TERRAIN (BOUGUER) DENSITY

One of the largest effects on gravity is the attraction of the topography near the gravity station. This is distinct from the Bouguer correction, which corrects for the attraction of the whole slab of material between the gravity station and the local geoid, using a **defined** Bouguer density of $2670 kg/m^3$, the mean density of crustal rock. The defined Bouguer density is most likely greater than the actual terrain density, which will mean that Bouguer gravity will reflect the topography. To look at a '*Bouguer gravity*'that avoids this we sometimes generate a Bouguer gravity using the terrain density and maybe only correcting to the lowest elevation in the survey rather than the geoid. There are a variety of ways to estimate the terrain density (everyone still calls this the Bouguer density).

TECHNIQUES TO ESTIMATE TERRAIN DENSITY

NETTLETON

SCATTERPLOT

PARASNIS

COVARIANCE

FIRST DIFFERENCE AND MULTI-SCALE FIRST DIFFERENCE

If the purpose of the terrain density is to do a correction for the attraction of the topography, then there are two things the correct terrain density should result in.

1) It will result in a minimum correlation between (corrected) gravity anomalies and elevations.

2) It will result in a smoother variation in (corrected) gravity. Part of the variance in gravity is due to the topography, so if we correct for this we should remove variance from the gravity.

One or both of these considerations are used to estimate the terrain density. The terrain density can also be estimated in the modeling stage.

SUBSURFACE STRUCTURE THAT CORRELATES WITH TOPOGRAPHY

BIASES THE ESTIMATED TERRAIN DENSITY

Free air gravity g_{fa} is due to the attraction of the topography, $T(x)$, and the attraction of subsurface structure, $h_s(x)$. h_s may correlate or anti-correlate with T. Then, if these are the only two sources of gravity

$$g_{fa}(x) = 2\pi G\rho_t T(x) + 2\pi G\Delta\rho h_s(x)$$

where ρ_t is the terrain density, and $\Delta\rho = \rho_s - \rho_t$ is the density contrast (+ if density increases going into the second layer.)

If we assumed that free air gravity was only due to topography then by the Parasnis method (explained later) the effective density, is

$$\begin{aligned}\rho_e =& \frac{g_{fa}(x)}{2\pi GT(x)} = \rho_t + \frac{2\pi G\Delta\rho h_s(x)}{2\pi GT(x)} \\ =& \rho_t + \Delta\rho\frac{h_s(x)}{T(x)}\end{aligned}$$

and this is the result that any of the density determination methods would yield. so ρ_e can be greater than or less than the terrain density depending on the sign of $\Delta\rho$ and the correlation (positive or negative) between surface topography and subsurface structure.

This means "correlates positively"

