Attenuation and dispersion

- Mechanisms:
  - *Absorption* (anelastic);
  - *Scattering* (elastic).
- *P*- and *S*-wave, bulk and shear attenuation
- Mathematical descriptions
- Measurement
- Frequency dependence
- Velocity dispersion, its relation to attenuation

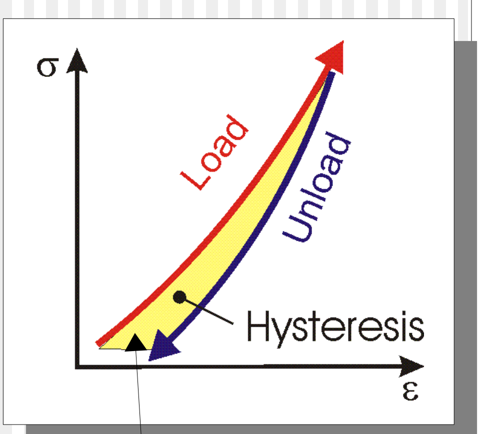
- Reading:
  - Shearer, 6.2, 6.6
  - Sheriff and Geldart, Sections 2.7; 6.5

Mechanisms of attenuation

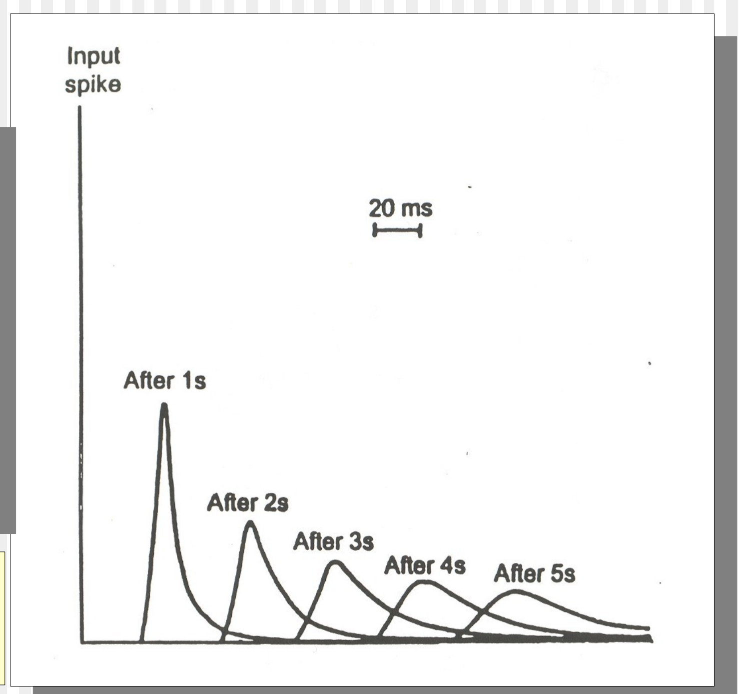
- Three processes lead to reduction of elastic amplitude as the wave propagate away from the source:
  - *Geometrical spreading* - total energy is conserved but distributed over larger wavefronts
    - In fact, not so easy to define mathematically
  - *Scattering* (elastic attenuation) – elastic energy is scattered out of the seismic phase of interest
    - In practice, can be hard to differentiate from geometrical spreading
  - *Anelastic* (intrinsic) attenuation, or *absorption* – elastic energy is converted to heat
    - Key distinction – frequency dependence

Absorption

- When an elastic wave travels through any medium, its *mechanical* energy is progressively converted to *heat* (through friction and viscosity)
  - On grain boundaries, pores, cracks, water, gas, *etc*.
  - Loss of elastic energy causes the amplitude to *decrease* and the pulse to *broaden*.



