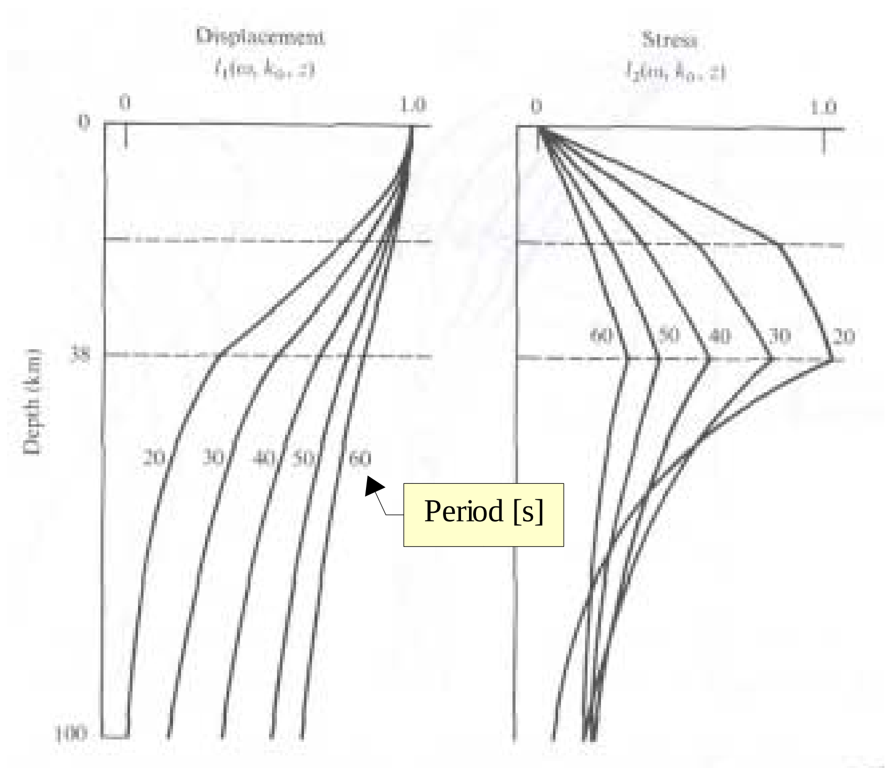
Love waves

- These are *SH*-type waves propagating along the free surface
  - particle motion is parallel to the surface;
- Because there is just one *SH* potential, *two* modes are required to satisfy the two boundary conditions ($\sigma_{xz}=\sigma_{zz}=0$ on the free surface)
- Thus, Love waves exist when the semi-infinite medium is overlain with a layer with different elastic properties.
  - ... this situation is quite common.

Love waves

- Love waves is dispersive:
- At high frequencies, its velocity approaches the S-wave velocity in the surface layer
- At low frequencies, velocity is close to that of the lower layer.

Displacement and stress in Love wave



Depth of sampling increases with period. This is common to all surface waves.

Dispersion

- Ideal Rayleigh wave (in uniform half-space) is non-dispersive
- However, all *real* surface waves exhibit dispersion.