if $\Delta\rho$ + and H_s correlates + with topography → density biased high

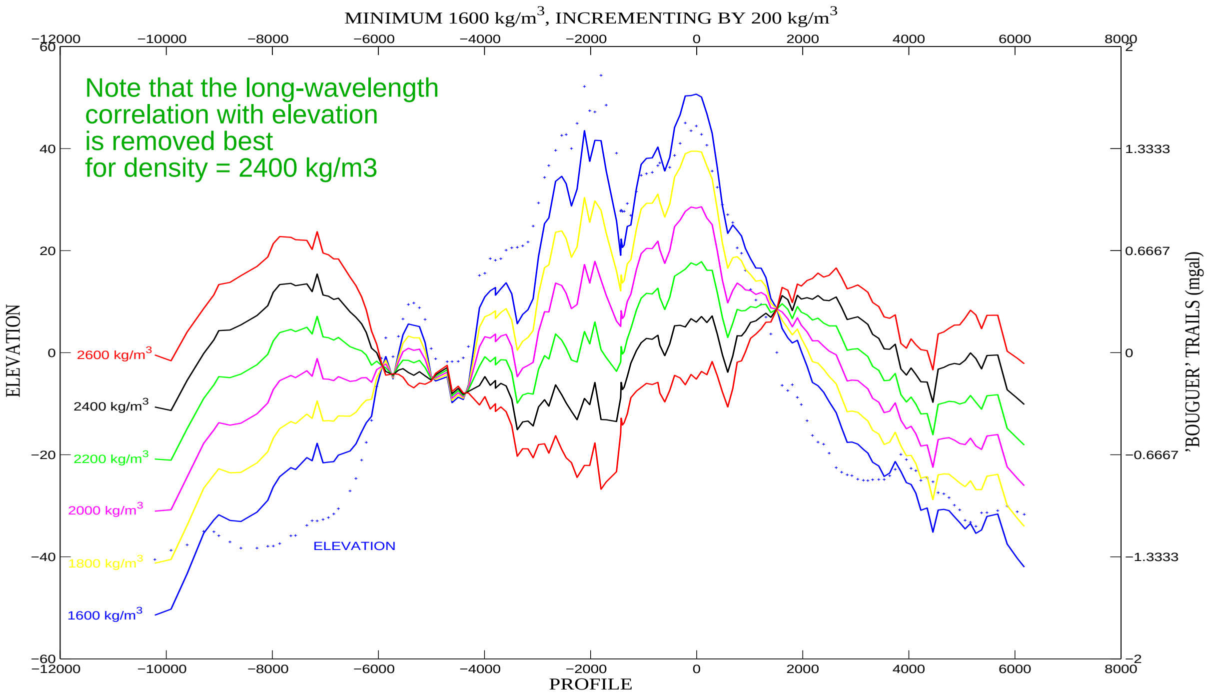
if $\Delta\rho$ + and H_s correlates - with topography → density biased low

"correlates negatively", anti-correlates

THE NETTLETON METHOD

The Nettleton method is a graphical method that uses a visual correlation between topography and Bouguer gravity with a number of trial densities. Bouguer gravity with a zero terrain density is the same as free air gravity, so a trial terrain density that is too low results in a trail Bouguer gravity that correlates positively with topography.

In the figure below a number of trial terrain densities are used. Small horizontal scale features are sometime the best to use, so looking at the topo high at about -6000m, a terrain density of about $2400 kg/m^3$ is indicated. The larger topo highs at about -2000 m and 0 m are somewhat contradictory (there is a correlation between topography and subsurface geology), but $2400 kg/m^3$ is good.



'If rho is fixed, this is the SCATTERPLOT' method in the labs

THE COVARIANCE METHOD

The covariance method uses all of the data (or a subset), and it can be modified to allow for a variation in terrain density over the survey.

"demeaned" might be a better term

Suppose Δg and h are detrended free air and topography, then

NOTE: this is not really a covariance of DELTAg and h but variance of the difference

Note that this is simply the least-squares fitting for model parameter rho

$$cov(g_{BT}) = \int\int [\Delta g_{FA}(x,y) - 2\pi G\rho h(x,y)]^2 dxdy$$

ie the integral of detrended Bouguer anomalies will be a minimum when the Bouguer anomalies are the smoothest. Think back to the Nettleton profile method: you look for the density that gives the smoothest Bouguer profile.

This will be a minimum when

$$2\int\int (\Delta g_{FA}(x,y) - 2\pi G\rho h(x,y))(-2\pi G h(x,y) dxdy$$

or

$$\rho = \frac{\int\int \Delta g_{FA}(x,y) h(x,y) dxdy}{2\pi G \int\int h^2(x,y) dxdy}$$

Instead of an integral we could write this as simply a sum over the stations.

$$\rho = \frac{\sum_i \Delta g_i h_i}{2\pi G \sum_i h_i^2}$$

where i is a station number.

Question: how can we replace the integral with a sum more precisely?