Area of the hysteresis curve is a measure of absorption



Scattering

- Wavelength- dependent;
- *Scattering regime* is controlled by the ratio of the *characteristic scale length* of the *heterogeneity* of the medium, *a*, to the wavelength.
- Described in terms of *wavenumber*, $k=2\pi/wavelength$:
  - *ka* << 0.01 (quasi-homogeneous medium) - no significant scattering;
  - *ka* < 0.1 (*Rayleigh scattering*) - produces *apparent Q* and anisotropy;
  - 0.1 < *ka* < 10 (*Mie scattering*) - introduces strong attenuation and discernible scattering noise in the signal.
    - typical for high-resolution seismic studies (boulder clay with 0.5-1 m boulders, $V_P \approx 2000$ m/s, $f \approx 500$ Hz

Quality Factor, *Q*

- Attenuation is measured in terms of *quality factor*, *Q:*
  - The logarithmic decrement of amplitude α is generally proportional to frequency

$$A(t) = A(0)e^{-\alpha x} = A(0)e^{\frac{-\pi f t}{Q}}$$

$\alpha \propto f$

$x = Vt$

Therefore, Q here is approximately frequency-independent

  - Amplitude and energy loss per cycle (wavelength):

$$\ln\left(\frac{A(t+T)}{A(t)}\right) = \frac{-\pi fT}{Q} = \frac{-\pi}{Q}$$

This value, in *dB*, is also often used to characterize attenuation

$$\ln\left(\frac{E(t+T)}{E(t)}\right) = \ln\left(\frac{E(t) - \delta E}{E(t)}\right) = \frac{-\delta E}{E(t)} = \frac{-2\pi}{Q}$$

  - Thus, *Q* measures relative energy loss per cycle:

$$Q = 2\pi \frac{E}{\delta E}$$

- Typical values:
  - $Q \approx 30$ for weathered sedimentary rocks;
  - $Q \approx 1000$ for granite.

Q_P and Q_S

- *P*- and *S*-waves have different *Q*'s
- Q_P and Q_S are thought to be related to the quality factors associated with the *K* and μ moduli of the medium:

$$Q_P^{-1} = L\,Q_\mu^{-1} + (1-L)\,Q_\kappa^{-1}$$

Shear attenuation

Bulk attenuation

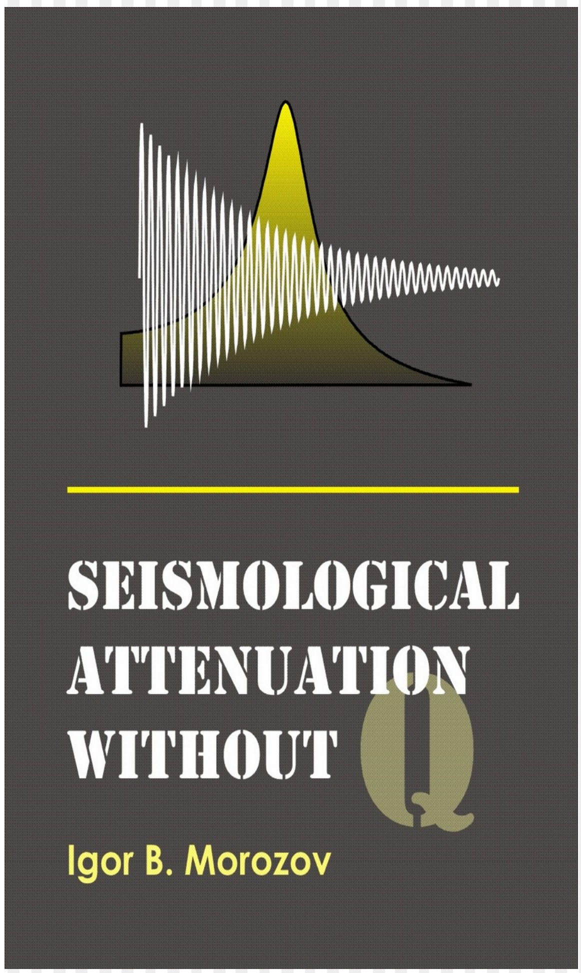
$$Q_S^{-1} = Q_\mu^{-1}$$

where: $$L = \frac{4}{3}\left(\frac{V_S}{V_P}\right)^2$$

- Q_κ is usually very high (assumed infinite)
- Because $\frac{V_S}{V_P} \approx \frac{1}{\sqrt{3}} \dots \frac{1}{2}$,

  typically: $$Q_P^{-1} \approx \left(\frac{1}{3} \dots \frac{1}{2}\right) Q_S^{-1}$$

This is not that simple though...



