Reflection seismic Method - 2D

- Zero-offset model
- Wavelets
  - Minimum, maximum, mixed phase
- Convolutional model
- Subsurface sampling

- <u>Reading:</u>
  - Sheriff and Geldart, Chapters 6, 8

Acoustic Impedance

What we image in reflection sections

- At *near-vertical* incidence:
  - *P*-to-*S*-wave conversions are negligible;
  - *P*-wave reflection and transmission *amplitudes* are sensitive to *acoustic impedance* ($Z=\rho V$) contrasts:

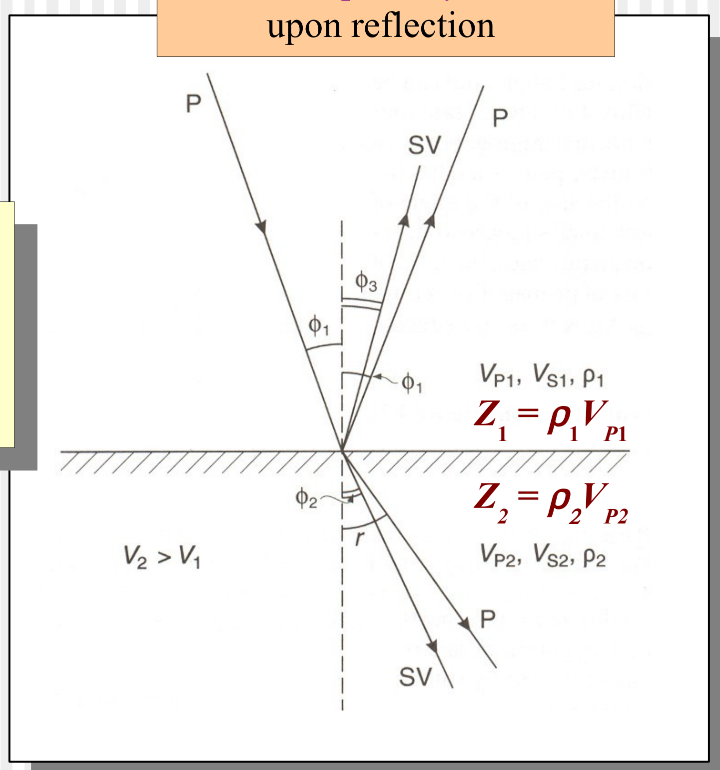
P-wave Reflection Coefficient

$$R_{PP}=\frac{A_{P_{reflected}}}{A_{P_{incident}}}=\frac{Z_2-Z_1}{Z_2+Z_1}$$

Note: if $Z_2 < Z_1$, $R<0$.
This means *polarity reversal* upon reflection

P-wave Transmission Coefficient (for amplitude)

