Tomography and Location

- Least-Squares inverse
- Generalised Linear Inverse
- Forward and Inverse travel-time problems
- Seismic tomography
- Location of seismic sources
- Reading:

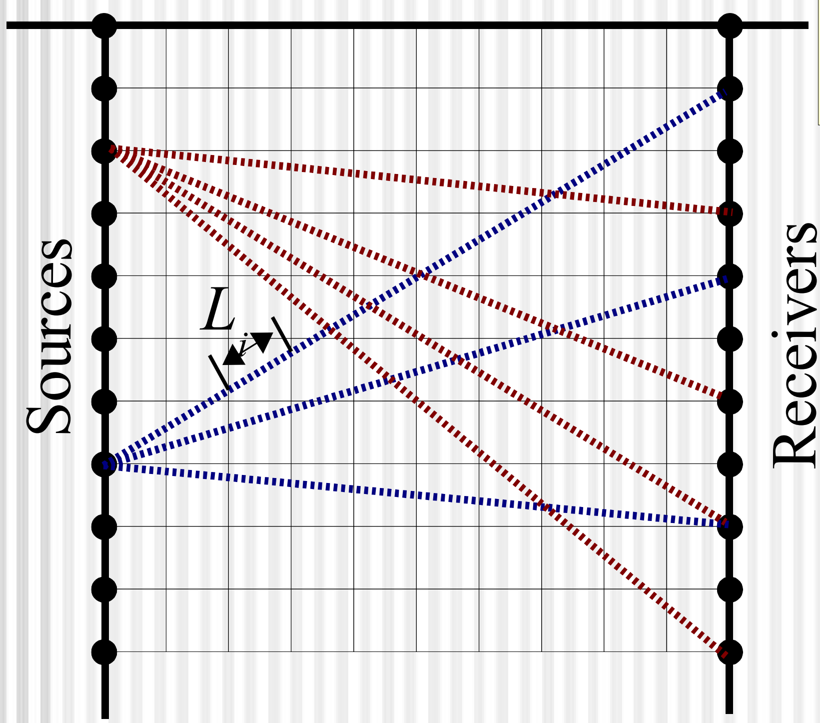
Seismic (velocity) tomography

- Tomography
  - The name derived from the Greek for “section drawing” - the idea is that the section appears *almost* automatically...
  - Using multitude of source-receive pairs with rays crossing the area of interest.
  - Looking for an unknown velocity structure.
  - Depending on the type of recording used, it could be:
    - *Transmission tomography* (nearly straight rays between boreholes);
    - *Reflection tomography* (reflected rays; in this case, positions of the reflectors could be also found);
    - *Diffraction tomography* (using least-time travel paths according to Fermat rather than Snell's law; this is actually more a waveform inversion technique).

Cross-well tomography

- Consider the case of transmission "cross-well" tomography first.
  - This is the simplest case – rays may be considered nearly straight, the data are abundant, and the coverage is *relatively* uniform.

These are the three principal concerns in tomography:
1) linearity of the problem;
2) density of data coverage;
3) good azimuthal coverage.



Travel-time inversion as a *linear inverse problem*

- First, we parameterize the velocity model
  - Typically, the parameterization is a grid of constant-velocity blocks (sometimes splines are used instead of the blocks).
  - This parameterization gives us a *model vector,* **m**.

$$\boldsymbol{m} = \begin{pmatrix} s_1 = 1/V_1 \\ s_2 = 1/V_2 \\ \dots \\ s_N = 1/V_N \end{pmatrix}.$$

- Second, we measure all travel times and arrange them into a data vector:

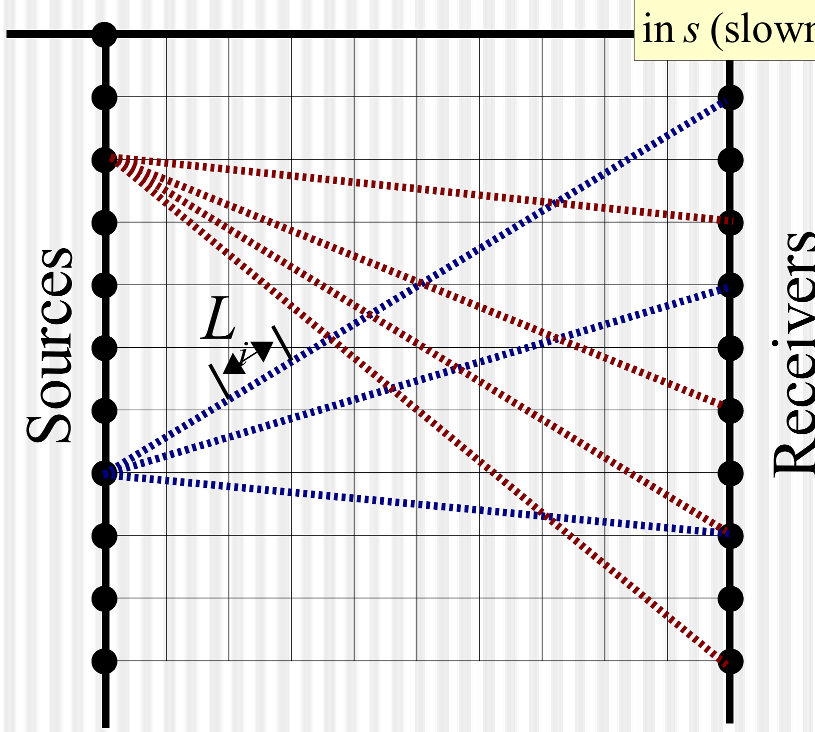
$$\boldsymbol{d}^{observed} = \begin{pmatrix} t_1 \\ t_2 \\ \dots \\ t_M \end{pmatrix}.$$