- *Q* may not really be a true medium property

Typical values of Q_P

Table 6.1 *Absorption constants for rocks*

	Q	δ (dB) = $\eta\lambda$
Sedimentary rocks	20–200	0.16–0.02
Sandstone	70–130	0.04–0.02
Shale	20–70	0.16–0.05
Limestone	50–200	0.06–0.02
Chalk	135	0.02
Dolomite	190	0.02
Rocks with gas in pore space	5–50	0.63–0.06
Metamorphic rocks	200–400	0.02–0.01
Igneous rocks	75–300	0.04–0.01

- For sandstones with porosity ϕ % and clay content C %, at 1 MHz and 40 MPa:

$$Q_P = 179C^{-0.84\phi}$$

General model for *Q* measurement

- The following model of seismic amplitudes is commonly used in attenuation measurements:

$$A_{recorded}(t,f) = A_{Source}(f)\,G(t)\,A_{Receiver}(f)\exp\left(\frac{-\pi ft}{Q}\right)$$

Source spectrum

Geometrical spreading

Receiver response

Attenuation

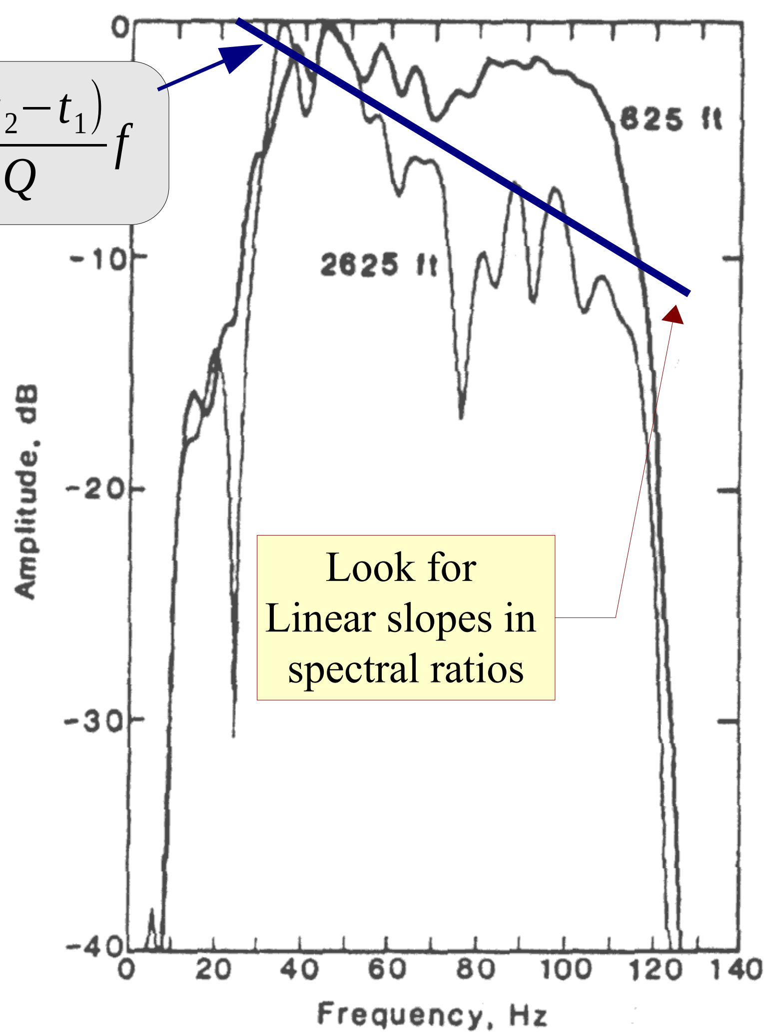
- Therefore, two basic approaches to measurement:
  1) Model-based correction for geometrical spreading $G(t)$ and $A_{source}(f)$
  2) Using ratios of spectral amplitudes

Spectral ratios

- Take *spectral ratios* of seismic spectra measured at two propagation times
  - The signal in the two windows must be the same in all other respects.