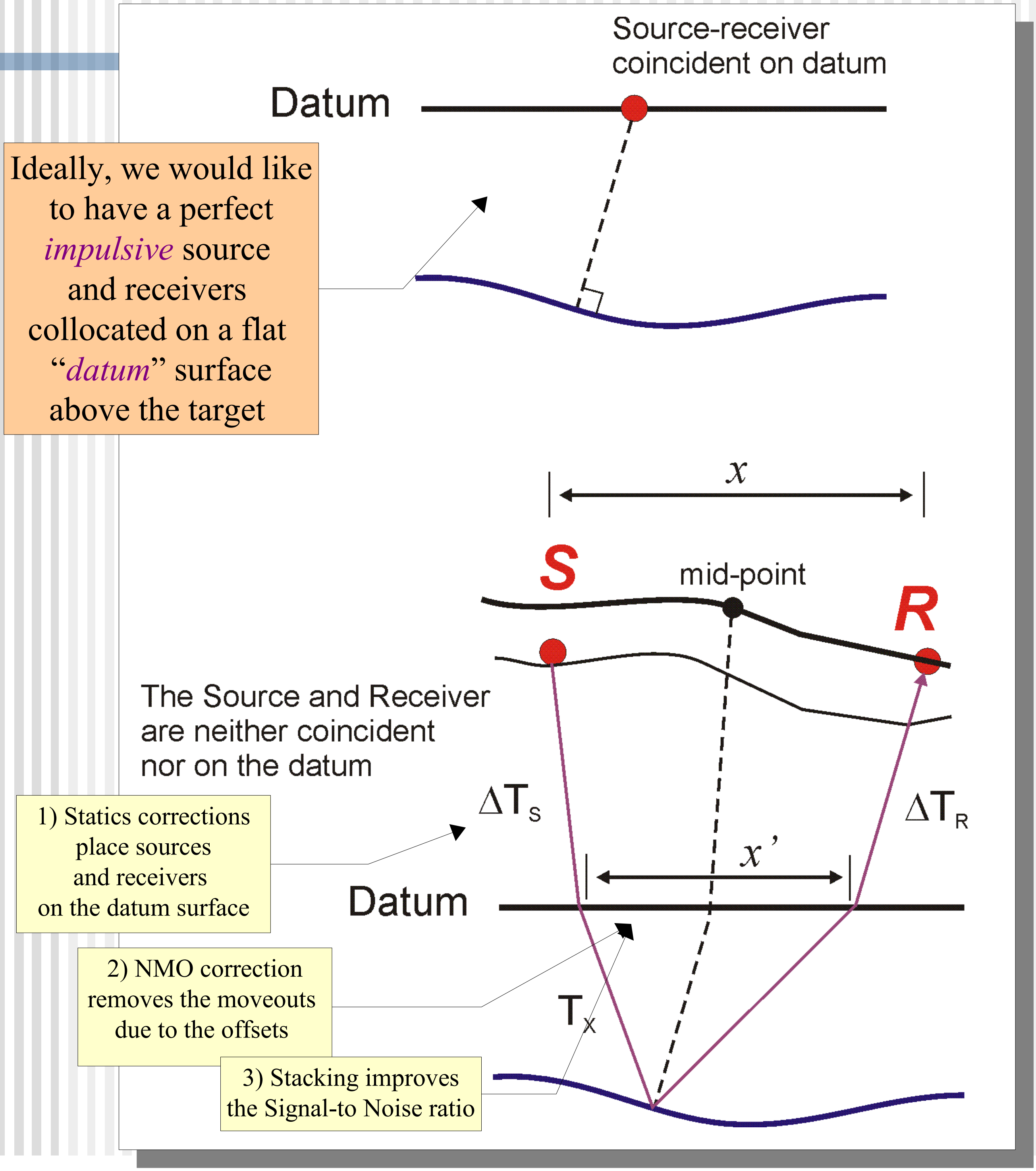
$$T_{PP}\equiv 1-R_{PP}=\frac{2Z_1}{Z_2+Z_1}$$



