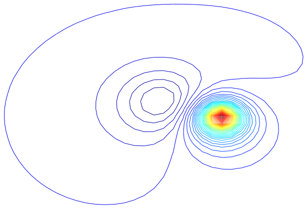

LAB 2

LATITUDE, FREE AIR BOUGUER CORRECTIONS

Gravity at the surface of the Earth varies by about 5 gal due to the centrifugal force of the earth's rotation and the attraction of the resulting ellipsoidal shape of the earth. This is a relatively large variation. At mid-latitudes it is about a mgal increase in gravity for every km moved in the direction North. Fortunately this effect is well known and easy to correct for. The current standard is defined by the world geodetic system (1984)

$$\gamma(\theta) = 978.03267714 \frac{1 + 0.00193185138639 cos^2\theta}{\sqrt{1 - .00669437999013 cos^2\theta}} \qquad \text{Gal}$$

where θ is the co-latitude (90^o minus latitude). An older standard is the 1967 International Gravity Formula

$$\gamma = 9.78031846(1 + 0.0053024 cos^2\theta - .0000058 sin^2 2\theta) m/s^2$$

One hundredth of a mgal (the reading level of the most common exploration gravity meters) is the seventh decimal place $9.780318\underline{4}\ 6$, so this formula is accurate enough for most exploration purposes, but note that there is almost a mgal difference between the two standards.

1) Correct your data for the latitude effect by subtracting the international gravity formula from your (tied to absolute) values.
There are two ways to do this. If you have the latitude of each station, you can simply use these in the IGF and subtract the result from each of your tied-to-absolute gravity values. Alternatively, if you have your station positions in m relative to the origin of your coordinate system, you can proceed as follows. Calculate the IGF at the latitude of the origin of your coordinate system, and differentiate the formula at this latitude as well, so you have the IGF at one point and its local rate of change. The latitude effect is then

$$978.03267714 \frac{1 + 0.00193185138639 cos^2\theta}{\sqrt{1 - .00669437999013 cos^2\theta}} + \frac{d\gamma}{dx} X \qquad \text{Gal}$$

where θ_o is the co-latitude of the zero of your coordinate system and X is the distance a station is north of the zero. Subtract t5his from your tied-to-absolute values. The latter method is good to $\pm 0.01 mgal$ over $\pm 25km$. You need Northings to $\pm 12m$, so you need to find the latitude of your origin to $\pm 0^o.0001$. The GPS monument is the origin of our local coordinate system. The latitude $52^o.1962$, or co-latitude $37^o.8038$, and longitude -$106^o.3983$.
The rate of change of the IGF67 (close enough to WGS84) with co-latitude is

$$\frac{\partial\gamma}{\partial\theta} = -978.04900 \times (0.0052884 sin2\theta + 0.0000118 sin4\theta) gal/rad$$

At colatitude 45^o this corresponds to

$$\Delta\gamma = -5.1722 gal/rad \Delta\theta$$

or

$$\Delta\gamma = -.8118 mgal/km \times S$$

where S is the distance south of the base station in km. At S'toon this is -.7805 mgal/km.

The derivative for WGS1984 (in gal/rad) is

$$\frac{\partial\gamma}{\partial\theta} = -978.03267714 sin2\theta[\frac{0.00193185138639}{\sqrt{1-0.00669437999013cos^2\theta}} + 0.00669437999013\frac{(1+0.00193185138639cos^2\theta)cos\theta sin\theta}{(1-0.00669437999013cos^2\theta)^{3/2}}]$$

or

Latitude dependence of gravity

$$\Delta\gamma = -978032.67714[\frac{0.00527904138145}{6378.137}sin(2\theta) + \frac{0.00001293254726}{6378.137}sin(2\theta)cos(\theta)^2] \quad mgal/km$$

$$\Delta\gamma = -0.81306 mgal/km \quad \text{at} \quad 45^o \quad \text{and} \quad -0.78668 mgal/km \quad \text{at Saskatoon.}$$

WGS 1984 includes the mass of the atmosphere (the earlier IGF definitions do not). That is, to construct WGS 1984, the atmosphere was condensed on the surface of the ellipsoid, so that WGS 1984 is actually gravity at the surface of the ellipsoid, but outside the atmosphere. The reason for this is that WGS 1984 is used in satellite gravity and satellites are outside the atmosphere. The convention is to correct WGS 1984 (GRS1980) by subtracting the correction below, or equivalently correct observations by adding it. For a station at sea level we would need to add the attraction of the full atmosphere (at a point just outside) and for a station outside the atmosphere we would need to add zero. For all other elevations the correction to be added to observed gravity (in mgal) is:

$$\delta g_A = 0.874e^{-0.000118h^{1.047}} mgal \quad \text{for } h \geq 0$$
$$\delta g_A = 0.874 mgal \quad \text{for } h < 0$$
$$\&\text{or}\delta g_A = 0.874 - 9.9\times10^{-5}h + 3.56\times10^{-9}h^2$$

where h is in m.
So the maximum effect is just under a mgal.
The Free Air Effect is the gravity variation that results from the vertical motion of the gravity meter in the nominal gravity field of the earth. The IGF and the world geodetic system also define a vertical gradient. Including corrections for the ellipsoidal shape of the earth this is IGF(1930) :

$$\frac{\partial\gamma}{\partial h} = -.308766[1-.0014665cos^2\theta]mgal/m + 1.442\times10^{-7}mgal/m^2\times h$$

