Magnetic field intensity

$\vec{H}$ is the force on a magnetic

monopole of strength +1

$$F = \frac{+m_1 m_2}{\mu r^2}\hat{r} \quad \vec{H} = \frac{\vec{F}}{(m_1 = +1)}$$

The sign convention is that $+m$ is a N pole, $-m$ is a south pole, which means that the geographic north pole of the earth is a magnetic south pole.

$$\begin{aligned} H_r =& (\frac{-m}{r_2^2}cos\alpha_2 + \frac{m}{r_1^2}cos\alpha_1) \quad r = r_1 cos\alpha_1 + acos\theta \\ =& + m(-\frac{r + acos\theta}{r_2^3} + \frac{r - acos\theta}{r_1^3}) \quad r_1^2 = r^2 + a^2 - 2racos\theta \end{aligned}$$

and

$$\begin{aligned} H_\theta =& m(\frac{sin\alpha_1}{r_1^2} + \frac{sin\alpha_2}{r_2^2}) \quad sin\alpha_1 = (a/r_1)sin\theta \\ =& m(\frac{asin\theta}{r_1^3} + \frac{sin\alpha_2}{r_2^3}) \quad r_2^3 = (r^2 + a^2 + 2arcos\theta)^{3/2} \end{aligned}$$

using the binomial expansion and dropping higher order terms in a/r $(a << r)$

$$\begin{aligned}\vec{H} =& \frac{4ma}{r^3} cos\theta \hat{r} + \frac{2ma}{r^3} sin\theta \hat{\theta} \\ =& 2\mathcal{M}\frac{cos\theta}{r^3}\hat{r} + \mathcal{M}\frac{sin\theta}{r^3}\hat{\theta}\end{aligned}$$

where $\mathcal{M} = 2ma$ is the magnetic moment - the monopole strength time the separation.

A volume density of dipole moments

$$\vec{M} = \frac{\partial \mathcal{M}}{\partial V}$$

produces an intensity

$$\vec{H} = 4\pi\vec{M} \quad \vec{M} \text{ is the } \textbf{MAGNETIZATION}$$

or
~~of~~ more properly the **INTENSITY OF MAGNETIZATION**

$\vec{M}$ is the source for $\vec{H}$ just as ρ is the source for $\vec{g}$, but in magnetics H can also induce M

$$\vec{M} = k\vec{H} \quad \text{(actually } k(H))$$

where k is called the **SUSCEPTIBILITY**

$$\vec{H} = \frac{4ma}{r^3} cos\theta\hat{r} + \frac{2ma}{r^3} sin\theta\hat{\theta}$$

by inspection

$$\vec{H} = -\vec{\nabla}A \quad A = \frac{\mathcal{M}}{r^2} cos\theta = \vec{\mathcal{M}} \cdot \vec{\nabla}\frac{1}{r}$$

so the magnetic intensity due to a dipole can be derived from a scalar potential

$$\text{Maxwell} \quad \vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{J}$$

where $\vec{J}$ is the current density

and if there is no current

$$\vec{\nabla} \times \vec{H} = 0$$

$$\rightarrow \vec{H} = -\vec{\nabla}A \quad \text{because} \vec{\nabla} \times \vec{\nabla}A = 0 \text{ for any A}$$

$$A(\vec{r}) = -\int_{v'} \vec{M}(\vec{r'}) \cdot \vec{\nabla} \frac{1}{|\vec{r} - \vec{r'}|} dv'$$

The quantity $\vec{M}(\vec{r'}) \cdot \vec{\nabla} = M \frac{\partial}{\partial m}$

and if the magnetization is uniform, we can take it out of the integral

$$A(\vec{r}) = -M \frac{\partial}{\partial m} \int_{v'} \frac{dv'}{|\vec{r} - \vec{r'}|}$$

and the field intensity is then

$$\vec{H}(\vec{r}) = -\vec{\nabla} A = \vec{\nabla}[M \frac{\partial}{\partial m} \int_{v'} \frac{dv'}{|\vec{r} - \vec{r'}|}]$$

The total magnetic intensity is the vector sum of the inducing field plus the field produced by magnetization