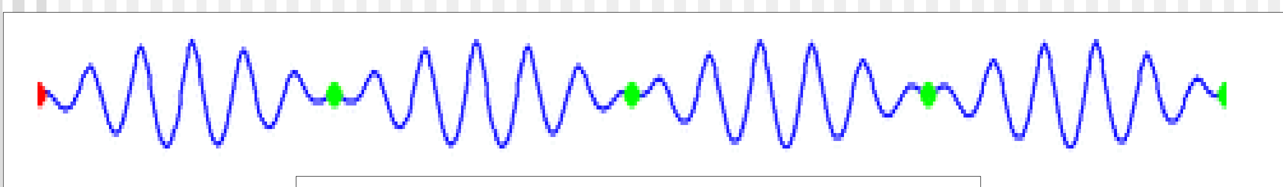
$$\ln\left(\frac{A(f,t_2)}{A(f,t_1)}\right)=\ln\frac{G(t_2)}{G(t_1)}-\frac{\pi(t_2-t_1)}{Q}f$$

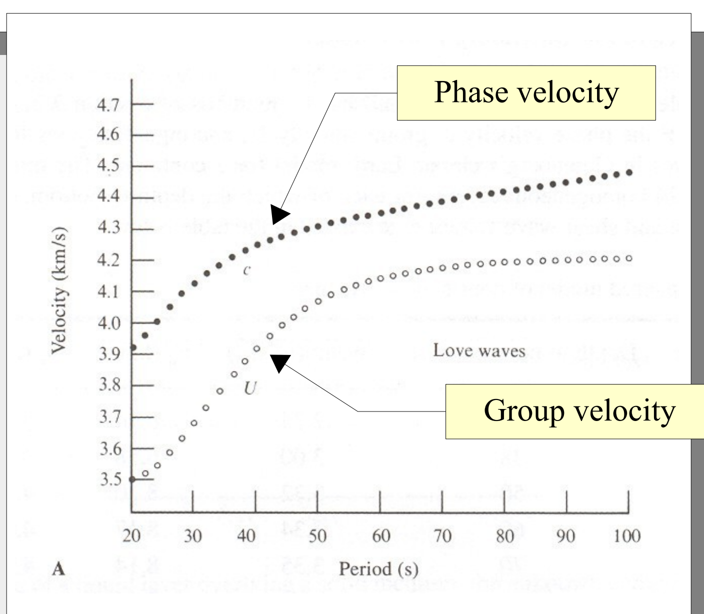
Ratio of spectral amplitudes does not depend on the source spectrum.



Phase-Velocity dispersion

- When phase velocity is dependent on frequency, the wave is called *dispersive:*
  - Wavelet changing shape and spreading out when traveling
  - *Group velocity* (velocity of wave packet, *U*) is different from phase velocity (*V*):
    - U < V – "*Normal dispersion*";
    - U > V – "*Inverse dispersion*".





Group and phase velocities

- Consider a plane harmonic wave:

$$u(x,t)=Ae^{i\varphi(x,t)}=Ae^{i[k(\omega)x-\omega t]}$$

where $k=\omega/V$ is the *wavenumber*.

  - Note that k is dependent on ω.

