



POTENTIAL FIELD METHODS

LAB 5 ANSWERS FOURIER TRANSFORMS

No, F1
is simply
a constant

- A1) It has amplitude only at the zero or DC frequency and zero amplitude everywhere else. A boxcar is supposed to transform to a sinc function, so what is wrong here? The answer is that a continuous Fourier transform will transform to a sinc, but the discrete transform gives just a spike at zero frequency. This is an important point - the discrete transform is not simply the continuous transform sampled at discrete points. This is particularly true at the long period end of any discrete spectrum.

The wave
length is
longer than
the FT interval,
hence it has
higher frequencies
when periodically
extrapolated

- A6) If you look at your tabulated spectrum, you will see that the frequencies in the discrete transform are: $0, \pm 0.125, \pm 0.250, \pm 0.375, \text{ and } +.5 \text{ cy/sec}$, so the lowest frequency we have available with this transform is $\pm 0.125 \text{ cy/sec}$, and this is not low enough to adequately describe this function. The transform compensates by trying to describe the sine wave as a sum of higher frequency waves, so the spectrum looks broad-band, but it just does not work. The signal we transform cannot have a wavelength, or a component with a wavelength, longer than the span of the data. A component with such a wavelength should be removed as part of the drift.

A7) This is the same signal we looked at in the prepackaged exercise, so no need to say anymore.

- A8) If the sampling interval is 1 meter, the Nyquist frequency is $.5 \text{ cy/meter}$. This is the highest frequency in the transform. The input wave had a frequency of $1/1.75 = .57 \text{ cy/meter}$, or higher than the Nyquist. As a result, instead of getting a nice spike at the frequency of the wave, we do not have that frequency at our disposal, and the signal is **aliased** into lower frequencies in fact to 0.43 cy/meter . This is not one of the Fourier frequencies, so the spectrum is not a spike but is spread out. This is somewhat like what happened at the other end of the frequency scale when we fed the routine a signal with frequency lower than it could handle. Always remember that there is an upper, and a lower, limit to the frequency scale in a discrete Fourier transform. The highest frequency is the Nyquist, or $1/(2 \times \text{the sampling interval})$, the lowest is $1/(\text{data span})$.

- A9) If the samples are a second apart, and we have 8 of them, then the Nyquist is 0.5 and the frequencies in the transform are $0.0, \pm 0.125, \pm 0.250, \pm 0.375, \text{ and } 0.5$, so the frequency of the wave 0.25 just happens to be one of the Fourier frequencies. If the same wave is sampled 8 times at 1.2 seconds, the Nyquist is .416 sec, the frequency increments are 0.104, and the Fourier frequencies are $0.0, \pm 0.104, \pm 0.204, \pm 0.304, \text{ and } 0.416$ so the frequency of the wave is not one of the Fourier frequencies, and the transform cannot describe the wave with just one spike. It "leaks" the signal to the nearest frequencies. Hence the term frequency leakage. So a pure sinusoid does not always appear in the transform as a spike at a single frequency.