- *P*- and *S*-wave *reflection* amplitudes *increase with incidence angle.*

Zero-Offset Section

the objective of pre-migration processing

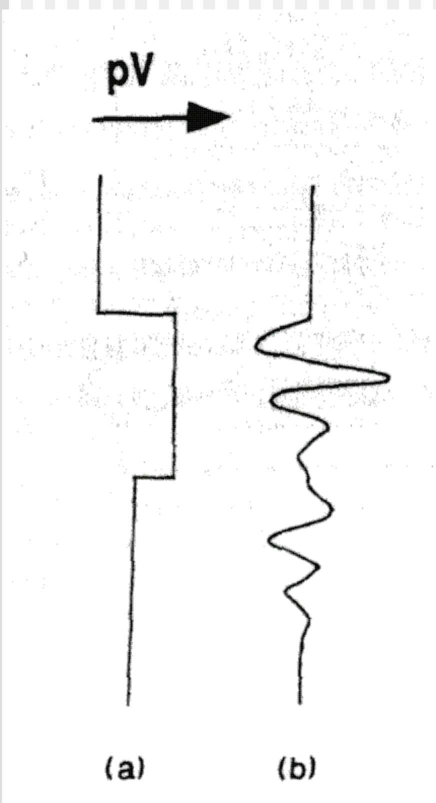


Reflection imaging

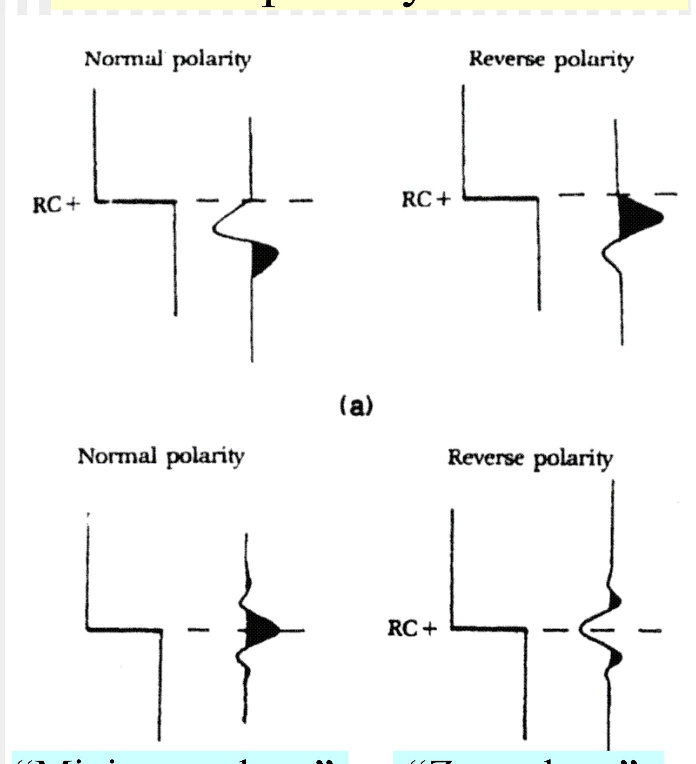
- Multi-offset data are transformed into a *zero-offset section*:
  - Statics place sources and receivers on a flat reference (datum) surface;
  - Deconvolution compresses the wavelet into a “spike” and attenuates “short-period” multiples;
  - Filtering attenuates noise and other multiples.
- *Migration* transforms the zero-offset section into a depth image

Wavelets

- Impedance contrasts are assumed to be sharp, yet the wavelet always imposes its signature on the record



Standard polarity convention



"Minimum-phase" wavelet

"Zero-phase" wavelet

Minimum-, maximum-, and zero-phase wavelets

Key facts

- Consider a wavelet consisting of two spikes: $w=(1,a)$:
  - For $|a| < 1$, it is called *minimum-phase*;
  - For $|a| > 1$, it is *maximum-phase*;
  - Note that its *z*-transform is $W(z)=1+az$, *and* $1/W(z)$ represents a convergent series near $z=0$. This means that there exists a filter that could convert the wavelet into a spike.
- A convolution of all minimum- (maximum-) phase wavelets is also a minimum- (maximum-) phase wavelet:

