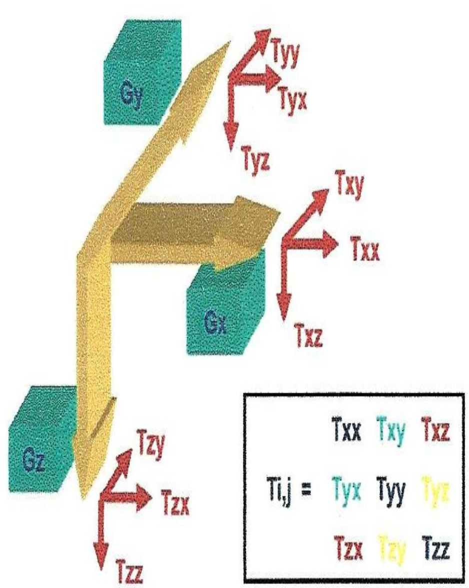
AIRBORNE GRAVITY

There are two types of systems in use, gravity and gradiometry. Airborne gravity, with GPS corrections for the acceleration of the aircraft, are good to about half a mGal at the present. Several companies offer this service in fixed wing, helicopter and zeppelin modes. The competitor is gravity gradiometry which has a few commercial configurations, including full tensor and partial tensor modes.

The gravity tensor measures the change in each component of gravity in each direction, so the change in the vertical component in the x, y and z directions etc. So getting each component from the gradient and taking a derivative of this we have.



$$\Gamma = \left[\begin{matrix} \frac{\partial^2 \Phi}{\partial x^2} & \frac{\partial^2 \Phi}{\partial y \partial x} & \frac{\partial^2 \Phi}{\partial z \partial x} \\ \frac{\partial^2 \Phi}{\partial x \partial y} & \frac{\partial^2 \Phi}{\partial y^2} & \frac{\partial^2 \Phi}{\partial z \partial y} \\ \frac{\partial^2 \Phi}{\partial x \partial z} & \frac{\partial^2 \Phi}{\partial y \partial z} & \frac{\partial^2 \Phi}{\partial z^2} \end{matrix} \right]$$

or

$$\Gamma = \left[\begin{matrix} \Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz} \end{matrix} \right]$$

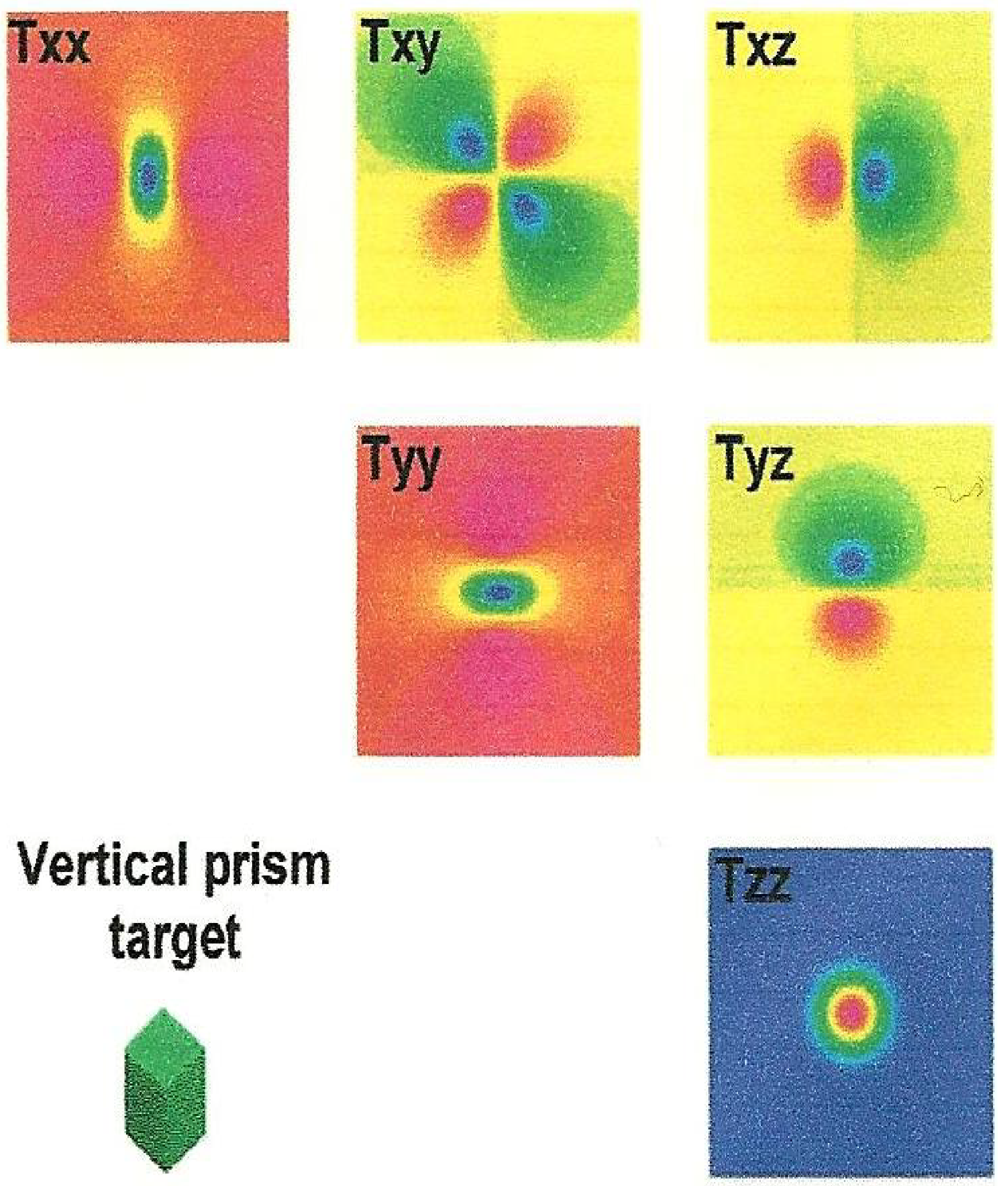
The first set indicates that the tensor is symmetric $\Gamma_{xy} = \Gamma_{yx}$, because it does not matter in which order the differentials are done. Thus, while there are nine components of the full tensor, only six are independent. Also, the trace of the tensor is zero, so

