

## Dipole (monopole pair) magnetic source

This is what is done in the dipole modeling script `DipoleFieldScript.m` you use in this lab. Note that the “dipole” with finite (several meters) length used in this lab is not truly a dipole (the true dipole has an infinitesimal length) but a pair of separate magnetic monopoles. As a good approximation, these monopoles can be viewed as ends of a strongly magnetized, thin rod (Figure 1).

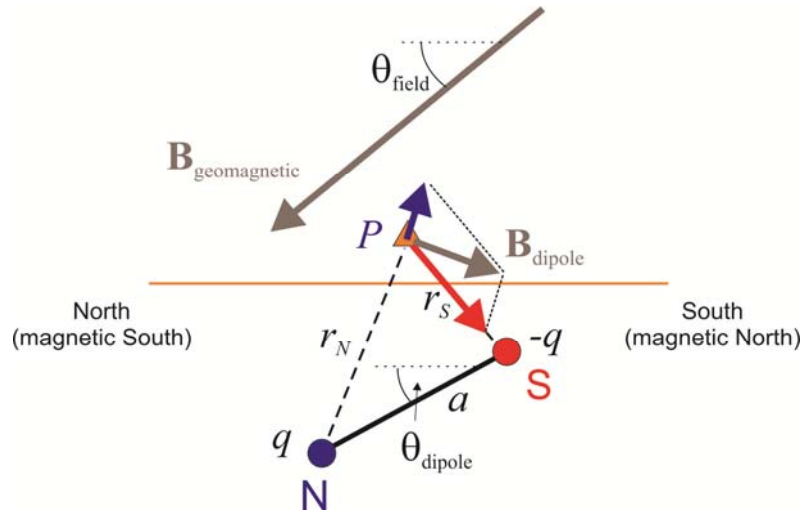


Figure 1. Combination of the geomagnetic field with fields produced by two magnetic monopoles with finite separation.

The magnetic potential produced by the two monopoles is (in SI units):

$$V(P) = \frac{4\pi}{\mu_0} \left( \frac{q}{r_N} - \frac{q}{r_S} \right),$$

where  $q = m/a$  is the “pole strength”, and  $m$  is the magnetic moment. The magnetic moment is fixed and equal  $100 \text{ A}\cdot\text{m}^2$  in the code. The magnetic field of these two monopoles equals:

$$\mathbf{B}_{\text{dipole}}(P) = -\vec{\nabla}V = \frac{4\pi}{\mu_0} \left( \frac{q}{r_N^2} \hat{\mathbf{r}}_N - \frac{q}{r_S^2} \hat{\mathbf{r}}_S \right).$$

This vector field is added to the geomagnetic field  $\mathbf{B}_{\text{geomagnetic}}$  (Figure 1).

The program outputs the “total” field, which is the projection of the field produced by the magnetization onto a unit vector in the direction of the geomagnetic field:

$$T = \mathbf{B}_{\text{dipole}} \cdot \frac{\mathbf{B}_{\text{geomagnetic}}}{|\mathbf{B}_{\text{geomagnetic}}|}.$$

For a dipole and geomagnetic field dipping downward to the north, this field looks like shown in Figure 2. Note that:

- 1) The low produced by the north pole of the dipole (blue) is smaller and broader than the high produced by the south pole (red). This is because the north pole is located deeper.
- 2) The magnetic high over the south pole (of the dipole) is shifted to the south.

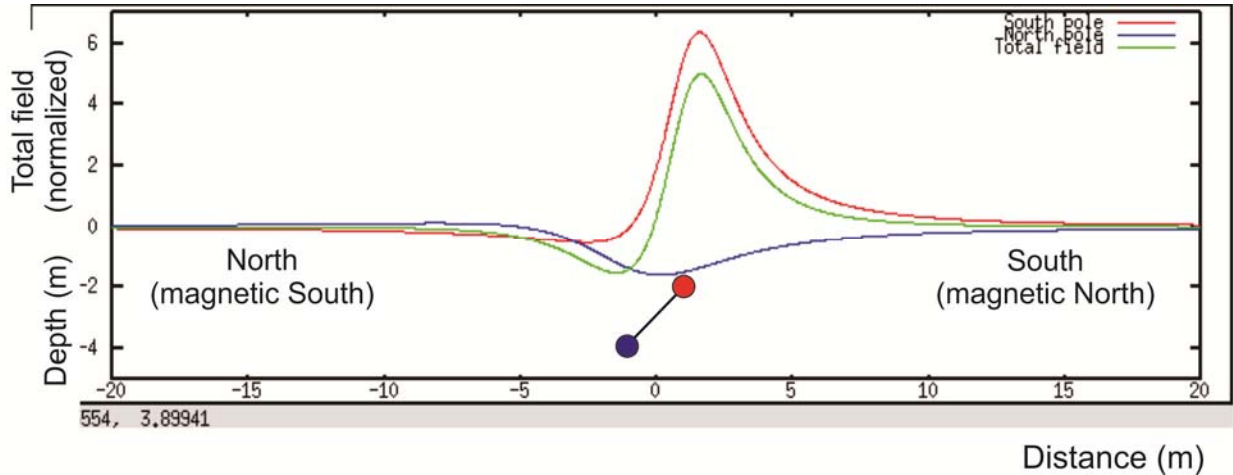


Figure 2. Total field  $T$  (green) produced by a geomagnetic field and dipole dipping at  $45^\circ$  to the north. For comparison, the individual contributions of the north and south poles are shown in blue and red, respectively. Note that for each of them, the peaks are shifted to the south because of the inclination of the main field. However, in the total field (green), this shift is only seen in the positive high over the south pole, and the negative low is slightly shifted northward.

Note that in the code (`DipoleFieldGui.m`), the inclination of the geomagnetic field is related to the latitude by:

$$\tan \theta_{\text{field}} = 2 \tan(\text{Latitude}).$$