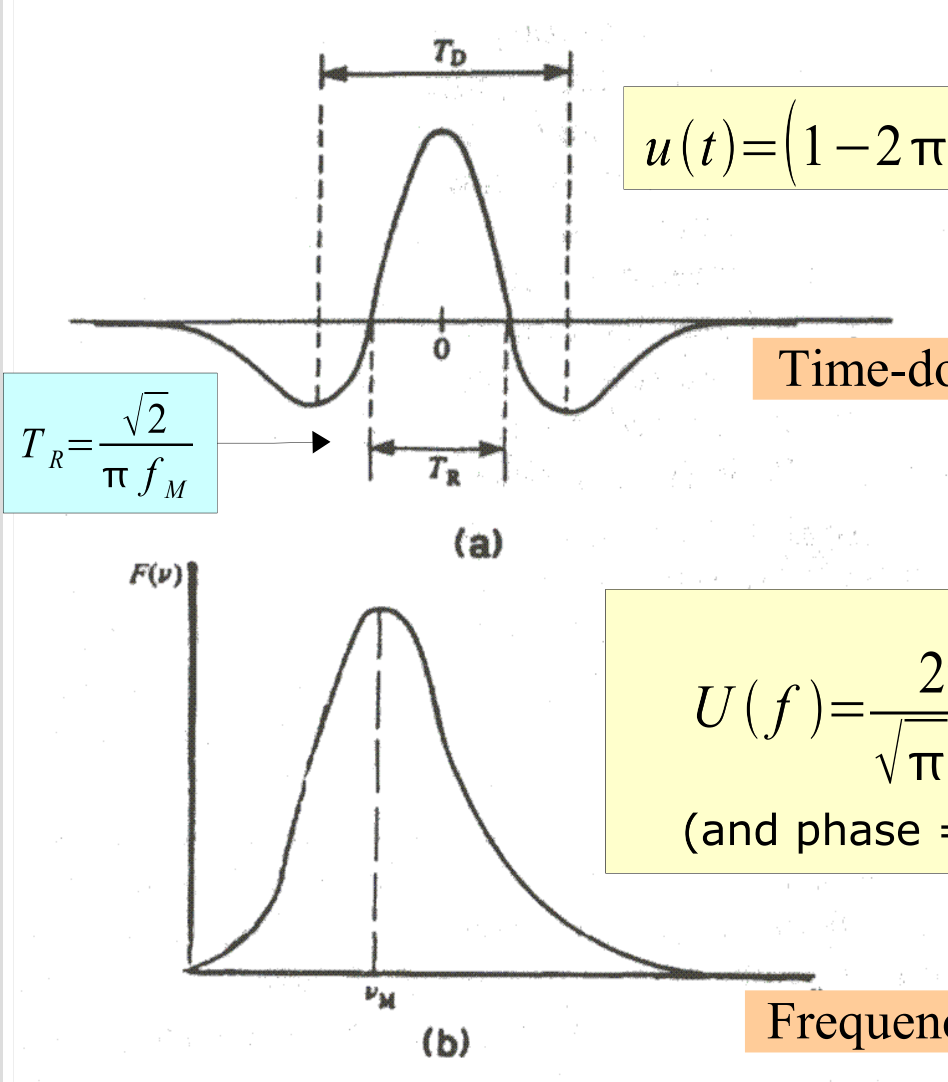
$$W(z)=\prod_{i=0}^{N}\left(1+a_i z\right)$$

- When minimum- and maximum-phase factors are intermixed in the convolution, the wavelet is called *mixed-phase.*
- Minimum- (maximum-) phase wavelets have the fastest (slowest) rate of energy build-up with time
- Minimum-phase wavelets are associated with *causal* processes.

Ricker wavelet

- A common zero-phase wavelet (Ricker):

  (f_M is the *peak frequency*)



$$u(t) = \left(1 - 2\pi^2 f_M^2 t^2\right) e^{-(\pi f_M t)^2}$$

$$T_R = \frac{\sqrt{2}}{\pi f_M}$$

Time-domain

$$U(f) = \frac{2f^2}{\sqrt{\pi} f_M^2} e^{-\left(\frac{f}{f_M}\right)^2}$$

(and phase = 0)

Frequency-domain

Convolution

- Convolution of two series, u_i, and w_i, denoted $u*w$, is:

$$\left(u*w\right)_i=\sum_k u_k w_{i-k}$$

For each i, the result is dot product of u and shifted and "reflected", or "folded" (i..e., running backwards) w

- In integral form:

$$u(t)*w(t)=\int_{-\infty}^{+\infty} u(\tau)w(t-\tau)d\tau$$

- As multiplication, it is symmetric (commutative):

$$u*w=w*u$$

- Note that to multiply two polynomials, with coefficients u_k and w_k, we would use exactly the first formula above. Therefore, **in *Z* or *frequency* domains, convolution becomes simple multiplication** of polynomials (show this!):

$$u*w\Leftrightarrow U(z)W(z)\Leftrightarrow U(f)W(f)$$

- This property facilitates efficient digital filtering.