Forward model

- Third, we formulate the *forward model* to predict **d** from **m**. To achieve this, we need to *trace rays* through the model and measure the length of every ray's segment in each model block, L_{ij}.
  - The travel time for *i*-th ray is then:

$$t_i = \sum_j L_{ij} \frac{1}{V_j} = \sum_j L_{ij} s_j .$$

Note that the expression is not linear in *V* but linear in *s* (slowness).



Generalized Linear Inverse

- The model for travel times: $t_i = \sum_j L_{ij} s_j$ can be written in matrix form:
$$d = Lm.$$
- Now, we want to substitute **d**=**d**^{observed} and solve for unknown **m**. This is called the *inverse problem*.
- Typically, matrix *L* is not invertible (it is not square), and so it is inverted in some *generalized* (averaged) sense.
- In particular, tomography problems are typically overdetermined (contain many more ray paths then grid model blocks), and the *Least Squares Generalized Inverse* works well:
  - multiply by transposed **L**^{T}:
  $$L^T d^{observed} = L^T L m,$$
  - hence:
  $$m = (L^T L)^{-1} L^T d^{observed}.$$

(This is used in the famous GLI method for refraction statics)

Least Squares Inverse

- Note that the solution is a linear combination of data values:

$$m = (L^T L)^{-1} L^T d^{observed} = L_g^{-1} d^{observed}.$$

The "generalized inverse" matrix

- The reason for its name of "Least Squares" is in its minimizing the mean square of data misfits:

$$Misfit(m) = (d^{observed} - Lm)^T (d^{observed} - Lm).$$

  - Exercise: show this!

Damped Least Squares

- Sometimes the matrix $\mathbf{L}^T\mathbf{L}$ is singular and its inverse is unstable.
  - This happens, e.g., when some cells are not crossed by any rays, or there are groups of cells traversed by the same rays only.
- In such cases, the inversion can be *regularized* by adding a small positive diagonal term to $\mathbf{L}^T\mathbf{L}$:

$$m=(L^T L+\varepsilon I)^{-1} L^T d^{observed} .$$

  - This is called the *Damped Least Squares* solution.
  - ε is chosen such that stability is achieved and the non-zero contributions in $\mathbf{L}^T\mathbf{L}$ are affected only slightly.

Resolution matrix

- Assessment of the *quality of inversion method* is often done by using the *Resolution Matrix*
  - Regardless of the selected form of the inverse, we can:
    1) Perturb 1 parameter (grid node) of the model;
    2) Perform forward modeling (generate synthetic data);
    3) Perform the inverse.
  - When repeated for each parameter, this process results in a resolution matrix:

$$R = L_g^{-1} L$$

- Note that **R** *does not* depend on the data values but depends on sampling
  - Crossing rays are VERY important in tomography.

Source Location Problem

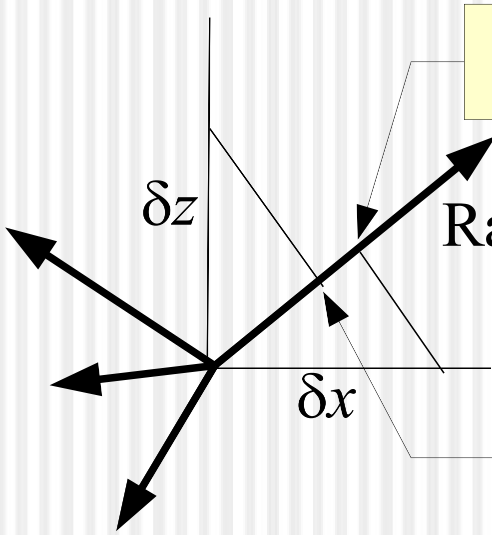
- When using a natural (yet impulsive) source, its location can also be determined by a similar approach.
  - This method is used for location of earthquakes worldwide...
  - for monitoring creep of mine walls (Potash Corp)
  - monitoring reservoirs while doing injection (Weyburn)
- To solve this problem, we:
  - Start from some reasonable approximation for source coordinates and solve the velocity tomography problem.
  - Add the coordinates of the source to model vector:

$$\boldsymbol{m} = \begin{pmatrix} s_1 = 1/V_1 \\ s_2 = 1/V_2 \\ \dots \\ s_N = 1/V_N \\ x_{source} \\ z_{source} \end{pmatrix}.$$

(In two dimensions)

Source Location (cont.)

- Include into the matrix **L** time delays associated with shifting the source by δx or δz:



$$\delta t = \frac{-\cos\alpha}{V}\delta x \equiv L_{i,N+1}\delta x$$

$$\delta t = \frac{-\sin\alpha}{V}\delta z \equiv L_{i,N+2}\delta z$$

- Now, when solved, the Generalized Inverse will yield the corrections to the location (δx, δz).

- This process is often <u>iterated</u>: with the new source location, velocities are recomputed, and sources relocated again, etc.