For comparison IAG 1975 is:

$$\frac{\partial\gamma}{\partial h} = -.308769[1-.0013797cos^2\theta]mgal/m + 1.442\times10^{-7}mgal/m^2\times h$$

and WGS80 is:

$$\frac{\partial\gamma}{\partial h} = -.3087691[1-.001424cos^2\theta]mgal/m + 1.442\times10^{-7}mgal/m^2\times h$$

In IGF(1930), the vertical gradient has a constant part (-.308766 mgal/m), and adjustments for the variation with latitude, and height. In exploration surveys, usually only the lead term is used. In the Saskatoon area (colatitude 38^o) WGS80 is

$$
\begin{aligned}
&= -.308422 + 1.442 \times 10^{-7} \times (580m) \\
&= -.308464 mgal/m
\end{aligned}
$$

Free-air effect

which differs by only 0.03 percent from the standard value -0.3086 mgal/m.
This (-.308464 mgal/m) is the free air *effect*. It causes observed gravity to decrease by .3 mgal per m of elevation above the ellipsoid. To correct for this we subtract the free air effect from raw gravity measurements, that is, add .3084 mgal for every m of elevation above the geoid. Note that the gradient is calculated at some height above the ellipsoid, but the correction is for height above the geoid. This convention arose because heights above the geoid are more readily available than heights above the ellipsoid (at least before GPS). It has only become possible since the 1970's to estimate the difference in height between the geoid and the ellipsoid to better than a few meters, but geophysicists still needed to do some sort of free air correction, so the use of height above the geoid in the free air correction became standard.

2) If we measured a vertical gradient during field school, compare this value with the above, and comment on any difference. Despite the fact that the measured gradient may differ from the standard value, we must still use the standard value in the free air correction.

3) Correct your gravity data for height above the local geoid.

The Bouguer correction is a correction for the attraction of the mass between the gravity station, and the local geoid. A simple Bouguer correction approximates the attraction of that mass with the attraction of a slab of the same mean density. The attraction of a slab of thickness h and density ρ is $g_b = 2\pi G\rho h$. If the density is $2.67gm/cm^3$ then this is $g_b = 0.1119mgal/m \times h$. Since Saskatoon is about 580 m above sea level, the Bouguer slab is entirely in the sedimentary column, so the mean density of material between the physical surface and the geoid is slightly less than the standard 2.67 gm/cc that would be appropriate in shield areas. However, it is common practice to use a standard Bouguer density of 2.67 gm/cc, and treat any variation from this in the modeling stage.

A Bouguer anomaly calculated from a simple slab model is called a **simple Bouguer anomaly.** A **complete Bouguer anomaly** also includes corrections for the curvature of the earth, and corrections for the attraction of the local topography (the terrain correction). The curvature correction is also called the Bullard B correction, and it is achieved by replacing the flat Bouguer slab with a curved spherical cap of the same thickness, and radius 166.738 km. The correction is normally done with the aid of tables, but a polynomial in height is adequate for most situations

Bullard B (Bouguer) effect on gravity

$$
\begin{aligned}
BB &\approx Ah + Bh^2 + Ch^3 \\
A &= 1.464 \times 10^{-3} \\
B &= -3.533 \times 10^{-7} \\
C &= 4.5 \times 10^{-14}
\end{aligned}
$$

mGal, and h is in meters

At elevations below 4 km the spherical cap has more downward attraction than the slab, so the simple Bouguer correction is too small. At greater elevations the attraction of the spherical cap is less than the Bouguer slab, so the Bullard B is negative. BB as calculated above would be added to the attraction of the slab to get the attraction of the cap, and subtracted from simple Bouguer anomalies.

4) Do the Bullard B correction.

The terrain correction can be done in a variety of different ways, but the easiest and most reliable, is to include the terrain as part of the modeling. We will return to this in a later lab. The traditional way to do terrain corrections is with a Hammer chart, although this is so tedious, I won't make you do it.

The terrain effect of short horizontal scale topography within 2 m of a station, is usually handled by an inners correction done in the field. In this context 'inner' means inside the smallest Hammer zone. If the elevation changes by h (always positive) over the inner Hammer zone, then the inners correction is:

$$\rho G h$$

which should be added to the Bouguer anomaly. Note that this is a factor 2π smaller than the Bouguer correction for a slab of the same thickness. Thus, inner topography that varies by a m over the inner hammer zone would have an inners correction of about 0.01 mgal. In the context of the field school inners corrections may be required at stations very near the road embankment. Whether these need to be done or not depends on their magnitude, and the repeatability of the survey.

Make a table of Free Air gravity and Bouguer gravity at each of your stations.

Add columns to your data matrix with the three effects
and subtract them (together with drift) from raw data values.