Gravity

1 Gal = 1 cm/s^2 = 0.01 m/s^2

Gravitational constant: $G = 6.67384(80)\cdot 10^{-11}\,\mathrm{N}\left(\frac{\mathrm{m}}{\mathrm{kg}}\right)^2$ (the standard uncertainty shown in parentheses)

Basic solutions:

Point or sphere of mass M at distance $\mathbf{r}$ (Newton's law): $\mathbf{g}(\mathbf{r}) = -G\frac{M}{r^2}\hat{\mathbf{r}} = -GM\frac{\mathbf{r}}{r^3}$

Line of linear density γ: $\mathbf{g}(\mathbf{r}) = -\frac{2G\gamma}{r}\hat{\mathbf{r}} = -2G\gamma\frac{\mathbf{r}}{r^2}$

Plane of surface density σ with normal $\mathbf{n}$: $\mathbf{g} = -2\pi G\sigma\mathbf{n}$

Relation of gravity strength to potential: $\mathbf{g} = -\vec{\nabla}U \equiv -\frac{\partial U}{\partial \mathbf{x}}$

Laplace equation: $\mathrm{div}(\mathbf{g}) = \nabla\cdot\mathbf{g} = -\nabla^2 U = 0$

Poisson's equation: $\nabla^2 U = 4\pi G\rho$

Gauss' theorem: $\oiint\limits_{\text{closed surface}} \mathbf{g}\cdot\mathbf{n}ds = -4\pi G\int\limits_{\text{volume}}\rho dV = -4\pi GM$

Ellipsoid:

figure flattening: $r(\theta) \approx r_e\left(1 - f\cos^2\theta\right)$

gravity flattening: $g(\theta) \approx g_e\left(1 + \beta\cos^2\theta\right)$

Standard density for Bouguer correction: $\rho = 2670\frac{\mathrm{kg}}{\mathrm{m}^3}$

Width of anomaly at half amplitude:

From a sphere at depth h: $w_{1/2} \approx \dfrac{h}{0.65} \approx 1.54h$

From a cylinder at depth h: $w_{1/2} = 2h$

Excess mass:

From integrated gravity: $M_e = \dfrac{\int g(x,y)dxdy}{2\pi G}$

With correction for limited survey area radius R: $M_e = \dfrac{1}{1-\dfrac{1}{\sqrt{1+(R/h)^2}}}\dfrac{\int g(x,y)dxdy}{2\pi G}$

From peak gravity (for a spherical source at depth h): $M_e = \dfrac{g_{\max}h^2}{G} \approx 0.42\dfrac{g_{\max}w_{1/2}^2}{G}$

Total buried mass: $M_{total} = \left(1+\dfrac{\rho}{\Delta\rho}\right)M_e$

Coordinates of the Centre of mass for gravity source:

$$x_{\text{Centre Mass}} = \frac{\int xg(x,y)dxdy}{\int g(x,y)dxdy}, \qquad y_{\text{Centre Mass}} = \frac{\int yg(x,y)dxdy}{\int g(x,y)dxdy}$$

Upward and downward continuation: $U(x,y,z) = \dfrac{1}{2\pi}\iint A(k_x,k_y)e^{ik_x x+ik_y y+|\mathbf{k}|(z-z_0)}dk_x dk_y$

where the Fourier transform of the field at $z=z_0$: $A(k_x,k_y) = \dfrac{1}{2\pi}\iint U(x',y',z_0)e^{-ik_x x-ik_y y}dx'dy'$

Magnetic field:

Magnetic potential for a dipole $\mathbf{m}$: $V(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\cdot\hat{\mathbf{r}}}{r^2}$

Poisson's relation between gravity and magnetic fields of a structure: $\mathbf{H}(\mathbf{r}) = \frac{\mu_0 m}{4\pi G\rho}\left(\mathbf{n}\cdot\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{g}(\mathbf{r})$

Induced magnetic field: $\mathbf{B} = \mu_0(1+\kappa)\mathbf{H} \equiv \mu\mathbf{H}$

Measurements:

Mean value for *N* measurements: $\overline{g}_N \equiv \frac{1}{N}\sum_{i=1}^{N} g_i$

Standard deviation for *N* measurements: $s_{N-1} \equiv \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(g_i - \overline{g}_N\right)^2}$

Mathematics:

Fourier series:

Sine- and cosine form: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi}{L}nx\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi}{L}nx\right)$

Exponential form: $f(x) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i\frac{2\pi}{L}nx}$

Forward transform (α_n from $f(x)$): $\alpha_n = \frac{1}{L}\int_0^L e^{-i\frac{2\pi}{L}nx} f(x)\,dx$

Dirac's delta function:

Principal property: $f(x) = \int_{-\infty}^{\infty} \delta(x-x') f(x')\,dx'$

Relation to Fourier transform: $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\,dk$