Time and Moveout Filtering

- Frequency filtering
- Wavelet shaping (deconvolution)
- Dip and Moveout (2-D) filtering
  - *f-k* (frequency-wavenumber)
  - τ-*p* (slant stack)

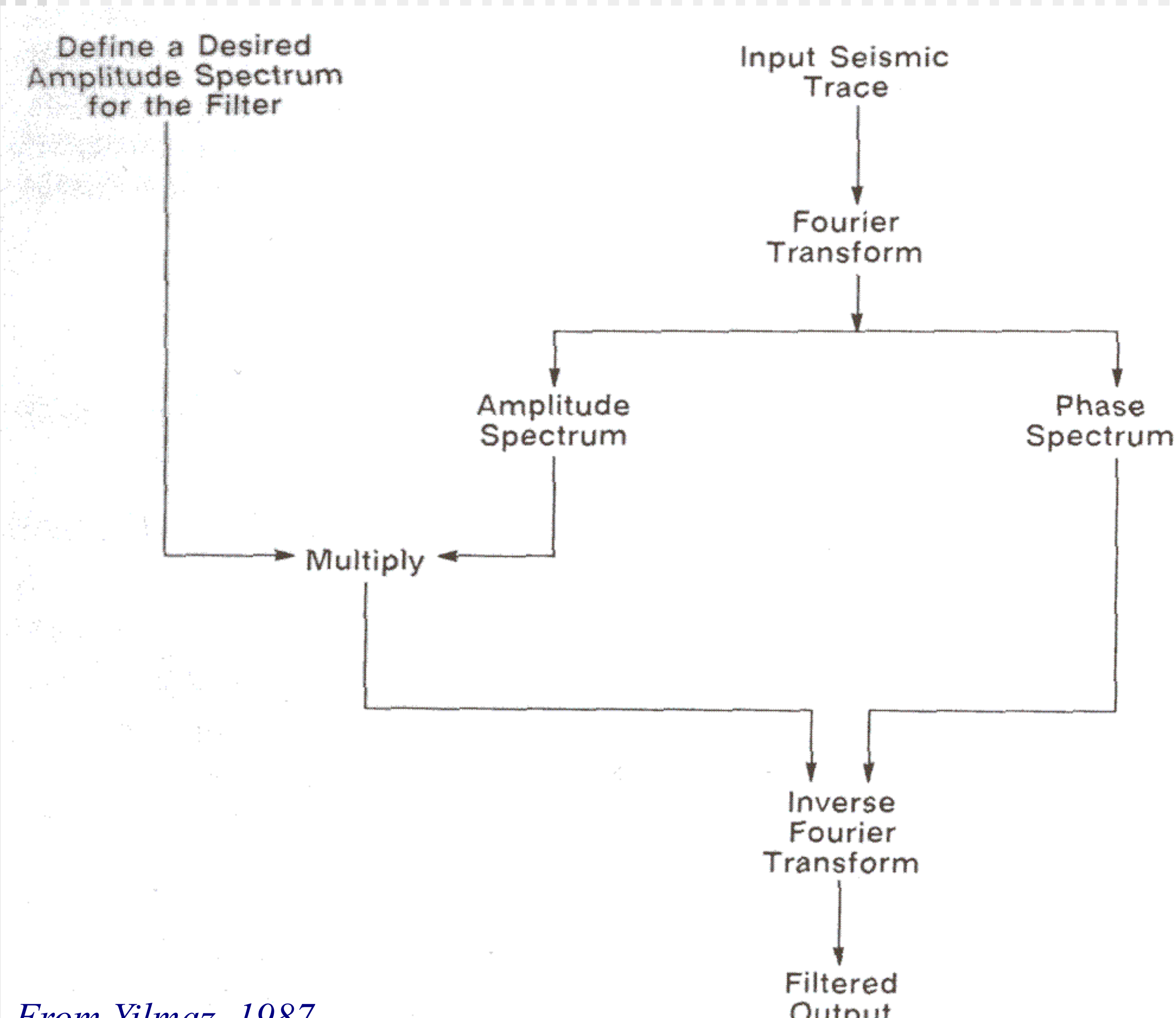
- Reading:
  - Sheriff and Geldart, Sections 9.5, 9.9, 9.11