Cross-Correlation

- Cross-correlation of two series, u_i, and w_i, is:

$$ccorr(u*w)_i = \sum_k u_k w_{i+k}$$

Unlike in convolution, no “folding” of w

- In integral form:

$$ccorr(u(t), w(t)) = \int_{-\infty}^{+\infty} u(\tau) w(t+\tau) d\tau$$

- It is anti-symmetric in the following sense (show this!):

$$ccorr(u, w)(t) = ccorr(w, u)(-t)$$

- **In *Z* or *frequency* domains**, cross-correlation is:

$$ccorr(u, w)(z) = \overline{U(z)} W(z)$$

Complex conjugate

- Cross-correlation is used as a *measure of similarity* between time series.

Autocorrelation

- Cross-correlation of a time function with itself is called *Autocorrelation*:

$$ccorr(u,u)_i = \sum_k u_k u_{i+k}$$

- In integral form:

$$Auto_u(t) = \int_{-\infty}^{+\infty} u(\tau)\, u(t+\tau)\, d\tau$$

- It always is an even function (show this!):

$$Auto_u(t) = Auto_u(-t)$$

- **In *Z* or *frequency* domains**, autocorrelation is:

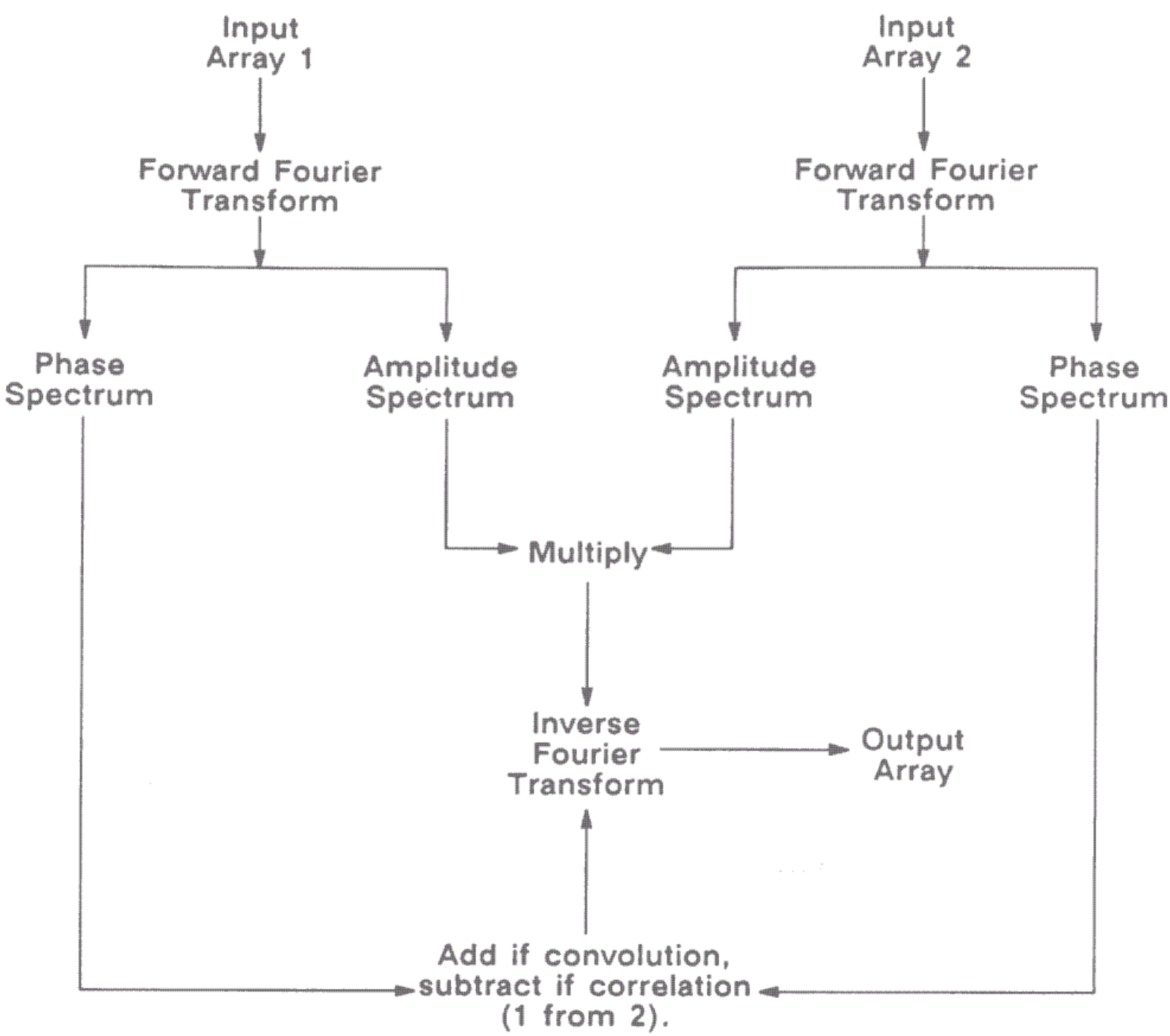
Always real value - **Energy Spectrum**

$$Auto_u(z) = \overline{U(z)}\, U(z) = |U(z)|^2$$

- Therefore, autocorrelation is also the Fourier transform of the energy spectrum of the signal
    - It is independent of the phase spectrum!
- Autocorrelation is used as a *measure of self-similarity* within a time series.

Practical convolution and cross-correlation

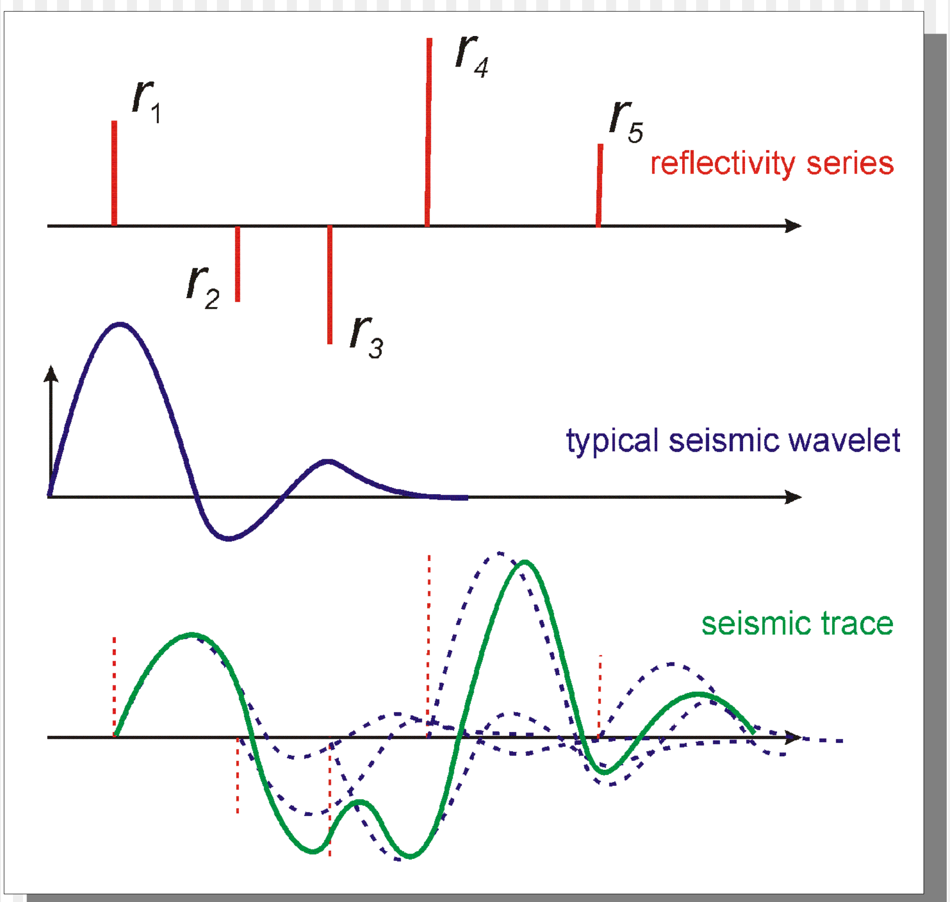
- Usually performed via Fast Fourier Transform



From Yilmaz, 1987

Convolutional model
basic idea

- Reflection seismic trace is a convolution of the source wavelet with the Earth's 'reflectivity series'
- The reflectivity series includes:
  - primary reflections;
  - multiples.



