Also see comments PPT file

WHAT IS A POTENTIAL FIELD?

A potential field (this is distinct from a potential) is a scalar field that satisfies Laplace's equation.

$$\nabla^2 U \equiv 0 \quad \text{Then U is a potential field}$$

Gravity measurements, total field and vertical component measurements of magnetic field are all potential fields, at least in a first approximation. I will return to the details of *why* and under what circumstances they may not be in a later class.

Most of the processing that we do with gravity and mag rests on the assumption that we are dealing with potential fields.

Note that here, functions "plane" and "surface" are meant as linear or quadratic functions of spatial variables.

Is a plane a potential field? A plane has the equation

$$PLANE(x, y, z) = ax + by + cz$$

Differentiating twice

$$\begin{aligned}\nabla^2 PLANE =& \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}(PLANE) \\ =& 0 + 0 + 0 = 0\end{aligned}$$

so a plane surface is a potential field.

Is a quadratic surface a potential field?

$$P = Ax^2 + By^2$$

is a polynomial whose Laplacian is

$$\nabla^2 P = 2A + 2B$$

This is not a potential field, except in the trivial case $A = -B$. But we measure gravity and mag in three dimensions, not two, so if I added a variation in z to the above equation that exactly canceled the second derivatives in x and y, the quadratic would be a potential field. If I add $-(A + B)z^2$ to P then it does satisfy Laplaces equation and $S = Ax^2 + By^2 - (A + B)z^2$ is a potential field.

There are an infinite number of 3-D surfaces whose 2-D expressions are $Ax^2 + By^2$. Only the one that also has a vertical variation like $(A + B)z^2$ is a potential field.

SO... if you fit a quadratic surface to gravity data estimate the regional you are implicitly assuming that the gravity varies vertically in a particular way.

Later we will see that the second vertical derivative is connected to the depth, so saying our quadratic surface is a potential field restricts the second vertical derivative and therefore the depth.

What about a sinusoid in (x,y)?

$$S = Asin(ax) + Bcos(by)$$

$$\nabla^2 S = -Aa^2 sin(ax) - Bb^2 cos(by) \neq 0?$$

So, sinusoids in x and y are not in general potential fields, except if we can find a suitable vertical variation as we did above.

- All polynomial surfaces, all planes, and most sinusoids in xy are therefore potential fields, but an appropriate z variation is immediately implied.

- In fact any continuous function in xy can be a section through a potential field with the appropriate z variation.

- Potential fields are always smooth. There are no discontinuities in the field, and no cusps.

The Laplacian is a linear operator, so if U and V are potential fields, so is their sum. This is more important than it seems. Suppose I measure some gravity, and then I do a regional residual separation in which the regional is a plane or quadratic. I know the measurements are of a potential field, but what about the decomposition into regional and residual? If the regional is a potential field, and this implies some specific variation with z, Then the residual is therefore also a potential field. Similarly, if the regional is not a potential field then neither is the residual. field.

WHY DO GRAVITY AND MAG?

1) CHEAP <\$100/line km compared to > \$1000/line km for seismic.

2) INTEGRATED SIGNAL Both gravity and mag measure the integrated signal from a great volume of earth. This makes them very useful as a reconnaissance tool, because stations can be relatively far apart and a small source may still be detected. With seismic an anomaly even slightly off the line will be missed entirely. This is sometimes a problem as well as it is difficult to get precise information on structure with gravity, and to a greater extent with mag.

3) Gravity responds only to density contrasts, mag only to magnetization contrasts, so they directly measure these two things. Seismic velocity measures a combination of density and elastic parameters, same with reflectivity.

I'd call this "Temporal and spatial variations"

ERRORS

Table based on

Rymer, H. 1989. A contribution to precision microgravity data analysis using Lacoste Romberg gravity meters. Geophys. Jour. **89,** 311-322.