Single-channel Filtering
Objectives

- Performed in order to increase the Signal/Noise ratio or to improve signal shape:
  - Modify the frequency band
  - Flatten ("whiten") the spectrum
  - Convert the wavelet into minimum- or zero-phase (*wavelet shaping*)
    - Minimum-phase wavelet is causal;
    - Zero-phase is better for display and interpretation
  - Normalize the effects of different sensors by bringing them to a common response (*matching filters*)
  - Remove reverberations (*deconvolution*)
- The Filter is always a time series *convolved* with the signal
  - This can always be done in *time* or *frequency* domain

Frequency filtering
Frequency-domain

- Most common
- Zero phase filter in order to preserve phase character



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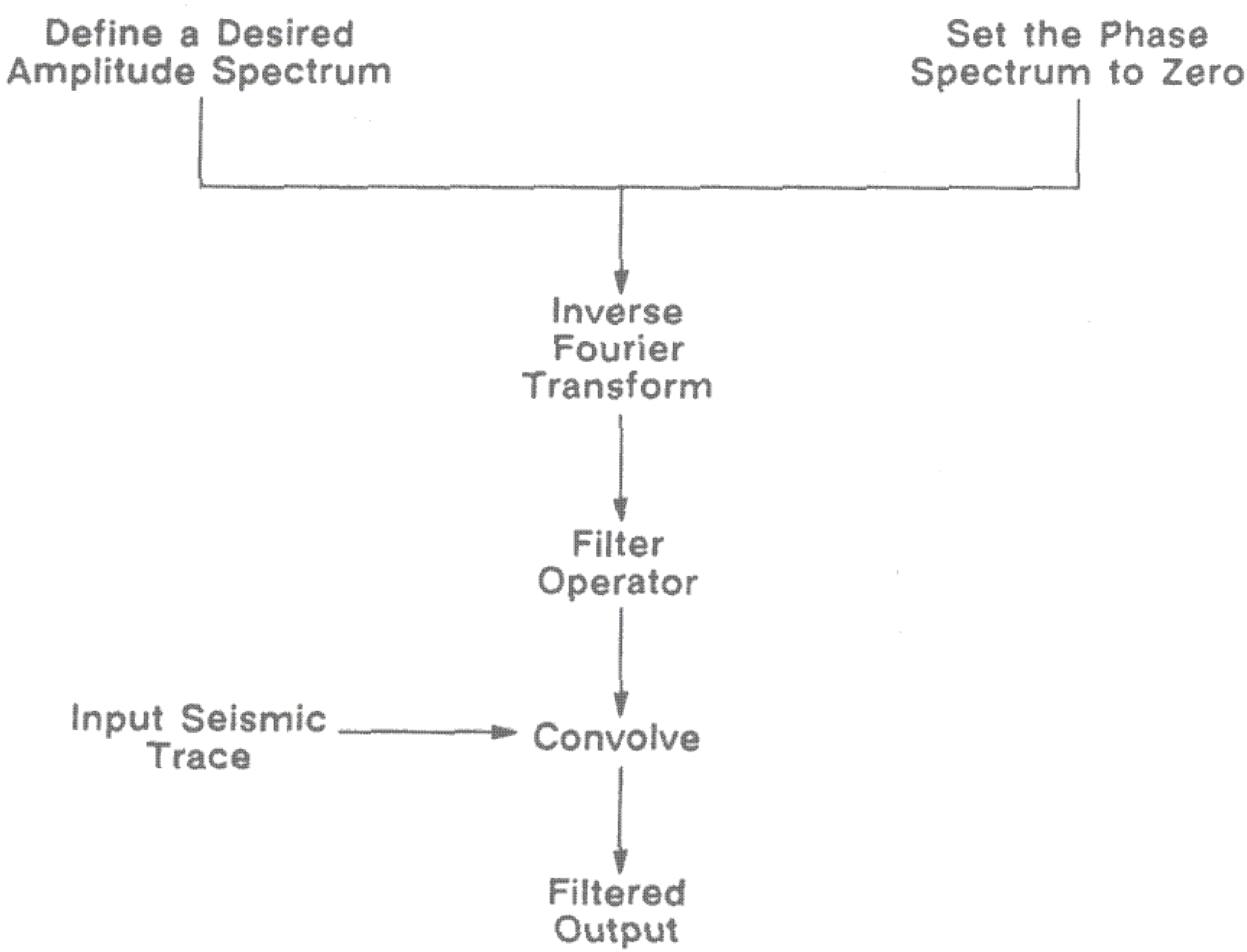
Frequency filtering

Time-domain

- This is used only for broad-band (short in time) filters when time-domain convolution is more efficient then forward and inverse FFT