$$\Gamma_{xx} + \Gamma_{yy} + \Gamma zz = 0$$

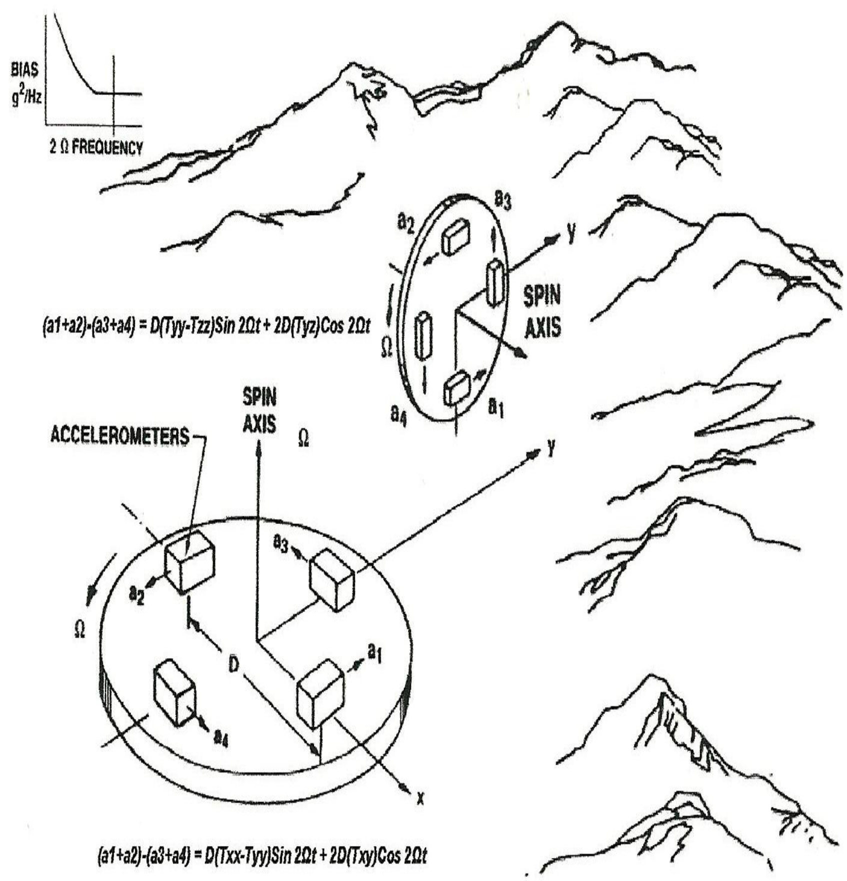
and so there are only five independent components.