Convolutional model
general

$$u = e * w + n$$

- u: Recorded signal
- e: Earth's response
- w: Source wavelet
- n: Noise

$$e = R * A * r * S$$

- R: Receiver site response
- A: Attenuation
- r: Reflectivity
- S: Source site response

$$n = n_{coherent} * w + n_{random}$$

Convolutional model
simplified for practical reflection imaging

$$u = r * w + n_{random}$$

- Assumptions and practical approximations:

$$acorr(r) = E_r \delta(t)$$

"Random" (uncorrelated) reflectivity

$$acorr(n) = E_n \delta(t)$$

Random noise

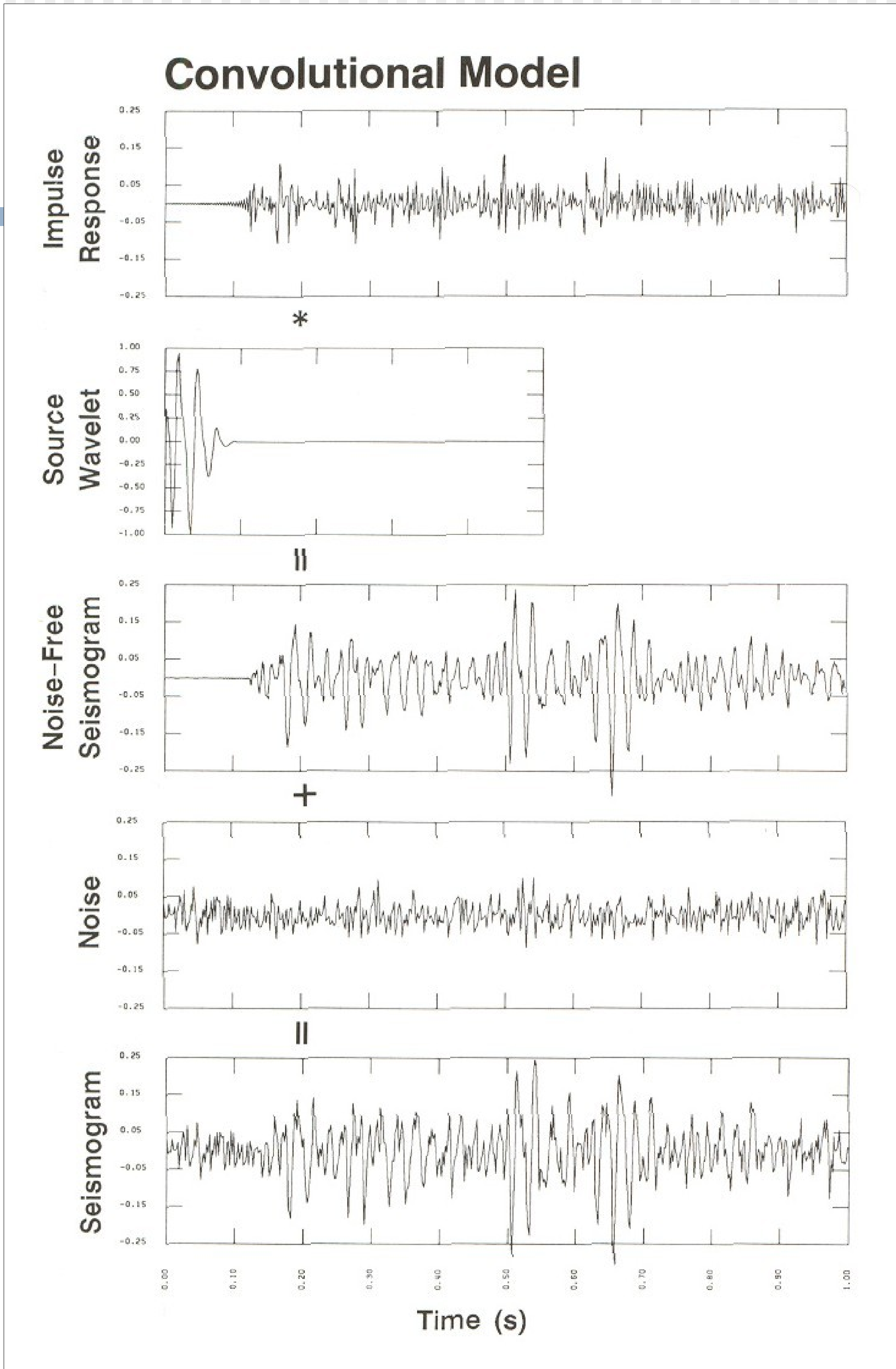
$$E_n \ll E_r$$

Noise attenuated by stacking and filtering

- Consequently, the autocorrelation function of the wavelet can be estimated from the data:

$$acorr(w) \approx acorr(u)$$

Convolutional model



From Yilmaz, 1987

Subsurface sampling

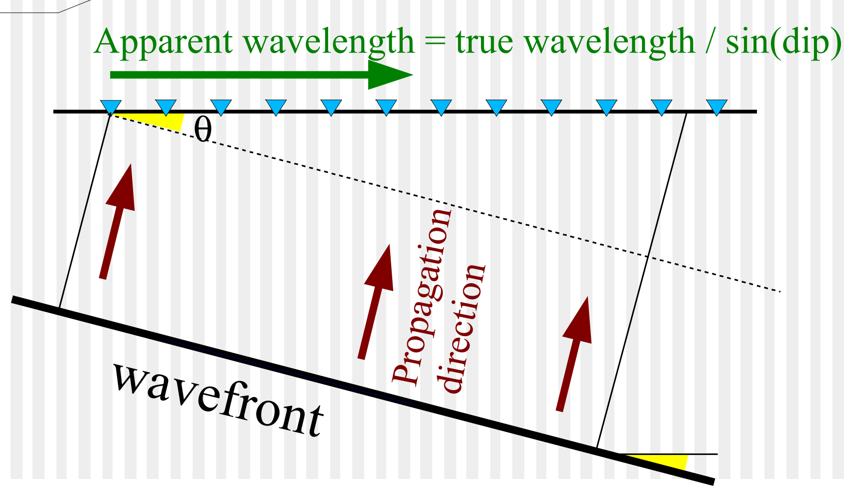
- Seismic surreys are designed with some knowledge of geology and with specific targets in mind:
  - Limiting factors: velocities, depths, frequencies (thin beds), dips.
- Maximum allowable geophone spacing in order to record reflections from dipping interfaces

$$Geophone\,Spacing_{max} < \frac{\lambda_{apparent}}{2} = \frac{\lambda_{min}}{2\sin\theta} = \frac{V_{min}}{2 f_{max} \sin\theta}$$

- The same, in terms of moveout *dt/dx* (sin θ = tan (*moveout*)):

$$Geophone\,Spacing_{max} < \frac{1}{2 f_{max} \frac{dt}{dx}}$$

More conservatively, this factor is usually taken = 4



Voxel
(Elementary cell of seismic volume)

- "Voxel" is determined by the spatial and time sampling of the data
  - For a typical time sampling of 2 ms (3 m two-way at 3000 m/s), it is typically 3 by 15 m^2 in 2D;
  - 3 by 15 by 25 m^3 in 3D.
- For a properly designed survey, voxel represents the smallest potentially resolvable volume
  - Note that the Fresnel zone limitation is partially removed by *migration* where sufficiently broad reflection aperture is available.
  - Migration is essentially summation of the amplitudes over the Fresnel zones that collapses them laterally.
  - Migration is particularly important and successful in 3D.