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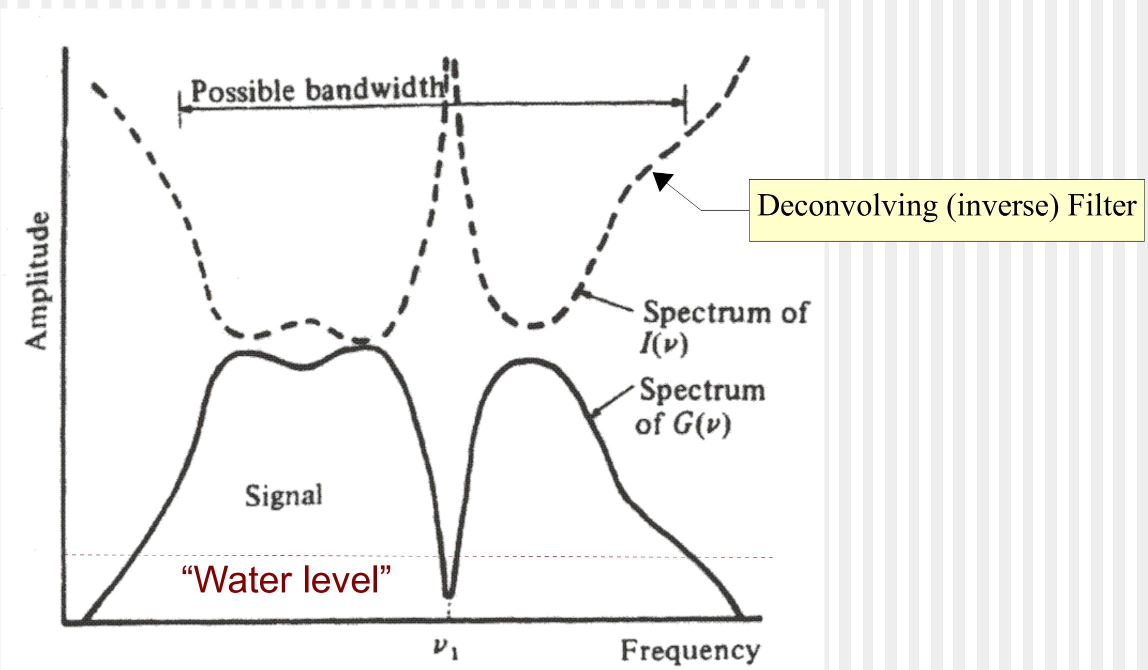
Deconvolution

- Time domain:
  - Changing the shape of the signal to some “desired” waveform
    - Spiking (to a spike)
    - Shaping (to a band-limited pulse).
  - Removal of short-period multiples
    - Prediction-error deconvolution.
- Frequency domain:
  - Flattening the spectrum
    - Spectral broadening, more spiky signal
    - Time-Variant Spectral Whitening (compensates attenuation)
  - Transformation to a zero-phase (symmetric) wavelet.

Deconvolution
Spectral whitening

- Frequency-domain
- The zero-phase inverse filter is constructed of the inverse of signal amplitude. “Spectral holes” corrected by adding 1-2% “pre-whitening” or “water level”



$$A_{inverse}(f)=\frac{1}{A(f)+A_{prewhitening}}$$

$$A_{inverse}(f)=\frac{1}{max\{A(f),A_{water\ level}\}}$$

Deconvolution
Wiener (least squares)

- Time-domain
- Changes the shape of the signal into some "desired" waveform:

$$u_i^{desired} = \sum_k f_k u_{i-k}$$

- This is solved for f_k by using the Least-Squares method:

$$\sum \left(u_i^{desired} - \sum_k f_k u_{i-k} \right)^2 \rightarrow min$$

- Gives rise to a broad group of techniques:
  - *e.g.*, for $u^{desired}$ being a spike, delayed spike, or a specified shape, this gives *spiking*, *optimal*, or *shaping* deconvolution

Deconvolution

Prediction-error (or "predictive")

- Time-domain
- Constructs a filter predicting the wavelet from its preceding values:

$$w_i = \sum_k f_k w_{i-k}$$

- Then, "prediction-error" filter:

$$f_k^{PE} = \delta_{k,0} - f_k$$

removes the reverberation from the signal.