PARASNIS

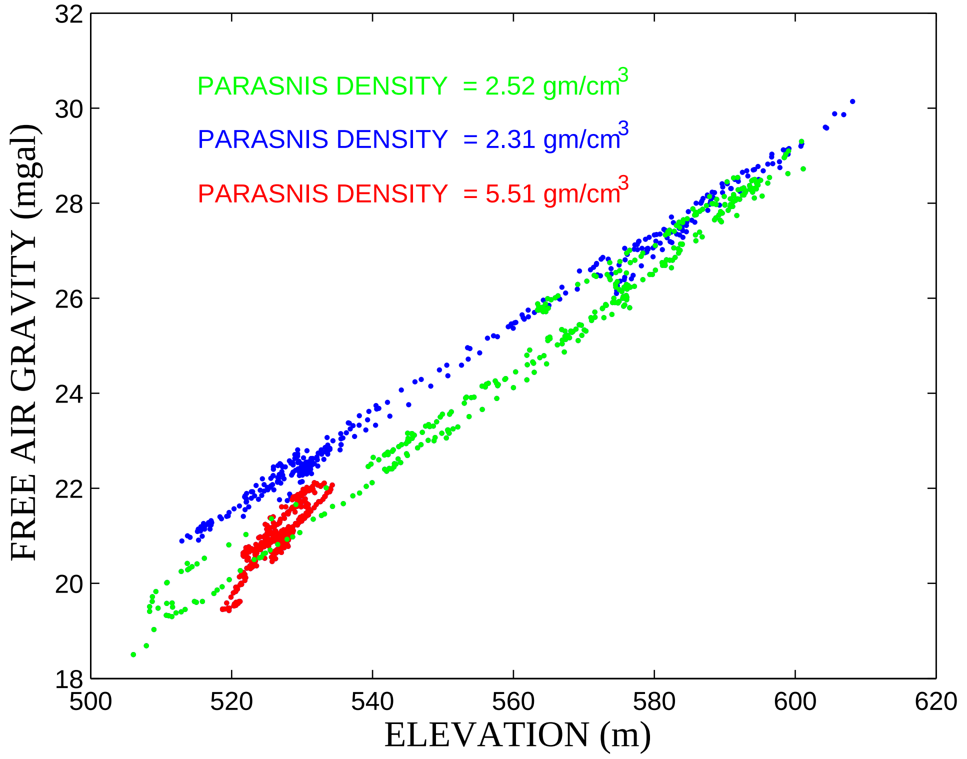
The Parasnis method consists in analysing the dependence of
~~and~~ free-air gravity on topography.

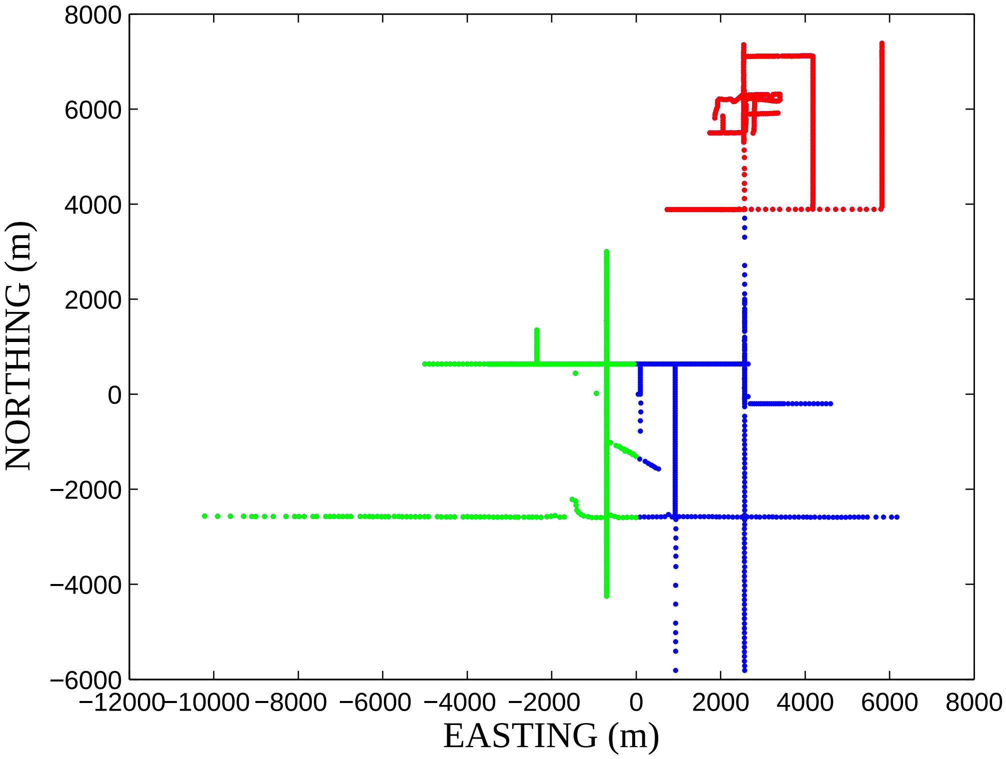
$$\Delta g_{FA} = \Delta g_B + 2\pi\rho G h$$

