Geol 483.3

Lab project #3 – Seismic source location

You will study grid search and iterative methods to solve a 2D seismic (earthquake) location problem. .

You are given a 13-station array that recorded the following arrival times from two earthquakes:

Station	x [km]	y [km]	t1 [s]	t2 [s]
1	9.0	24.0	14.189	20.950
2	24.0	13.2	13.679	21.718
3	33.0	4.8	13.491	21.467
4	45.0	10.8	14.406	21.713
5	39.0	27.0	13.075	20.034
6	54.0	30.0	15.234	20.153
7	15.0	39.0	13.270	18.188
8	36.0	42.0	12.239	16.008
9	27.0	48.0	12.835	15.197
10	48.0	48.0	14.574	16.280
11	15.0	42.0	12.624	16.907
12	18.0	15.0	13.496	21.312
13	30.0	36.0	10.578	16.664

This table is also available in Excel format from the lab web page.

Theory

Before staring with the exercise, please read section 5.7 in Shearer's text.

Location

Location is based on minimization of the misfit between the observed travel times and those predicted from the model. In a 2D case (only the epicenter coordinates (x,y) are unknown and assuming the hypocenter depth to be zero), the predicted times are:

$$t_i = t_S + \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{V},$$

where (x_i,y_i) are coordinates of the i-th station, t_S is the time of the source, and V is the velocity. The total travel-time misfit, measured using the L_2 (also often called the RMS, Root Mean Square) norm is:

$$\Phi(x, y \mid t_S, V) = \sum_i \left(t_i - t_i^{observed}\right)^2 = \sum_i \left(t_S + \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{V} - t_i^{observed} \right)^2. \qquad (1)$$

The best location (x,y) is the one minimizing the function above. For simple problems like the one below, this optimum location can be found by computing function (1) at all grid points, contouring it, and finding the minimum.

Algorithm

Values for V and t_S are typically not included in grid search. In order to estimate them, make several steps of the following iteration:

1) Make an initial guess about the time t_S and V;
2) Perform grid search for (x,y) using formula (1);
3) Calculate the average time residual: $\delta t = \frac{1}{N}\sum_{i=1}^{N}\left(t_i^{observed} - t_i\right)$;
4) Add the residual to t_S, this is your updated location time;
5) Plot all the observed times ($t_i^{observed}$-t_S) versus the source-receiver distances in one plot. Check the slope and see if you can improve your V;
6) Repeat steps (1-5).

Confidence ellipse

There always is some error in the data, and so you will not be able to achieve zero misfit. From the residual time misfits, estimate the data variance:

$$\sigma^2 = \frac{\sum_{i=1}^{N}\left(t_i^{observed} - t_i\right)^2}{n.d.f.} \qquad (2)$$

where n.d.f.=(N-2) is the “number of degrees of freedom” (the number` of data minus the number of adjusted model parameters; see eq. 5.30 in Shearer). Square root of this value (σ) represents an estimate of data (travel time) error.

To measure the associate uncertainty of location, compute and contour the following function of location (x,y):

$$\chi^2(x, y) = \frac{\sum_{i=1}^{N}\left(t_i^{observed} - t_i\right)^2}{\sigma^2.}.$$

His function represent the normalized travel-time error associated with a location selected at (x,y). Values of χ^2 are tabulated (see table 5.1 in Shearer) and can be used to deerine the probability that the trave-time error for location at (x,y) is still caused by random data errors. To determine the ***confidence ellipse***, you will need to find the contour in $\chi^2(x,y)$ corresponding to 95% percentage points of the distribution (that is, only 5% probability of random error).

Assignments

[20%] Write a Matlab program to perform grid search to find the best location for each earthquake. Try every point in a 100-km by 100 km box, at 1-km increments (0 to 100 km in both x and y). At each point, assume a source at depth equal 0 and P-wave velocity of 6 km/s.

Note that the travel times t_1 ns t_2 in the table above assume the origin times to be *approximately* 0, which is, however, not exactly true. The task of your inversion is to also estimate the origin times. For each earhquake, just assume the origin times to be 0 to begin with, then average the residuals from all stations. This average is your best estimate of the origin time.

For each earthquake:

1. [20%] Find the best location and origin time by making several iterations 1)-6) described above;
2. Find the variance (standard deviation) of the residuals at the best-fitting point using eq. (2). This is your estimate of the overall data uncertainty.
3. [10%] Compute χ^2 for each grid point using the expression above. What is the χ^2 for the best-fitting point?
4. Make a contour plot of χ^2(x,y). Identify those values that are within the 95% confidence range.
5. [5%] Make a plot showing the station locations, the best location, and the range within the 95% confidence region.
6. [5%] Comment on the shapes of the confidence regions, similarities and differences between the two earthquakes.

Hand in:

Codes, plots, and report by uploading to seisweb.