- To find the predictive filter f_k, note its action on the auto-correlation of the wavelet ϕ:

$$\Phi_i = \sum_k f_k \Phi_{i-k} \qquad (*)$$

- Wavelet's auto-correlation is approximately equal the total signal auto-correlation (the "white reflectivity" hypothesis)
- From "normal equations" (*), f_k is obtained.

Deconvolution

F-X (predictive in space domain)

- X- or XY-domain
- Operates for each frequency independently
- Note that any linear event...

$$u(x,t)=\delta(a+bx-t)$$

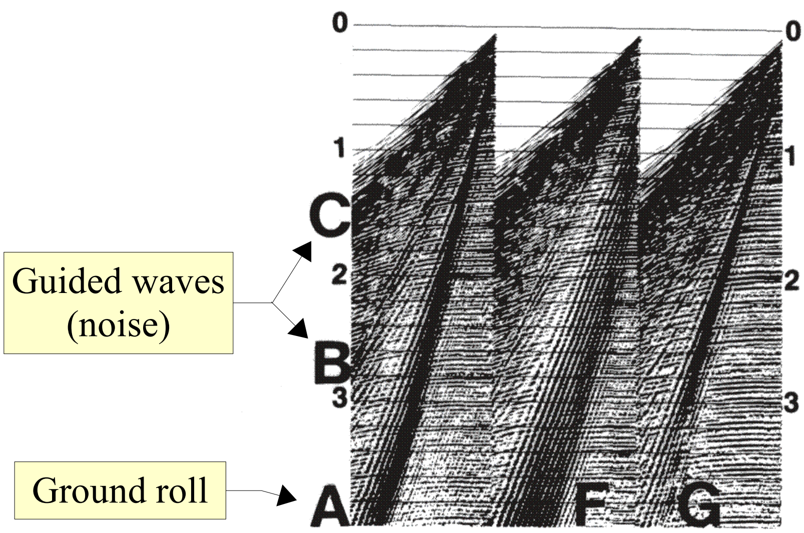
- ...after Fourier transform, becomes periodic in *X*:

$$u(x,\omega)=e^{i\omega a}e^{i\omega bx}$$

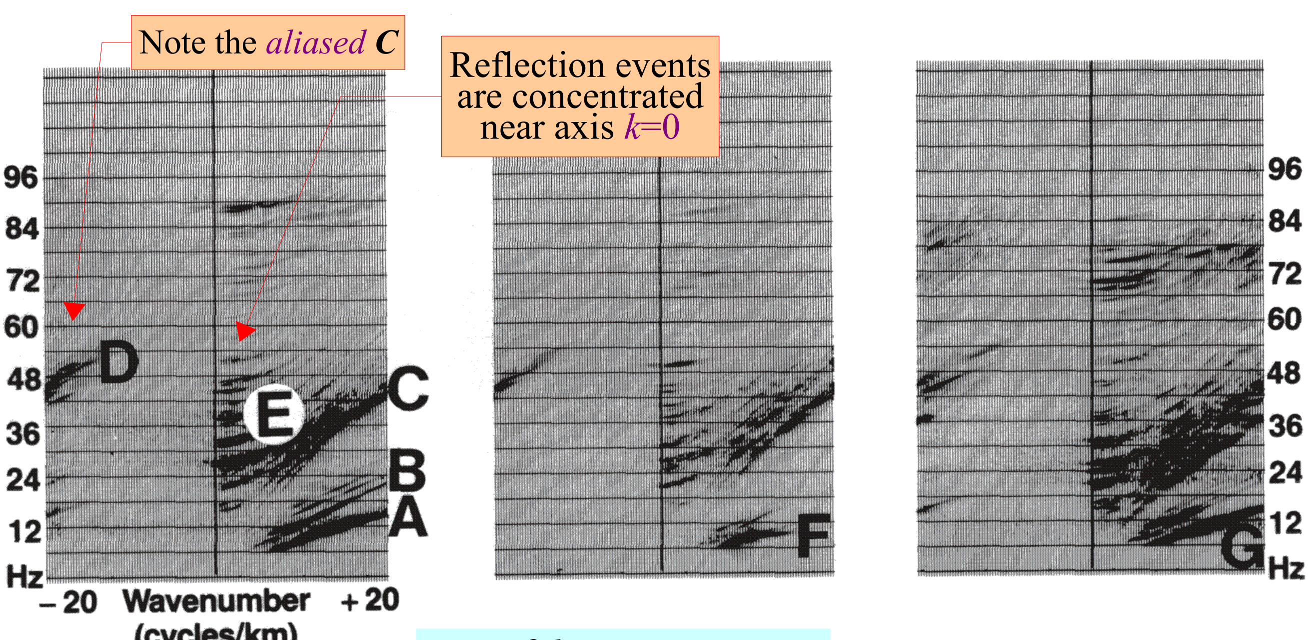
- Such periodic events can be enhanced by a predictive filter in X.
- Application:
  - Partition the data into windows small enough for the events to appear linear;
  - Fourier transform each window;
  - Calculate two prediction filters: one forward and one backward in X;
  - Sum the two predictions and transform back into the time domain.

