Common Concepts

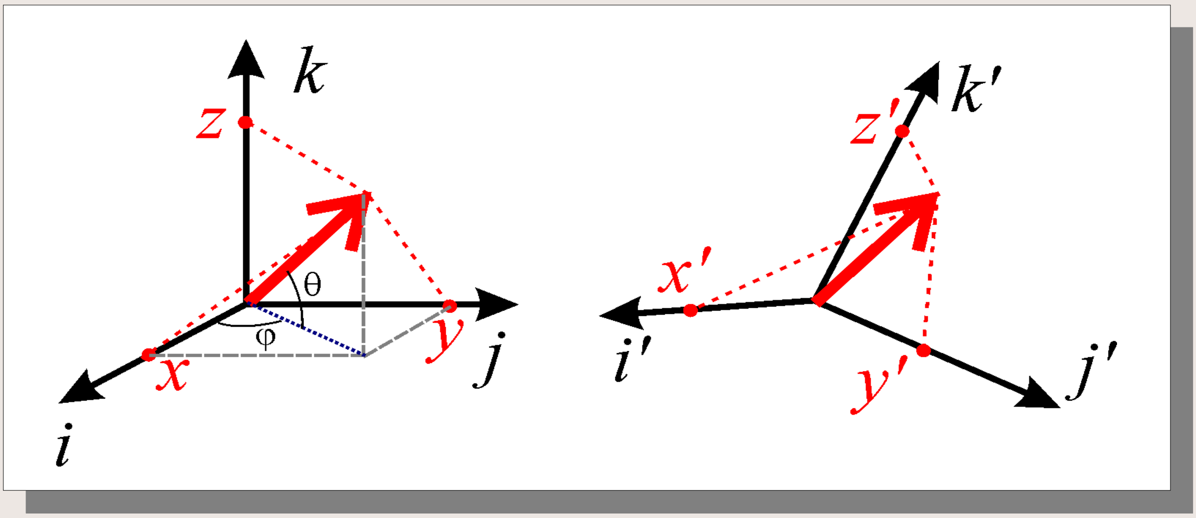
(parlance of geo-Physics; for information only)

- Scalars, Vectors, Tensors
- Matrices, Determinants
- Fields
- Differential equations
- Complex numbers
- Waves

- Reading:
  - Telford et al., Sections A.2-3, A.5, A.7
  - Sheriff and Geldart, Chapter 15.1

