



POTENTIAL FIELD METHODS

LAB 5

FOURIER TRANSFORMS

Make sure you have the local directory and everything below it and that your matlabpath is set to look in /local, /local/481, /local/userm, local/gravmag etc. To add a directory to your matlab path do `addpath('type in the full path to a directory here')` and repeat for every directory you need. Get `FourierTransfromDemoI.m` and `FourierTransfromDemo2.m` from PAWS and put them in a directory in the matlab path. Enter `FourierTransformDemoI` at the command line prompt. This will take you through an introductory session on the FFT. Run it a second time if you need another look to understand everything. Enter `FourierTransformDemoII` and run it until you understand as well. After you have finished read the material below and perform the exercises.

type 'load LABFTI'

Get `LABFTI.mat` from PAWS and load `LABFTI.mat` into matlab. This will create a number files in your workspace called F1, F2,F3,F4,F5,F6,F7,F8,F9,F10,F11 and F12.

QFT is a 1-D discrete Fourier transform program which you can operate on F1, F2 etc with. Just type QFT on the Matlab command line, and work in the figure axis. Enter the name of the variable to be Fourier transformed, (this must exist in your workspace), and select the tapering mean removal and drift removal options. Click OK and QFT execute with whatever steps you have selected (taper etc). The raw data will be displayed in the top left axes, the modified data in the top right axes, that is tapered etc, and the amplitude spectrum in the bottom left axes. The amplitude spectrum is

$$\sqrt{((\text{real part})^2 + (\text{imaginary part})^2)}$$

QFT will also create `QFT_out` and `QFT_f` in your workspace. `QFT_out` is the amplitude spectrum, and `QFT_f` is a vector of frequencies.

Like all DFT programs QFT just operates on a string of numbers, without knowing if they were sampled in time, or distance, or whatever. It simply assumes that the first sample is at zero time, (or zero distance or whatever). The frequencies in `QFT_f` are in units of cy/sample interval, where the sample interval is one. To convert these to engineering units, divide the frequencies by the sample interval in, m, or s, or whatever.

To see how QFT works, run it on the data in F6.

F6 is a sine wave of unit amplitude sampled every 22.5° , for a total of 32 samples, which makes two full cycles, and it consists of a column of 32 numbers which represent the amplitude of the sine wave at those intervals. Let's say the sampling interval is 1 second, so the period is 16 seconds and the frequency $1/16$ cy/sec= 0.0625 cy/sec.

QFT reads the data and counts how many samples there are (32). Since this is an FFT it needs to operate on 2^N numbers. If there are not 2^N QFT will try 3^N , or 5^N , or some combination, but we will always work with 2^N .

In most exercises, do not taper, remove a mean or remove a drift unless the instructions are otherwise. The drift removal routine is a simple least squares fit of a straight line, but in most cases you would want to do something better than this.

Although you and I know the samples are at 0 sec 1 sec 2 sec etc, QFT does not know, nor does it need to. The sample interval is used only to construct a frequency scale, and QFT would spit out the same numbers for the amplitude spectrum regardless of what you selected for the sampling interval. In most of these exercises enter 1 for dt unless the exercise asks for something different. The routine computes the Nyquist frequency as 0.500e+00 cy/sample (and we know it means cy/sec, because the samples are 1/sec), and the increments to the frequency scale are 0.0313E+00 (again cy/sec).

Try to make sense of the spectrum in terms of the data. Run this a few more times with other options and note the changes.

Here are ten exercises, some of which require answers. Hand your responses in at the end of the lab.

- 1) F1 is a boxcar function of unit amplitude, 16 samples (alternatively, a DC signal of unit amplitude). Transform F1.DAT (do not remove the DC, the drift, or taper). Look at the spectrum, and explain why it looks like this (I thought a boxcar transformed to a SINC function?)
- 2) F2 is a ramp function ... 0,1,2,3,4,5,6,7 ... 15. This is also called a linear drift. Transform it (no taper, no drift removal) and look at the spectrum. It has amplitude at every frequency (in technical jargon it is "broad-band").
- 3) F3 is a sine function of four second period sampled every second. Look at its transform (no taper etc). It is exactly what we would expect - a positive spike at frequency +.25 cy/sec, and zero everywhere else. This spectrum is easy to interpret because it is so clean, we look at it and immediately recognize the sine wave. Since this is an amplitude spectrum, a cosine of the same period would give the same results.
- 4) F4 is the sum of all three of the above signals. Transform it and look at the spectrum (no taper no drift removal). The interpretation is now not so easy because the DC and the drift complicate the interpretation. Remember that adding two or more signals together is the same as adding their individual spectra.

The lesson is that we should remove the DC and drift before transforming. The DC is not such a problem because it only contributes a real spike at zero frequency, but if this were a continuous transform the DC would convolve a sinc function with every other frequency in the spectrum and result in an absolutely unrecognizable mess. With discrete transforms, it is not so important to remove the DC, (provided you taper properly), but as a rule we do it anyway.

- 5) Ok, just for a laugh, re-run the ~~F4.DAT~~ but this time answer Y(es) to the prompt for drift removal. This is OK for a quick and dirty drift removal, but if you really wanted to do a good job on drift removal, you would need to nearly custom design a drift filter for every signal.
- 6) F5 is a sine wave of 18 second period sampled for 16 seconds. Its frequency is therefore 0.055 cy/sec, but if you transform this function you do not get the spike we had from the last sine wave. Why not?
- 7) F6 is also a sine wave, this time with period 16 seconds, but sampled every second for 32 seconds. Transform it and sketch the spectrum.
- 8) F7 is a sinusoid of period 1.75 meters, sampled every meter for 32 samples. Transform it and explain your results.
- 9) F8 is the same 4 second period wave that is in ~~F3.dat~~, except now it is sampled every 1.2 seconds. Last time you got a nice spike at the appropriate frequency what has happened here?
- 10) F9 is a gravity anomaly of 1 mgal from a sphere at depth of 50 sample intervals. Run it through QFT. F10 is a sphere at 1 sample interval depth. Run it and note the difference with the spectrum of F9. F11

is a sphere at 4 sample intervals depth, and finally, F12-~~DAT~~ is F11 with random noise of 0.1 mgal. See how the spectra of these last two compare.