F-K spectra (shot gathers)

- By performing Fourier Transform in both time and space, the *f-k spectra* are obtained
- The physical significance is in decomposition of the wavefield into *harmonic plane waves*



Shot gathers



f-k spectra
of the same gathers

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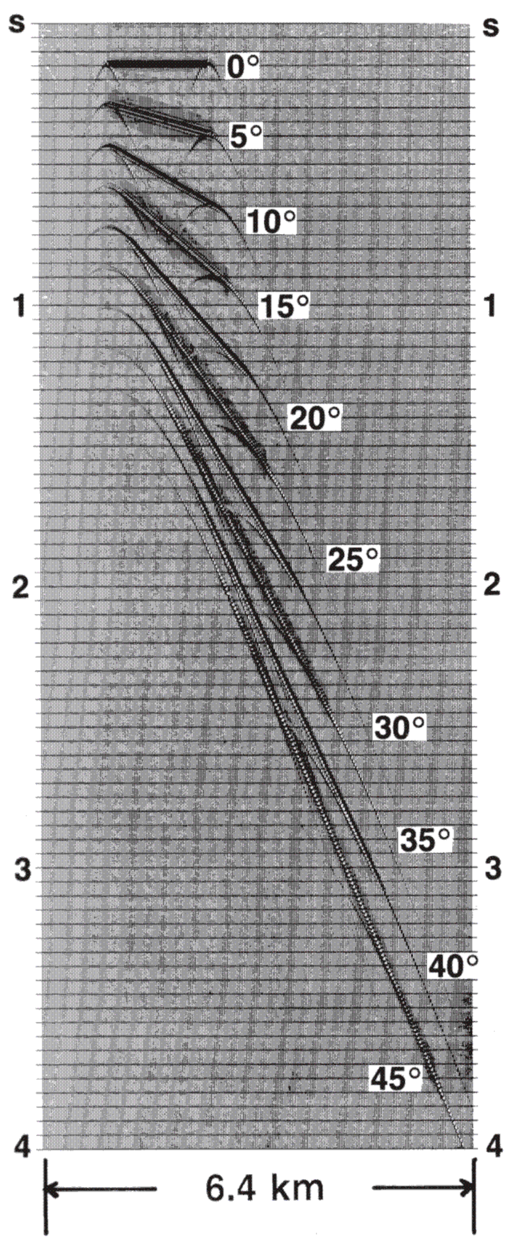
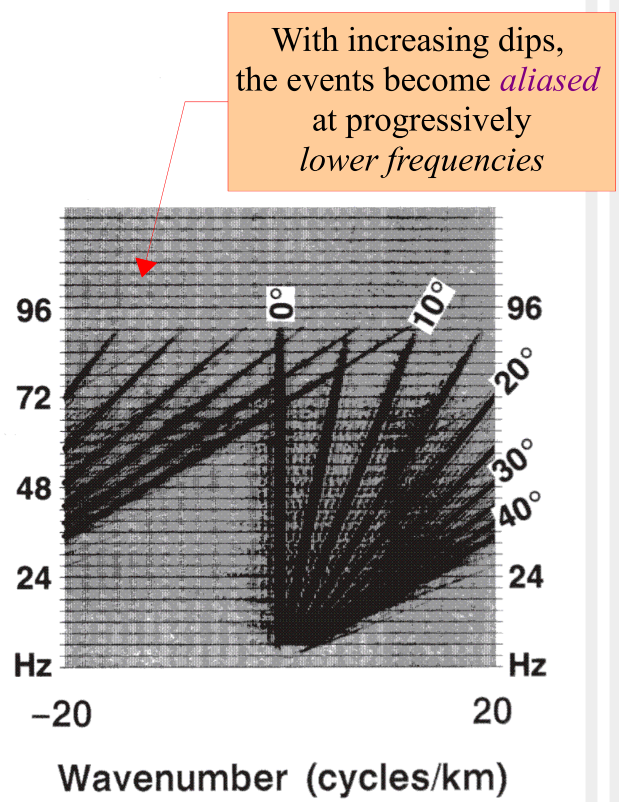
F-K spectra (dipping events in a zero-offset section)