CAUSE	COMMENTS	MAX ERROR	MIN ERROR
EXTERNAL			
a) tide	Effect is always significant but depends on the precision of the time, latitude, longitude and elevation. Greatest effect is approximately 50 μ gal /hr. Well known and accurate correction	$< 200\mu gal$	$< 1\mu gal$
ii)ocean	Attraction and loading of ocean tide. Poorly known and rarely performed correction.	$< 10\mu gal$	$< 10\mu gal$
b)atmosphere	pressure increase decreases gravity $-0.3\mu gal/mb$	$30\mu gal$	$1\mu gal$
c)noise	Low frequency (1-50Hz) distrubances. May cause tares	$< 50\mu gal$	$< 1\mu gal$

CAUSE	COMMENTS	MAX ERROR	MIN ERROR
READER			
a)leg length	$-3\mu gal/cm$	$< 9\mu gal$	$< 1\mu gal$
b)sensitivity	Sensitivity can be varied manualy. Failure to level, esp. along the long level, changes the value of gravity and the sensitivity	$< 20\mu gal$	$< 1\mu gal$
c)dial	Slack in the gears causes error unless the reading line is approached from the same side every time.	$< 40\mu gal$	$< 1\mu gal$
d)timing	No effect on G meters	$< 1\mu gal$	$< 1\mu gal$
INSTRUMENT			
	The rms error is greater if the (stationary) meter is clamped between readings than if it is not clamped. Transfer of oil from beam to clamp?	?	?
	The rms error is greater if the meter is clamped and moved between readings than if it is not moved.	?	?
a)calibration		0.1%	0.1%
b)tares	Thermal instabilities and mechanical shock	$\sim 10mgal$	$\sim 1mgal$

SURVEY ACCURACY is usually estimated from the repeats. What we would like is a statistical estimate of the error at any one station, due to operator error, poor drift correction etc. If N repeat measurements $R = reading_1 - reading_2$ are taken then

$$\sigma = \sqrt{\frac{\sum R^2}{N}}$$

is the standard deviation of the repeats, and it is an *estimate* of the standard deviation of all of the measurements. It is usual to take about 20% repeats. I will explain the drift correction in more detail later.

USES OF GRAVITY

1) **Recon for oil and gas.** Basin thickness and shape.

2) **Estimates of ore mass.**

3) **Outline the shape of ore mass.**

4) **Geotechnical - location of tunnels voids etc.**

USES OF MAGNETICS

1) **Recon for base metals.**

2) **Alluvial gold and heavy minerals.**

3) **Archaeology.**

4) **Recon for oil and gas.**

5) **Environmental geophysics.**

Gravity modeling is a much more reliable procedure than mag modeling, but a lot of mag modeling is still done. Gravity modeling is generally more reliable because there is only one cause - **a density contrast.** Mag may be caused by induced or remanent. In modeling mag you must specify the direction of magnetization, which would be the magnetic field if the magnetization were all induced, but you do not know this without drilling. Also, the density contrast in a structure is more likely to assume just one value (there are obvious exceptions). A mag source which is lithologically a single structure may have varying polarizations throughout.

Spatial scale-length

RANGE SENSITIVITY

MAGNETICS		GRAVITY	
deg	FIELD	deg	FIELD
0		0	980 gal
1	30000 nT	1	
2	2000 nT	2	3 gal
3	1000nT	3	7 mgal
4	500nT	4	3 mgal

Mag maps often look 'busier' than gravity maps. This is because of differences in range and resolution.

RANGE AND RESOLUTION

	GRAVITY	MAGNETICS
RESOLUTION	0.01 mgal	0.1 nT
RANGE	1 mgal	1000 nT
RANGE/RESOLUTION	100	10,000

Gravity is a weak force compared to magnetic.

You need a relatively large volume of material to give you a measurable gravity anomaly. A ball of till 1 m in radius with the meter sitting right on top, or equivalently a 1 m radius void in till, would give 0.05 mgal, and this would fall off as $1/r^2$ if the ball was at some depth. This also assumes a density contrast of 2 gm/cc, which is a huge density contrast for the subsurface. Even 0.2 gm/cc is a relatively large density contrast, so large structures are needed to produce a measurable gravity anomaly.

Relatively small volumes of magnetic material give large anomalies. Our field school targets are only several kg, and much less than a cubic m, but the anomalies are 1000 nT. Even a pin flag might have an anomaly this large.