so if Δg_B does not correlate with topography

$$\frac{1}{2\pi G}\frac{\partial \Delta g_{FA}}{\partial h} = \rho_B$$

The two figures on the next page are (top) a Parasnis plot of all the Bergheim field school data 1996 - present. There are three separate trends and hence three indicated densities. The three densities are the result of the true terrain density being biased by a correlation between subsurface geology and surface topography. The colours in the bottom figure indicate where the colours in the top figure came from. On the east side of the hill topography correlates positively with Bouguer gravity (with any reasonable terrain density used in the Bouguer correction), and the Parasnis density is biased low. On the west side of the hill the correlation is negative and the Parasnis density is biased high. The red data are all from the NE corner where topography is fairly flat, but starting to dip low into the river valley.

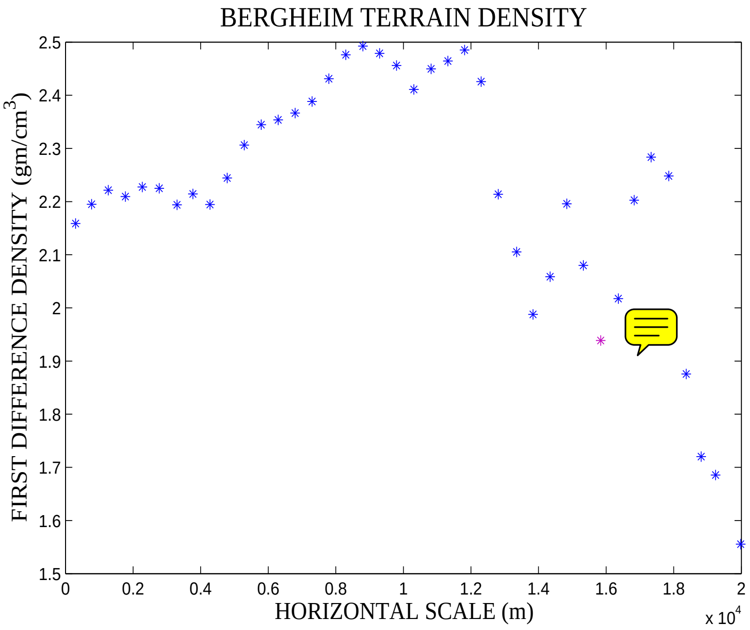

PARASNIS DENSITY = 2.52 gm/cm^3
PARASNIS DENSITY = 2.31 gm/cm^3
PARASNIS DENSITY = 5.51 gm/cm^3
FREE AIR GRAVITY (mgal)
18
20
22
24
26
28
30
32
ELEVATION (m)
500
520
540
560
580
600
620


NORTHING (m)
−6000
−4000
−2000
0
2000
4000
6000
8000
EASTING (m)
−12000
−10000
−8000
−6000
−4000
−2000
0
2000
4000
6000
8000