- *Phase velocity* is the velocity of propagation of the constant-phase plane ($\varphi(x,t)$=const):

$$\boxed{V_{phase}=\frac{\omega}{k}}$$

- *Group velocity* is the velocity of propagation of the amplitude peak in the wavelet
  - this is the point where the phase is *stationary* (independent on ω):

$$\frac{d[k(\omega)x-\omega t]}{d\omega}=\frac{dk(\omega)}{d\omega}x-t=0$$

  - hence: $$\boxed{U_{group}=\left[\frac{dk}{d\omega}\right]^{-1}=\frac{d\omega}{dk}}$$

Group velocity

- Example: two cosine waves with

$$\omega_1=\omega_0-\Delta\omega,\, k_1=k_0-\Delta k$$
$$\omega_2=\omega_0+\Delta\omega,\, k_2=k_0+\Delta k$$

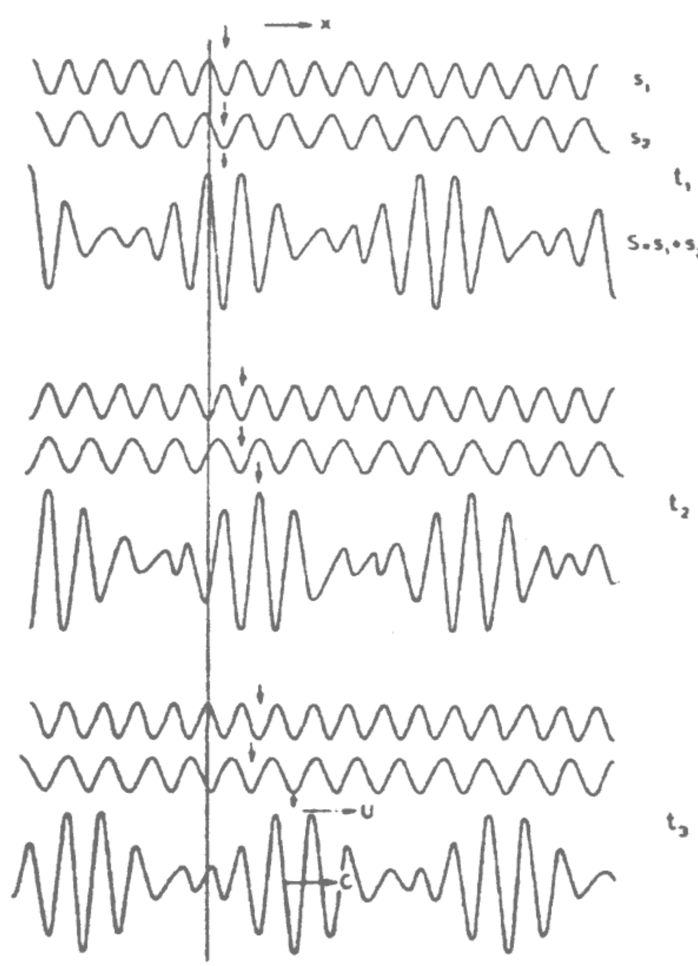
superimpose to form beats:

Show that the envelope of these beats travels with group velocity:

$$U=\frac{\Delta\omega}{\Delta k}.$$

...while within the beats, peaks and troughs propagate at approximately:

$$V=\frac{\omega}{k}.$$



Normal and Inverse dispersion

- When phase velocity is frequency-dependent, group velocity differs from it:

$$U=\frac{d\omega}{dk}=\frac{d(kV)}{dk}=V+k\frac{dV}{dk}=V-\lambda\frac{dV}{d\lambda}\approx V+\omega\frac{dV}{d\omega}.$$

because $k = 2\pi/\lambda = \omega/V$.

