LAB 6

FOURIER TRANSFORMS II ANSWERS

1) You should get diurnal, semidiurnal, terdiurnal, quartdiurnal, monthly, and semimonthly signals. The monthly is just barely short enough period to be handled properly. There are annual and semiannual signals as well which over this short record constitute a small drift.

2) The linear decrease in sea level is probably mostly post-glacial rebound. This area is still rebounding from the weight of the Fenno-Scandian ice sheet. To work out the aliased frequency suppose we found two sinusoids with frequencies ω_a and ω_h where ω_a is the aliased version of some high frequency ω_h, that satisfied

$$e^{i(\omega_a \Delta T)} = e^{i(\pm\omega_h + \frac{2n\pi}{\Delta T}) \times \Delta T}$$

where n and m are integers, so that we are sampling at integer multiples of ΔT, and at each sample the sinusoids are identical. The exponentials give the change in phase of each wave. Therefore,

$$(\omega_a \Delta T) = (\pm\omega_h \times + \frac{2n\pi}{\Delta T}) \times \Delta T$$

or, with circular instead of radial frequencies

$$(f_a \Delta T) = (\pm f_h \times + \frac{n}{\Delta T}) \times \Delta T$$

or

$$f_a = \pm f_h + \frac{n}{\Delta T}$$

or

$$f_a = -f_h + \frac{2n}{f_N}$$

where f_n is the Nyquist frequency. The choice of the + sign would lead to aliased frequencies outside of our Fourier bandwidth, so we drop this choice. The file alias.m in geol481 calculates the aliased frequency of a high frequency signal.

In other words the aliased frequencies of ω_h, are ω_a, as given by this formula, because they result in the same samples of the two sinusoids.

Rearranging

$$f_a = f_N - (f_h - f_N) \quad \text{with n=1}$$

So f_a is obtained by folding the frequency axis at f_N and f_h folds over to f_a. If f_h is high enough that folding the frequency axis at f_N aliases F_h to negative frequencies, the folding the axis again at 0 will bring the alias back to the positive side of the axis. The twice folded frequency may still be above the Nyquist, so folding again about the Nyquist, (and again if necessary about 0) and so on will eventually bring the aliased frequency between 0 and the Nyquist, where it may cause a problem.

If the Chandler frequency is .845 cy/yr, and we are sampling at 1 year, then the aliased frequency is

$$\sigma_a = .5cy/yr - (.845\, cy/yr - .5cy/yr) = \pm 0.155cy/yr$$

There is an m-file *alias.m* that will calculate aliases. You will probably not see a peak at the aliased Chandler frequency unless you do the stacking suggested in the assignment.

3) There is of course an annual signal, and the transform reveals a semi-annual signal as well, which is not that easy to see in the data. There does not appear to be much else, other than the seasonal fluctuations, and the secular increase. There are atmospheric phenomena in this period range, including El-Niño, but they do not seem to involve CO_2.

4) This motion is dominated by the 14 month Chandler wobble, which is the only peak you see in the 10 year data set, but when you use all the data, the single peak splits into a 14 month (Chandler) term, and a yearly term. These are too close in period for the transform to separate with only ten years of data. Why? We have about 200 samples in ten years, so the frequency increments are

$$\Delta f = \frac{1}{(2 \times 1/20\,(yr) \times 100}) = .4\,cy/yr$$

The Chandler frequency is .845 cy/yr and the annual 1, so they are closer together (.155 cy/yr) than the spacing on the frequency scale! If we increase the number of samples to 1520, or 76 years at 20^{th} yr sampling rate, the frequency increments are .05 cy/yr, so we can resolve two separate peaks. At this sampling rate (20/yr) we would need 30 or more years to resolve the two peaks. You may also see a semi-annual peak, caused (like the annual) by the atmosphere moving about, and perhaps a 24 year period, which some people say is caused by the ocean, others by the fluid core, and others by the observers themselves!

5) A seiche is a 'bathtub' oscillation of a lake, bay, or harbour. The period depends on the length and depth of the body of water. They are most often caused by wind storms that pile a lot of water up at one end of the lake. If the storm abates faster than the water can flow back, the lake will go into a few cycles of sloshing back and forth. For an east west seiche in lake Ontario the period is about four or five hours, with a maximum amplitude of about half a meter, and three cycles is typical. There is also a small amplitude tide at 12 and 24 hours. This is not evident compared to the seiche in the raw record but has a similar amplitude to th seiche in the spectrum. The reason for this is that the tide is pure harmonics that persist through the whole record while the seiche is transient. Would lake Diefenbaker have a tide? Blackstrap? A glass of beer? That tenth glass of beer?

6) The amplitude spectrum reveals an 11 year cycle, as well as a 100 year cycle and a 5 or 6 year cycle. The 11 year cycle is the famous sunspot cycle, but the 100 year and 6 year cycles are less well known, and much less prominent in the spectrum.

7) The amplitude spectrum should tail off to the noise level in the data by the time you get to the Nyquist. What did you estimate from the repeats, and what do you estimate here. They are probably not the same, because the repeats measure only one component of 'noise' that due to operator, or poor drift correction. The 'noise' in the spectrum estimates the actual short scale gravity variations that may be present as well, and are 'noise' for your survey.