FINDING THE CENTER OF MASS

In the northern hemisphere a mag high (induced) is displaced to the south. The greater the depth the greater the displacement and the greater the latitude the smaller the displacement (0 at pole).
To understand the origin of this you need to consider the geometry of an induced dipole. A high is associated with a south pole a low with a north pole. See the additional material in lab #8

With an induced dipole, the north pole is deeper than the south pole, **so the low produced by the north pole is smaller in magnitude and broader than the high produced by the south pole.**

What about gravity? Is a high centered over the center of mass?

We will now show that not the peak but the average coordinates of an anomaly are located over the center of gravity of the source.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\xi)d\Delta g(x,y)dxdy = 0$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(y-\eta)d\Delta g(x,y)dxdy = 0$$

The anomaly due to dm is symmetric about $(x=\xi, y=\eta)$ because the integrand at $x=\xi+a$ is cancelled by the integrand at $x=\xi-a$ because $d\Delta g(\xi-a,y) = d\Delta g(\xi+a,y)$ by symmetry.

Therefore

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x d\Delta g(x,y)dxdy = \xi\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta g(x,y)dxdy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y d\Delta g(x,y)dxdy = \eta\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta g(x,y)dxdy$$

But,by the excess mass theorem

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xd\Delta g(x,y)dxdy = \xi 2\pi G dm(\xi,\eta)$$

$dm(\xi,\eta)$ is the excess mass at (ξ,η)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yd\Delta g(x,y)dxdy = \eta 2\pi G dm(\xi,\eta)$$

and integrating over the whole source

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta g(x,y)d\xi d\eta dxdy = \hat{\xi} 2\pi GM$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\Delta g(x,y)d\xi d\eta dxdy = \hat{\eta} 2\pi GM$$

where $\hat{\xi}$ and $\hat{\eta}$ are the coordinates of the centre of mass.

Thus,

$$\hat{\xi} = \frac{1}{2\pi GM}\int_{-\infty}^{\infty} x\Delta g(x,y)dxdy$$
$$\hat{\eta} = \frac{1}{2\pi GM}\int_{-\infty}^{\infty} y\Delta g(x,y)dxdy$$

Here Δg is the total anomaly due to all the excess mass, whereas $d\Delta g$, is the partial anomaly due to the excess mass at a point in the subsurface source.

Of course to calculate the centre of mass we need to know the excess mass, which recall is

$$M = \frac{\int_{-X}^{X} \int_{-Y}^{Y} \Delta g(x, y) dx dy}{4G Tan^{-1} \frac{XY}{Z_o R}}$$

Where the Tan^{-1} is a correction for the missing flanks.

There is also a correction for the missing flanks on the c of m integral, which is similar to that on the excess mass integral, with the result that the c of m calculation corrected for flanks on both is

$$\hat{\xi} = \frac{\int_{-X}^{X} \int_{-Y}^{Y} \Delta g(x, y) x dx dy}{\int_{-X}^{X} \int_{-Y}^{Y} \Delta g(x, y) dx dy}$$

$$\hat{\eta} = \frac{\int_{-X}^{X} \int_{-Y}^{Y} \Delta g(x, y) y dx dy}{\int_{-X}^{X} \int_{-Y}^{Y} \Delta g(x, y) dx dy}$$