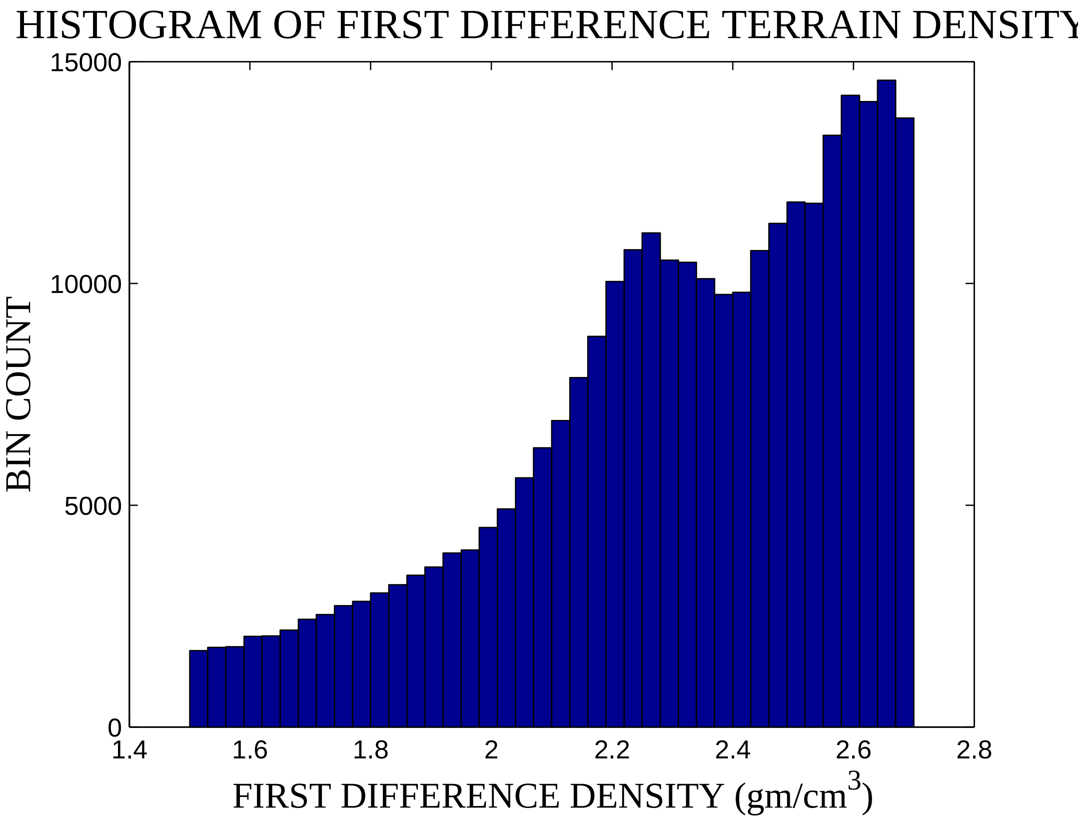
In southern Saskatchewan, the common situation is till with a density of about $2gm/cm^3$ overlying shale, with a density of as high as $2.6gm/cm^3$. If the topography correlates perfectly with the shale, so that the till has a uniform thickness, then any of the Bouguer density methods will give a density for the shale, not the till. In practice, there is some correlation, perhaps more at longer scales ie long compared to the thickness of the till, so that at short scales, less than a hundred m you measure the till density, and at longer scales you are more biased towards shale densities. This is probably part of the reason it is difficult to get a Bouguer density that meets the survey requirements - the effective terrain density is simply variable. It is probably better to always look at the shortest scale features, get a terrain density for till, and worry about the shale in the modeling stage.

First differences method

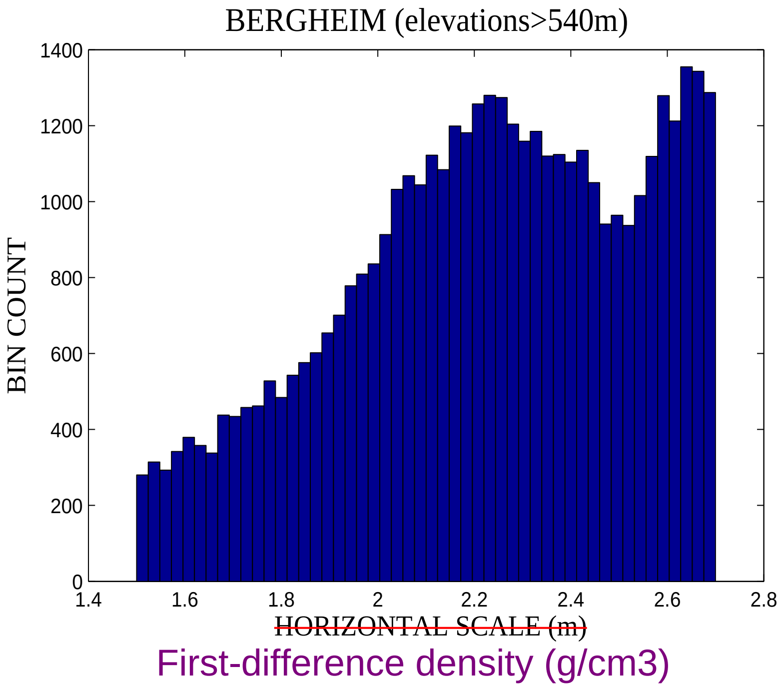
First differences are usually done at the station spacing, but it is also possible to do it at all scales, so the free air gravity difference between two stations is divided by the height difference between two stations and the resulting density is the effective terrain density at the horizontal scale of the separation between the stations. Here are the results using all the Bergheim data.