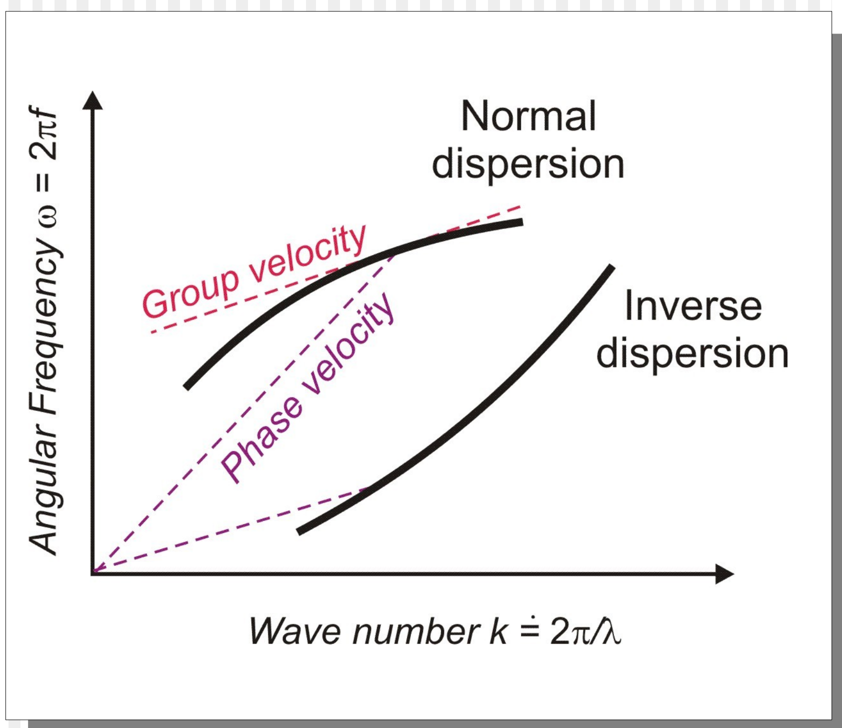
- Therefore:

$$\frac{dV}{d\omega}<0.$$

Normal dispersion
(typically observed in ground roll)