$\vec{H}_t(\vec{r}) = \vec{H}_o(\vec{r}) + \vec{H}_m(\vec{r})$

$\vec{H}_o$ and $\vec{H}_m$ are not necessarily in the same direction

The component of the induced field in the direction of the inducing field is

$$\frac{\vec{H}_o}{H_o} \cdot \vec{H}_m = \hat{h}_o \cdot \vec{H}_m = -\,\vec{h}_o \cdot \nabla A_m$$
$$= -\,\frac{\partial A_m}{\partial h_o}$$

where h_o is an element of distance in the direction of H_o

$\rightarrow \vec{h}_o \cdot \vec{H}_m = M \frac{\partial^2}{\partial h_o \partial m} \int \frac{dv'}{|\vec{r}-\vec{r}'|}$

if $\vec{H}_o$ and $\vec{M}$ are in the same direction (ie no remanence)

$$\vec{h}_o \cdot \vec{H}_m = M \frac{\partial^2}{\partial h_o^2} \int \frac{dv'}{|\vec{r}-\vec{r}'|}$$

and if

$$\vec{M} = k\vec{H}_o$$

$$\hat{h}_o \cdot \vec{H}_m = kH_o \frac{\partial^2}{\partial h_o^2} \int \frac{dv'}{|\vec{r}-\vec{r}'|}$$

The inducing field $\vec{H}$

induces a magnetization $\vec{M}$

which causes an additional field H'

$$\vec{M} = k\vec{h}$$
$$\vec{H}' = 4\pi\vec{M}$$

The sum of these two fields is called the **MAGNETIC INDUCTION** $\vec{B}$

$$\begin{aligned}\vec{B} &= \vec{H} + \vec{H}' \\ &= \vec{H} + 4\pi\vec{M} \\ &= \vec{H} + 4\pi k\vec{H}' \\ &= (1 + 4\pi k)\vec{H}\end{aligned}$$
$$\vec{B} = \mu\vec{H}$$

where μ is called the **permeability** in emu $\mu = 1$ and has no dimension

in mks $\mu = 4\pi \times 10^{-7}$ Henry/m (in space)

magnetic units are a little confusing so here is a summary

	mks	emu
H	amp/m	$oersted$
B	$weber/m^2$	$gauss = 10\gamma$
	or $Tesla = 10^9 nT$	
μ	$4\pi \times 10^{-7} Henry/m$	1

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Henry (H) is the unit of Inductance
(e.m.f./(dCurrent/dTime) in a circuit)
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Going back a few pages to a more general relation

$$\vec{H}(\vec{r}) = -\vec{\nabla} A = \vec{\nabla}[M \frac{\partial}{\partial m} \int_{v'} \frac{dv'}{|\vec{r} - \vec{r'}|}]$$

and

$$\vec{g}(\vec{r}) = \vec{\nabla} G\rho \int_{v'} \frac{dv'}{|\vec{r} - \vec{r'}|}$$

so

$$\vec{H}(\vec{r}) = \frac{M}{G\rho} \frac{\partial}{\partial m} \vec{g}(\vec{r}) \quad \text{POISSONS RELATION}$$

Poisson's Relation is a vector relation, so it works for components

$$Z = \frac{M}{G\rho}\frac{\partial}{\partial m}\Delta g$$

and if the magnetization is vertical, or nearly so

$$Z = \frac{M}{G\rho}\frac{\partial}{\partial z}\Delta g$$

and this is the most common form

PHYSICAL EXPLANATION FOR POISSON'S RELATION

The magnetic field of a dipole falls off as $1/r^3$, with a polarization in the direction of the dipole. The gravity field falls off as $1/r^2$. Differentiating $1/r^2$ gives $1/r^3$ and differentiating in a particular direction imposes that direction on the result.

$$Z = H_z = 4m_1 a/r^3 \hat{r} = 2m/r^3 \hat{r}$$

$$g = -\frac{GM}{r^2}\hat{r}$$

$$\frac{\partial g}{\partial r} = \frac{2GM}{r^3}\hat{r}$$

so

$$\begin{aligned} \frac{m/dv}{G\rho}\frac{\partial g}{\partial r} &= \frac{m}{GM}\frac{\partial g}{\partial r} \\ &= \frac{m}{GM}\frac{2GM}{r^3} \\ &= \frac{2m}{r^3}\hat{r} = Z \end{aligned}$$

As we have already seen

$$\begin{aligned}\vec{H} =& 4\pi\vec{M}\\ \vec{\nabla}\cdot\vec{H} =& 4\pi\vec{\nabla}\cdot\vec{M}\\ \nabla^2 A =& -4\pi\vec{\nabla}\cdot M \quad \text{POISSONS EQUATION}\end{aligned}$$