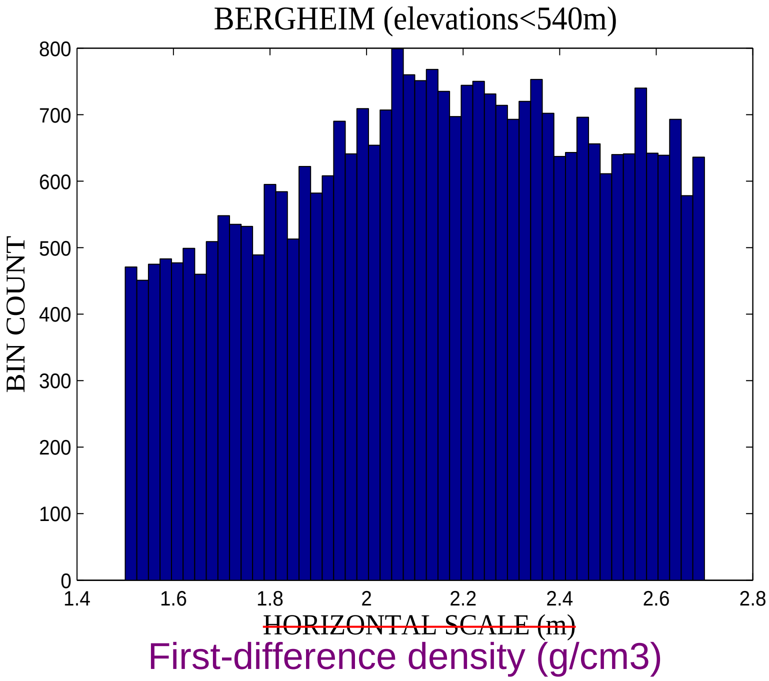
The horizontal scales are from 500m to 20km. The short scales have generally lower density, around 2.2 gm/cc, but at longer scales, from about 4 km to 12 km the indicated density jumps up to 2.5 gm/cc. At the longest scales, the density is more erratic. The Bouguer anomaly (geology) is more influential at longer scales and this is probably the reason for the erratic values. The short scale densities may result from both stations being down in the valley, so biased towards the Haultain sediment density. The higher densities at intermediate scales are close the density of the Floral till, which is what we get from the other techniques.



This is a histogram of results. Higher recovered densities are more common, but note the blip at 2.24-2.3 gm/cc



Here is the Histogram using only elevations above 540m in the Bergheim data. Densities of 2.25 gm/cc are indicated.



Here is the histogram using elevations below 540 m. Densities of about 2 gm/cc are indicated.

BOUGUER DENSITY SUMMARY

1) Determine the accuracy you need in the Bouguer density.

2) Use at least two methods, and as much data as possible. Be aware of the two elements you are looking for. The correct Bouguer density will produce a minimum correlation of Bouguer anomaly with topography, and a minimum variance, or smoothest Bouguer anomaly.

3) You will probably not be able to get an accuracy that meets the requirements. This is probably due to variations in the terrain density.

4) Be wary of correlations between subsurface geology and topography.

5) Parasnis is fast and reliable. Nettleton is more work, but even more reliable.

6) First Difference, especially multi-scale First Difference, can add an extra dimension, namely is the terrain density scale dependent, or spatially variable.