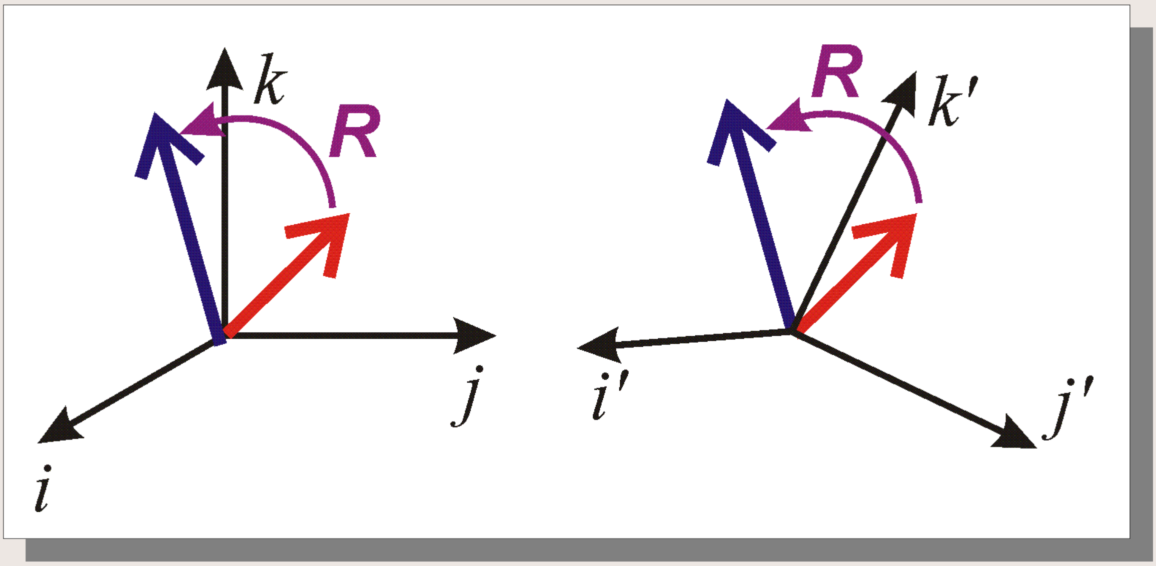
Vector

- Directional quantity
  - Possesses 'amplitude' and 'direction' and nothing else...
    - Thus it can be described by its amplitude and two directional angles (e.g., *azimuth* and *dip*).
  - Characterized by projections on three selected axes: (*x*,*y*,*z*)...
    - ...plus an agreement that the projections are transformed appropriately whenever the frame of reference is rotated.



Tensor
(informal)

- Bi-Directional quantity
  - 'Relationship' between two vectors;
  - Represented by a *matrix:*
    - 3×3 in three-dimensional space, 2×2 in two dimensions, etc.
    - ...this matrix, however, is transformed whenever the frame of reference is rotated.
- Examples:
  - Rotation operator, ***R*** in the plot below;
  - Stress and strain in an elastic body.



Vector operations

- Summation: $\boldsymbol{c} = \boldsymbol{a} + \boldsymbol{b}$

$$c_x = a_x + b_x, c_y = a_y + b_y, c_z = a_z + b_z.$$

- Scaling: $\boldsymbol{c} = \lambda \boldsymbol{b}$

$$c_x = \lambda b_x, c_y = \lambda b_y, c_z = \lambda b_z.$$

- Scalar (dot) product:

$$c = \boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z.$$

- Vector (cross) product:

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b} = \begin{Vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{Vmatrix}.$$

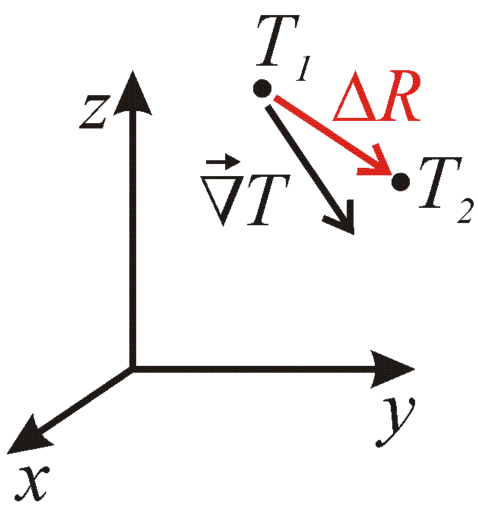
Field

- Physical quantity which takes on values at a continuum of points in space and/or time
  - Represented by a function of coordinates and/or time:
    - Scalar: $f(x, y, z, t)$ or $f(\boldsymbol{r}, t)$
      - Examples: temperature, density, seismic velocity, pressure, gravity, electric potential
    - Vector: $\boldsymbol{F}(\boldsymbol{r}, t)$
      - Examples: particle displacement, velocity, or acceleration, electric or magnetic field, current
    - Tensor
      - 'relation' between two vectors
      - Examples: strain and stress, electromagnetic field in electrodynamics
      - The only way to describe *anisotropy*
  - Always associated with some *source*, carries some kind of *energy*, and usually able to propagate *waves*
- *Everything in physics is fields!*

Scalar Fields

Some characteristics

- Gradient
  - Spatial derivative of a scalar field (say, temperature, T(x,y,z,t))
  - It is a Vector field, denoted ∇T ('nabla' T):



$$\Delta T = T_2 - T_1$$

$$= \vec{\nabla} T \cdot \Delta \vec{R}$$

$$= \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\partial T}{\partial z}\Delta z$$

