REGIONAL RESIDUAL SEPARATION

A suggested procedure for regional residual separation

1) Do a second vertical derivative. This enhances short scale features.
2) Remove a plane. This gives you a preliminary look at what the residual might look like.
3) If the location and size of anomalies are apparent decide on an appropriate technique for separation.
4) Remove the regional by:

a) Fitting a polynomial surface.
b) Spectral Factorization.
c) Graphical smoothing.
d) Upward continuation.
e) Convolution.

f) Another popular recent method is the "Empirical Mode Decomposition".

Be aware of local and regional geology.

What the regional looks like may be just as important as what the residual looks like. If the residual looks good, but the regional is inconsistent with regional geology, then the residual is no good.

Gauss' Theorem and EXCESS MASS

The work performed in moving a mass around a closed path is

$$Work = \int \vec{F} \cdot d\vec{s} = 0$$

Using Stokes' Theorem

$$\int \vec{F} \cdot d\vec{s} = \int \int \vec{\nabla} \times \vec{F} \cdot \vec{k} dA$$

which works for any vector field and any closed path. But if $\vec{F}$ is conservative

$$\int \int \vec{\nabla} \times \vec{F} \cdot \vec{k} dA = 0$$

or

$$\vec{\nabla} \times \vec{F} = 0 \quad \text{by shrinking A to a point}$$

vector fields satisfying this equation are called **CONSERVATIVE** or **IRROTATIONAL FIELDS**

But

$$\vec{\nabla} \times \vec{\nabla} = 0 \qquad \text{for any scalar field}$$

so

$$\vec{F} = \vec{\nabla} U$$

So a conservative field can be expressed as the gradient of a scalar field. The scalar field U is the potential for the vector field $\vec{F}$

Note that we have not yet shown the $\nabla^2 U = 0$ ie that U is a potential field.

For a point source of gravity

$$\vec{F} = -\frac{Gm}{r^2}\hat{r} \quad \text{so} \quad U = \frac{GM}{r}$$

LAPLACE AND POISSON

$$\int_S \vec{F} \cdot \hat{n} dS = \int_V \vec{\nabla} \cdot \vec{F} dV \quad \text{Divergence Theorem}$$

and since $\vec{F} = \vec{\nabla} U$

$$\int_S \vec{F} \cdot \hat{n} dS = \int \nabla^2 U dV$$

Suppose we integrate the normal component of this over a closed surface

$$\int \vec{F} \cdot \hat{n} dS = \int_S \int_V \frac{-G\rho}{r^2} \hat{r} dV \cdot \hat{n} dS$$

Where the inner integral is $\vec{F}$

$$
\begin{aligned}
&= -\int_S \int_V \frac{G\rho}{r^2}\hat{r}dV \cdot \hat{n}dS \\
&= -\int_S \int_V \frac{G\rho}{r^2}\hat{r}\cdot\hat{n}\frac{r^2 d\Omega}{cos\theta}dV \quad dS = \frac{r^2 d\Omega}{cos\theta} \\
&= -\int_S \int_V G\rho d\Omega dV \quad \hat{r}\cdot\hat{n} = cos\theta \\
&= -4\pi G\int_V \rho dV = \int_V \nabla^2 U dV
\end{aligned}
$$

and since this is true for any volume

$$\nabla^2 U = -4\pi G\rho$$

or

$$\nabla^2 U(x,y,z) = -4\pi G\rho(x,y,z) \quad \text{POISSON'S EQUATION}$$

Gravitational potential satisfies Poisson's equation at points where there is source of gravity field ρ

If there are no local sources

$$\nabla^2 U(x,y,z) = 0 \quad \text{Laplace's Equation}$$

Returning to the case where S encloses a large volume V

$$\begin{aligned}\int_S \vec{F} \cdot \hat{n} dS &= -\,4\pi G \int_V \rho dV \qquad \text{Gauss' Theorem} \\ &= -\,4\pi G M \quad \text{where M is the total mass inside S}\end{aligned}$$

So we can calculate the total mass inside an enclosing surface from its gravity field.