Density of a composite material (till)

Arithmetic mean, called "Voigt average" in rock mechanics

Another option would be
geometric mean,
or "Reuss average"
(for moduli, not for density)

grain pore

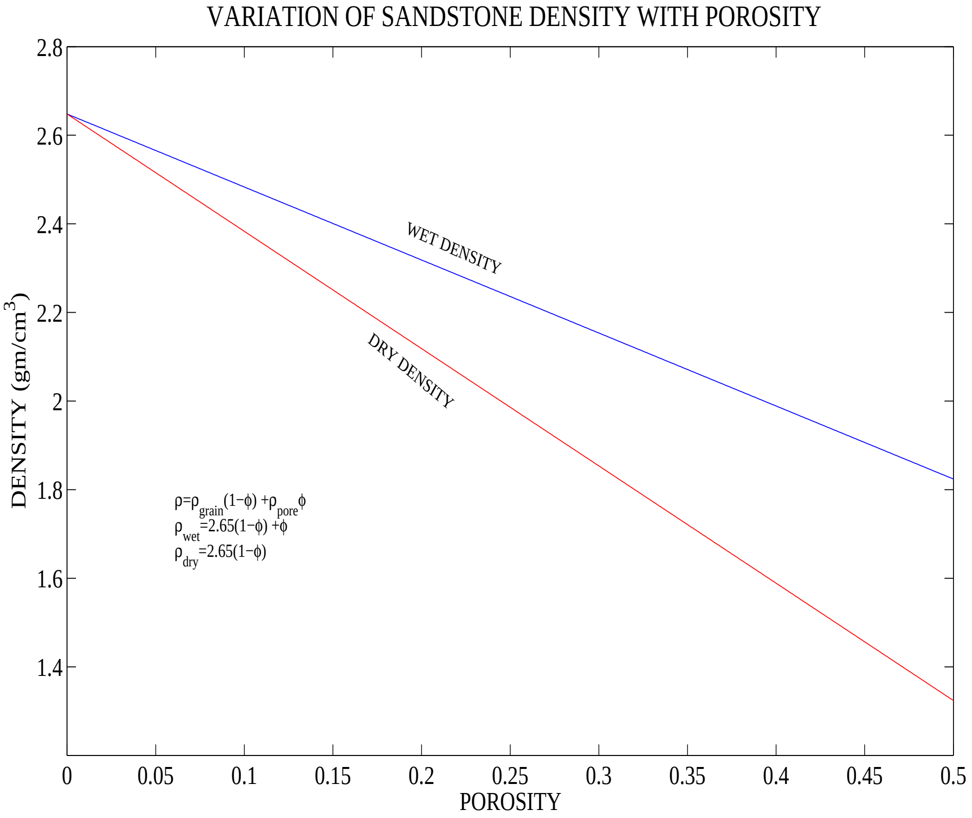
$$\rho = \rho_g(1-\phi) + \rho_p\phi$$

$$\rho_{wet} = 2.6(1-\phi) + \phi$$

pore density = 1 g/cc (water)

$$\rho_{dry} = 2.6(1-\phi)$$

Here is a graph assuming a grain desnity of $2670 kg/m^3$



and here is a table with some data from Kristian Hermann's thesis on local geology.

LITHOLOGY	DRY DENSITY kg/m^3	POROSITY %
glaciolacustrine	1398	48
ablation till	1663	38
subglacial till	1919	29
sand	1667	38

The Battleford till is an ablation till, so a dry density of $1663 kg/m^3$, a porosity of 38% and therefore a wet density of just under $2000 kg/m^3$. The gravity profile over the Bradwell Hill indicated a density of $1600 kg/m^3$, which is a dry density. The wet density would be about $2000 kg/m^3$, so keep these figures in mind when modeling. The subglacial till (Floral and Sutherland) would have dry densities of about $1919 kg/m^3$. The porosity is about 29%, so the indicated wet density is $2200 kg/m^3$, lower than the $2400 kg/m^3$ indicated by our gravity profiles over the Bergheim Hill. Finally the glaciolacustrine data (The Haultain sediments) indicate a dry density of only $1398 kg/m^3$ and a wet density of $1800 kg/m^3$