Tensors can be rotated mathematically into the principal axis frame, so that only the diagonal components are non zero. xy and z are defined by the aircraft frame, and so some other technique must be used to provide an absolute xyz, perhaps gps or gyroscope.

The units for the gradient in gravity are $1E = 10^{-4} mGal/m = 10^{-9} s^{-2}$. Current systems claim an accuracy of about 5E.



The most popular design has pairs of accelerometers mounted on a spinning platen. This system was designed in the seventies for submarine navigation and outed in the book *The Hunt for Red October* published in 1984. It was declassified in 1994 and commercialized soon after. The rotation changes the static signal into a signal that changes at twice the rotation frequency and allows each of the four accelerometers to measure the same acceleration at different locations. Rotation also allows for a cancelation of the calibration differences of the four accelerometers. Subtracting orthogonal pairs



$$a_1 - a_3 = (1/2)D(\Gamma_{xx} - \Gamma_{yy})sin(2\omega t) + D\Gamma_{xy}cos(2\omega t)$$
$$a_2 - a_4 = (1/2)D(\Gamma_{xx} - \Gamma_{yy})sin(2\omega t) + D\Gamma_{xy}cos(2\omega t)$$

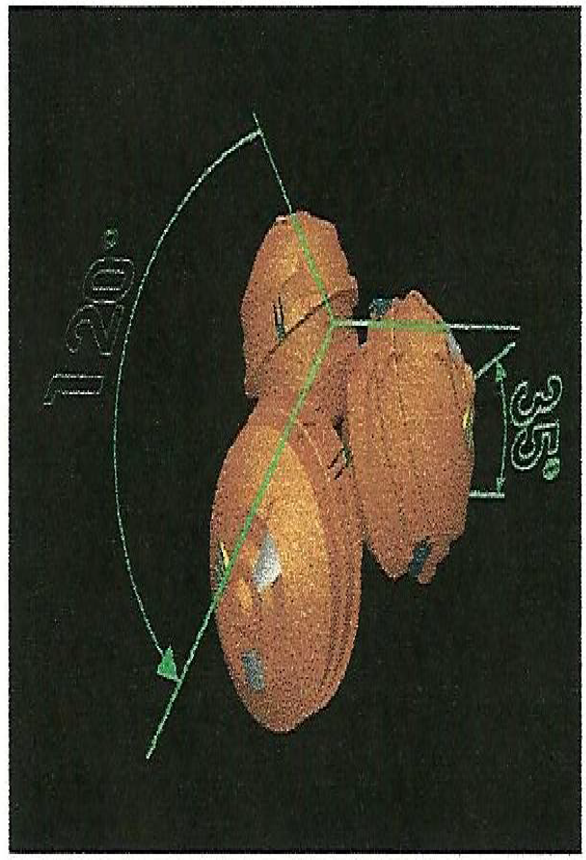
or

$$(a_1 + a_2) - (a_3 + a_4) = (\Gamma_{xx} - \Gamma_{yy})sin(2\omega t) + 2D\Gamma_{xy}cos(2\omega t)$$

Adding two more platens with their axes orthogonal to the first, so rotation along x and along y