AND IN THE ABSENCE OF MAGNETIZATION

$$\nabla^2 A = 0 \quad \text{LAPLACE}$$

MAGNETIC POTENTIAL SATISFIES LAPLACE'S EQUATION IN THE ABSENCE OF LOCAL MAGNETIZATION, OR WHERE MAGNETIZATION IS UNIFORM

and

$$\begin{aligned}\vec{\nabla}\nabla^2 A =& 0\\ \nabla^2 H_x =& 0\\ \nabla^2 H_y =& 0\\ \nabla^2 H_z =& 0\end{aligned}$$

SO THE THREE VECTOR COMPONENTS OF MAG SATISFY LAPLACE AND ARE THEN POTENTIAL FIELDS

INTEGRATING POISSON'S EQTN OVER A SPHERICAL VOLUME ENCLOSING A SOURCE

$$\int \vec{\nabla} \cdot \vec{H} dv = 4\pi \int \vec{\nabla} \cdot \vec{M} dv$$
$$\int \vec{H} \cdot \hat{n} dS = 4\pi \int \vec{\nabla} \cdot \vec{M} dv$$

AND USING THE SAME REASONING WE USED FOR GRAVITY

$$\int Z dS = -2\pi \int \vec{\nabla} \cdot \vec{M} dv$$

IN GRAVITY THIS RESULT WAS

$$\int g dS = 2\pi GM \quad \text{EXCESS MASS}$$

SO IT APPEARS THERE IS NO EXCESS MAGNETIZATION THEOREM. IF WE APPLY THE DIVERGENCE THEOREM TO THE RHS

$$\int Z dS = -2\pi \int \vec{M} \cdot \hat{n} dS$$

but the survey plane is outside the magnetized region (assuming the magnetized region is an isolated source at depth) so $\vec{M} = 0$ on it

$$\int Z dS = 0$$

SO THE INTEGRAL OF THE VERTICAL COMPONENT OF MAG IS IN FACT ZERO

$$V(\vec{r}) = \int \frac{G\rho(\vec{r'})dv'}{|\vec{r} - \vec{r'}|} \quad \text{GRAVITY POTENTIAL}$$

$$A(\vec{r}) = \int \vec{M}(\vec{r'}) \cdot \vec{\nabla} \frac{1}{|\vec{r} - \vec{r'}|} dv' \quad \text{MAG POTENTIAL}$$

WE DERIVE g_z, Z, T THE GRAVITY ANOMALY, VERTICAL MAG AND TOTAL FIELD MAG FROM POTENTIALS. DO THESE SATISFY LAPLACE'S EQUATION ALWAYS?

$$\vec{g} = -\frac{GM}{r^2}\hat{r}$$

Writing Laplace in spherical polar coordinates

$$\nabla^2 = \frac{1}{r^2 sin\theta}[sin\theta \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{\partial}{\partial \theta}(sin\theta \frac{\partial}{\partial \theta}) + \frac{1}{sin\theta}\frac{\partial^2}{\partial \phi^2}]$$

and substituting above

$$\nabla^2 g_r = \frac{2GM}{r^4} \neq 0$$

SO THE RADIAL COMPONENT OF GRAVITY DOES NOT SATISFY LAPLACE AND IS NOT A POTENTIAL FIELD

g_r IS NOT MEASURED IN A UNIFORM DIRECTION SO g_r IS NOT A POTENTIAL FIELD.

g_z IS MEASURED IN A UNIFORM DIRECTION, SO g_z IS A POTENTIAL FIELD.

A VECTOR COMPONENT OF A SCALAR POTENTIAL FIELD MUST BE EVALUATED IN A UNIFORM DIRECTION FOR IT TO BE A POTENTIAL FIELD ITSELF.

IS g_z ALWAYS MEASURED IN A UNIFORM DIRECTION ?

NOT IF THE SURVEY AREA IS LARGE!

ALSO, AN ANOMALY CHANGES THE DIRECTION OF LOCAL GRAVITY (THE INSTRUMENT IS LEVELED).

BY HOW MUCH?

$$\frac{1mGal}{10^3Gal} \approx 10^{-6}rad \approx 0.22seconds$$

SO NOT MUCH.

THE SAME COMMENTS APPLY TO Z.

WHAT ABOUT TFM?