Mass outside the surface does not count because it integrates to zero

Gauss' theorem is not very convenient for exploration geophysics because we do not have a closed surface to integrate anomalies over. However, we can make it work with a little work.

$$\int \Delta\vec{g} \cdot \hat{n} dS = -4\pi GM$$

If the radius of the sphere is large compared to the depth to the target, then by symmetry the integral over the upper hemisphere must equal the integral over the lower hemisphere, so

$$\int_{upper} \Delta\vec{g} \cdot \hat{n} dS = -2\pi GM$$

and since the upper hemisphere plus the base of the upper hemisphere form a closed surface containing no mass, the integral over the upper hemisphere plus the base of the upper hemisphere must be zero.

The result is

$$\int_{base} \Delta\vec{g} \cdot \hat{n} dS = 2\pi GM$$

where the integral is over the base of the upper hemisphere, or in our case the survey area (there are some limitations) and $\Delta\vec{g} \cdot \hat{n}$ is the vertical component gravity, which is exactly what we measure.

There is a quick and dirty version of the excess mass theorem. If we had a spherical source the maximum gravity would be

$$g_{max} = \frac{GM}{h^2}$$

where h is the depth to source. This gives

$$M = \frac{g_{max}(.65w_{1/2})^2}{G}$$

or

$$M = \frac{g_{max}}{G}.42w_{1/2}^2$$

Note that this formula is exact for a sphere, pretty good for a 'blocky' source and overestimates the mass of a horizontal cylinder by a factor of 2.

The excess mass is actually

$$M_e = \Delta\rho V$$

and the total mass (what you would have to dig out of the ground) is

$$M_t = (\rho + \Delta\rho)V$$

where ρ is the density of the host rock.

$$M_t = (\frac{\rho + \Delta\rho}{\Delta\rho})M_e$$

$$M_t = \frac{1}{2\pi G}(1 + \frac{\rho}{\Delta\rho}) \int \Delta g dx dy$$

CORRECTION FOR MISSING FLANKS

The integration for the excess mass extends from $-\infty$ to $+\infty$, but field data cannot realistically do this. Hence the need for a correction of the missing flanks of the anomaly. The correction consists of generating synthetic data on the flanks that is generated by a point source of the same mass as the anomaly.

The anomaly of a point source is

$$\Delta g = \frac{GM}{l^2} cos\theta = \frac{GM}{l^2}\frac{h}{l}$$

Integrating over a circular survey plane

$$\int \Delta g dS = 2\pi GMh \int_0^R \frac{rdr}{(r^2+h^2)^{3/2}}$$

and changing variables

$$u = r^2 + h^2, du = 2rdr$$

$$\begin{aligned}\int \Delta g dS =& 2\pi GMh \int \frac{du}{u^{3/2}} \\ =& -2\pi GMhu^{-1/2}]_{r=0}^{R} \\ =& 2\pi GMh(\frac{1}{h} - \frac{1}{(R^2+h^2)^{1/2}}) \\ =& 2\pi GM(1 - \frac{1}{[1+(R/h)^2]^{1/2}})\end{aligned}$$

Of course, here M is the true excess mass. Making this explicit by calling it M_{true} and solving for it

$$M_{true} = \frac{1}{(1 - \frac{1}{[1+(R/h)^2]^{1/2}})} \frac{1}{2\pi G} \int \Delta G dS$$

So

$$M_{true} = \frac{M_e}{(1 - \frac{1}{[1+(R/h)^2]^{1/2}})} \qquad (*)$$

M_e is what we get from an excess mass calculation on observed data and the term it is divided by is the flanks correction.

The survey plane we integrated over was circular. For a square survey plane the correction i s

$$M_{true} = \frac{\frac{\pi}{2} M_e}{Tan^{-1}(W/2\sqrt{2}h)}$$

where W is here the width of the survey plane. There is little practical difference between the two corrections.

CORRECTION FOR MISSING FLANKS

How big is the correction? The following table shows the magnitude of the correction factor as a function of the survey radius and the depth. If the survey is four times the anticipated depth, as is the nominal design, then R=2d gives a correction factor of 0.55, so the true mass is nearly twice the calculated mass in this case.

denominator in (*)

	CORRECTION FACTOR
R=d	0.29
R=2d	**0.55**
R=3d	0.68
R=4d	0.76
R=5d	0.80
R=6d	0.84
R=7d	0.86
R=8d	0.87
R=9d	0.89
R=10d	0.90
...	...
R=100d	0.99