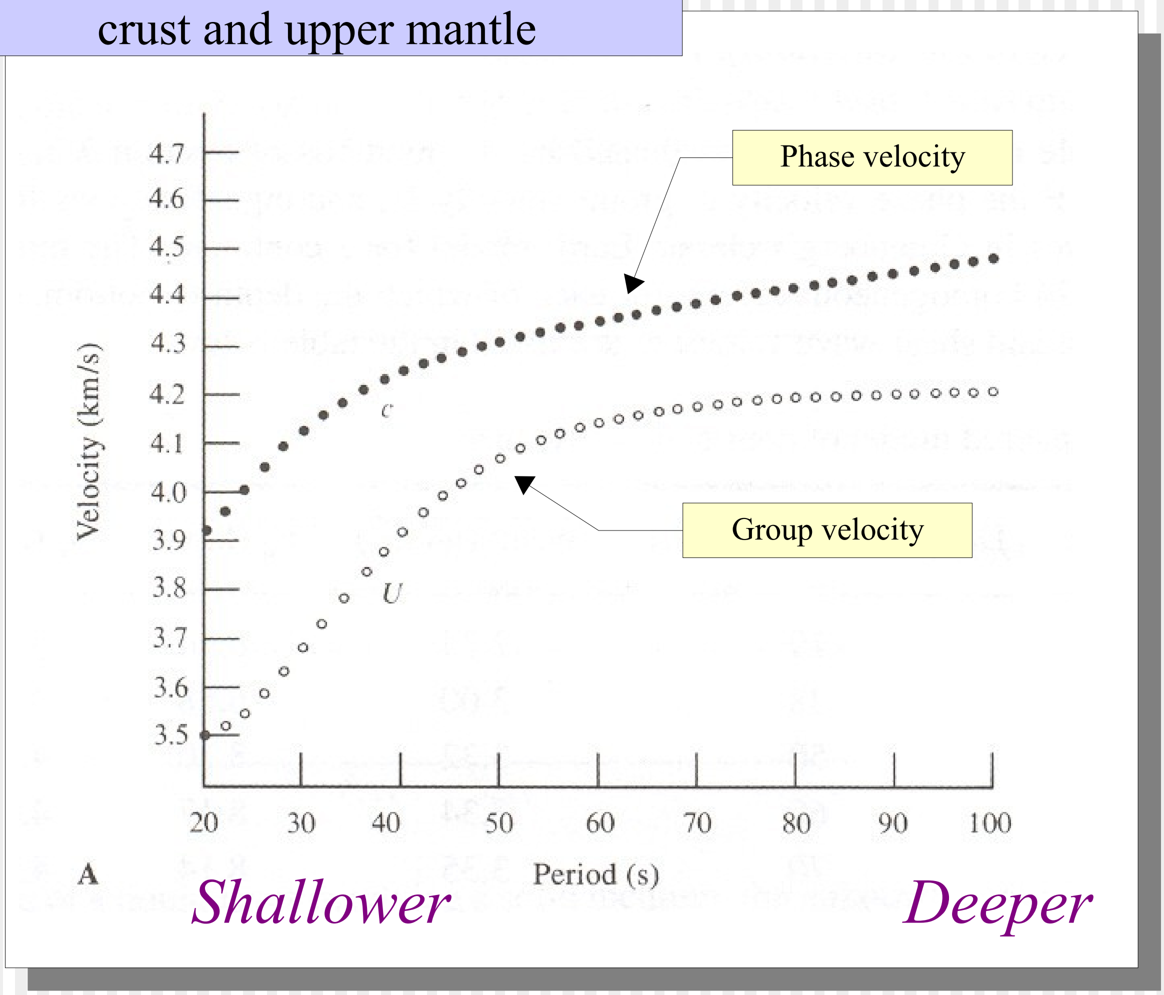
$$\frac{dV}{d\omega}>0.$$

Inverse dispersion



Example:
normal dispersion of surface waves

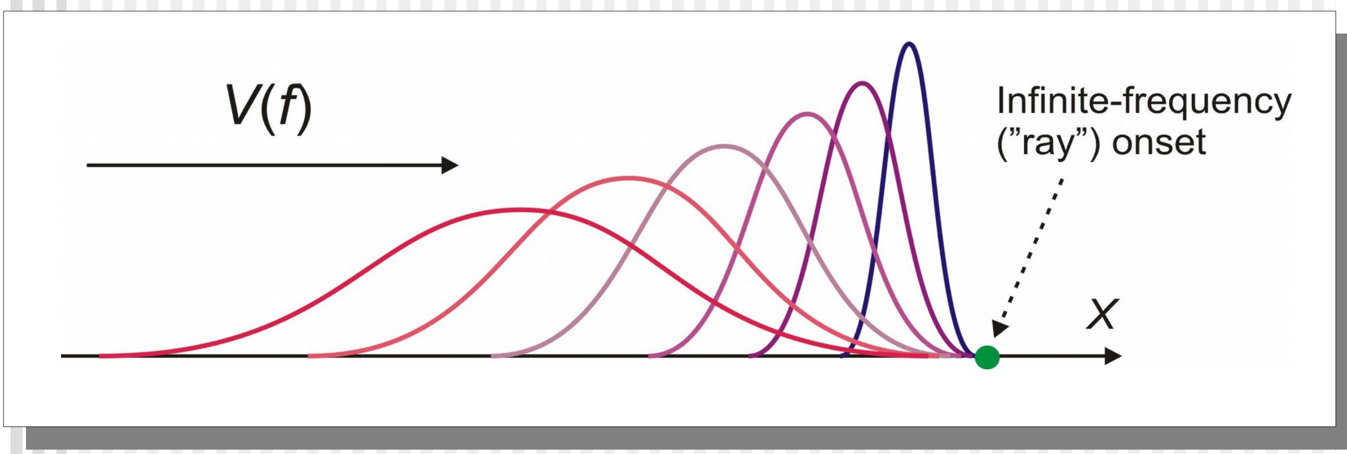
Long-period Love waves within the crust and upper mantle



- Normal dispersion occurs because the deeper layers are generally faster

Attenuation and Dispersion

- Attenuating medium is **always** dispersive
  - Example: ground roll is quickly attenuated and shows strong normal dispersion.
- *Causality* requires that lower-frequency wave components travel slower (*i.e.*, *inverse dispersion*):



- Mathematically, this is expressed by the so-called "Kramers-Krönig relations"
- For example, in a constant-Q medium,

$$c(\omega)=c(\omega_0)\left[1+\frac{1}{\pi Q}\ln\frac{\omega}{\omega_0}\right]$$

Inverse dispersion