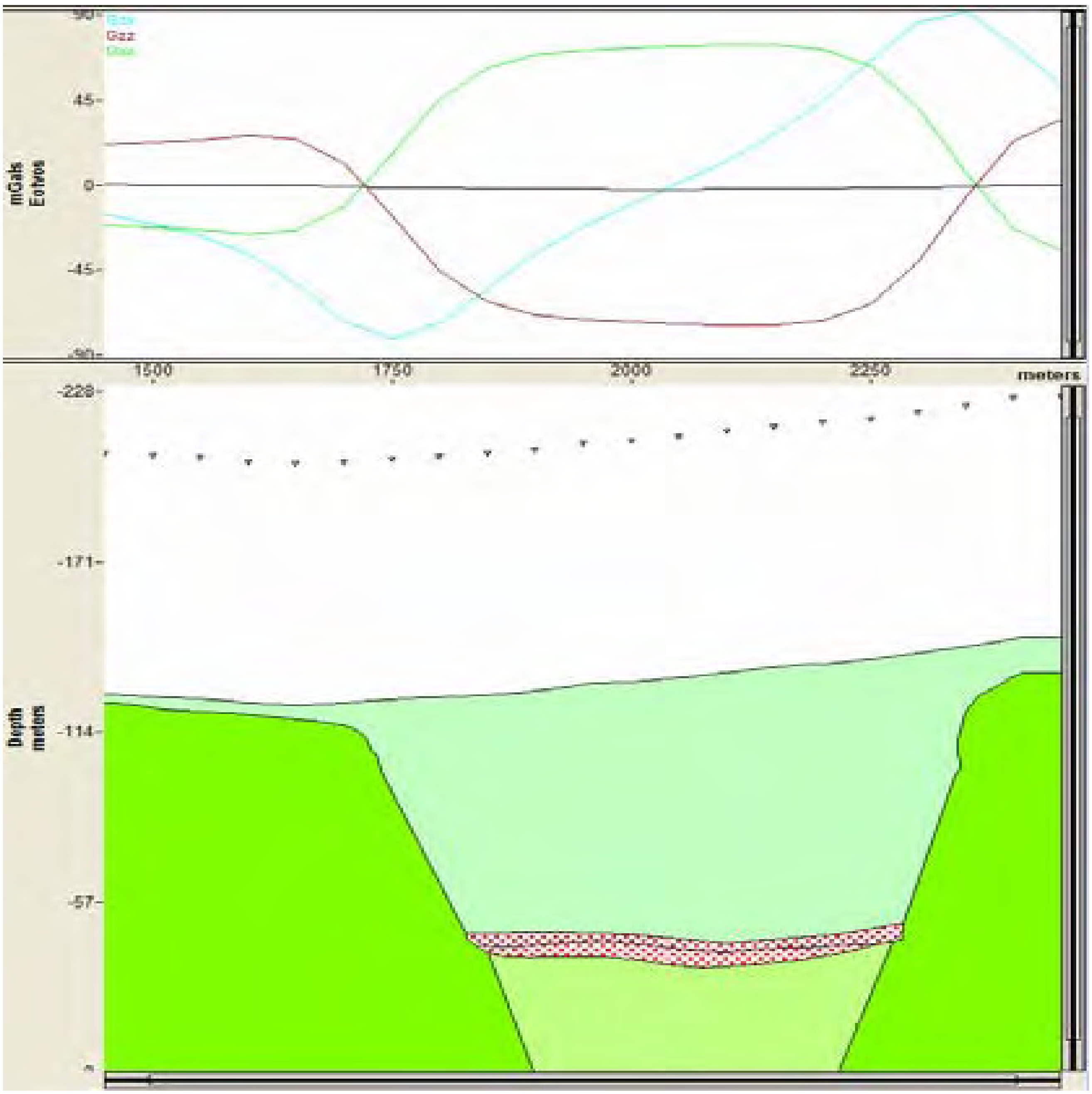
$$(b_1 + b_2) - (b_3 + b_4) = (\Gamma_{zz} - \Gamma_{yy})sin(2\omega t) + 2D\Gamma_{zy}cos(2\omega t)$$
$$(c_1 + c_2) - (c_3 + c_4) = (\Gamma_{zz} - \Gamma_{xx})sin(2\omega t) + 2D\Gamma_{zx}cos(2\omega t)$$

In the above equations the off diagonal components have a cos phase and the differences of diagonal components have a sin phase, so there are really six equations in six unknowns and the full tensor can be solved for.



In the Bell aerospace system, the platens are not rotating about x y and z, instead the rotation axes are all at 35 degree to the horizontal and viewed from above 120 degrees apart.

The nice thing about gradient instruments is the acceleration of the aircraft is canceled if all sensors see the same acceleration vector. What happens if this is not true? For example if the aircraft is yawing, or diving, or flexing in its airframe? The instrument is located in the centre of the airplane (centre of mass) to minimize this effect.



GRAVITY METER	SENSITIVITY	TIME ON SITE
	mGal	
Lacoste G	0.01	5 min
Lacoste D	0.001	10 min
CG5	0.001	5 min
Burris	0.01	5 min
ABS G	0.001	hours
SG	0.0000001	stationary
Bell	few E	s
ribbon	??	?
Cesium fountain	.000001	?
neutral bouyant		
vibrating string	1 mgal	
quantum latice clock	?	