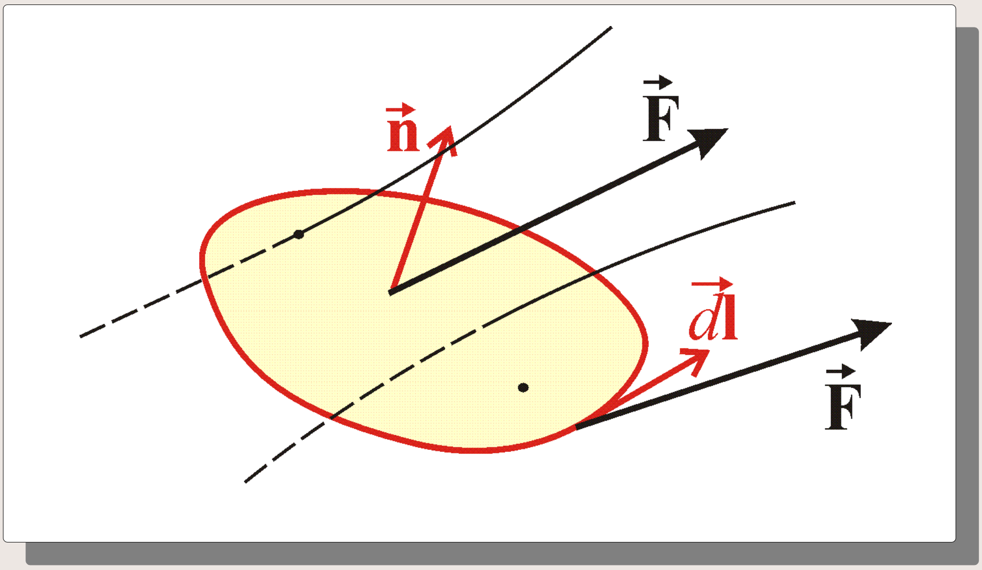
$$\Delta \vec{R} = \vec{i}\,\Delta x + \vec{j}\,\Delta y + \vec{k}\,\Delta z$$

$$\vec{\nabla} T = \vec{i}\,\frac{\partial T}{\partial x} + \vec{j}\,\frac{\partial T}{\partial y} + \vec{k}\,\frac{\partial T}{\partial z}$$

Vector Fields

Some characteristics

- Flux through a loop =
  - (Average normal component of **F**) times (Surface area of the loop)
  - $Flux(\boldsymbol{F}) = \iint \boldsymbol{F} \cdot \boldsymbol{n}\, dS$
- Circulation along a loop =
  - (Average tangential component of F) times (Length of the loop)
  - $Circulation(\boldsymbol{F}) = \int \boldsymbol{F} \cdot \boldsymbol{dl}$



Vector Fields

Two Important Results

- Gauss's Theorem - relates a flux out of a closed surface to a 'volume' integral:

$$\underset{\text{Surface}}{\oiint} \boldsymbol{E} \cdot \boldsymbol{n}\, ds = \underset{\text{Volume}}{\iiint} \underline{\nabla \cdot \boldsymbol{E}}\, dV$$

Divergence of **E**

  - Divergence is associated with sources and sinks of the field

- Stoke's theorem: relates a circulation around a closed loop to a surface integral:

$$\underset{\text{Loop}}{\oint} \boldsymbol{A} \cdot d\boldsymbol{l} = \underset{\text{Surface}}{\iint} \underline{(\nabla \times \boldsymbol{A})}\, dS$$

Curl of **A**

Vector Fields

Two Important Identities

- Divergence of a curl is always zero:

  div(**curl**(**U**)) ≡ 0.

- Curl of a gradient is zero:

  curl(**grad**(T)) ≡ 0.

- These properties are easily verified using the 'nabla' notation (try this!):

$$\vec{\nabla} T = \vec{i}\frac{\partial T}{\partial x} + \vec{j}\frac{\partial T}{\partial y} + \vec{k}\frac{\partial T}{\partial z}$$

$$\mathbf{curl}\,\mathbf{U} = \nabla \times \mathbf{U} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U_x & U_y & U_z \end{vmatrix}$$

Static Fields and Waves

- Fields in geophysics typically exhibit either *static* or *wave* behaviour:
  - Static – independent on time:

$$\frac{\partial T}{\partial t} = 0.$$

Stationary temperature distribution (geotherm).

  - Wave – stable spatial pattern propagating with time:

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0.$$

Acoustic (pressure) wave.

This is the typical form of wave equation; c is the velocity of propagation.

$$p = f(x - ct)$$

Plane wave propagating along the *X*-axis.

f() is the waveform, at time *t*, it is centered at $x = ct$