- Events with different (apparent) dips occupy different parts of the *f-k* spectrum, regardless of their positions in time or space





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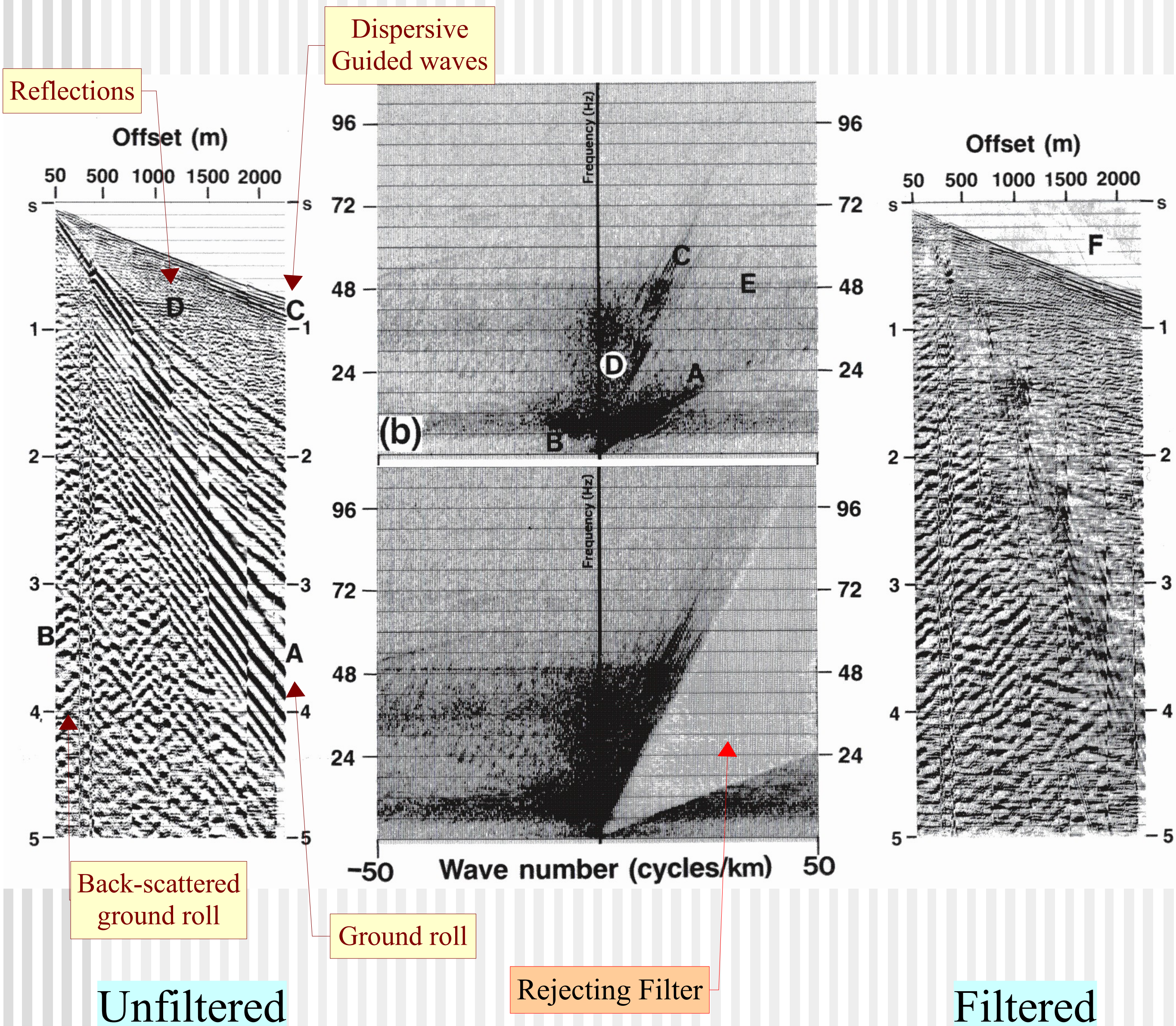
F-K filtering

- Here, only forward-propagating ground roll is rejected by the filter.



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Plane-wave decomposition
t-p transform

- Instead of *f-k* transform, plane waves can be extracted from the section by *slant-stacking:*

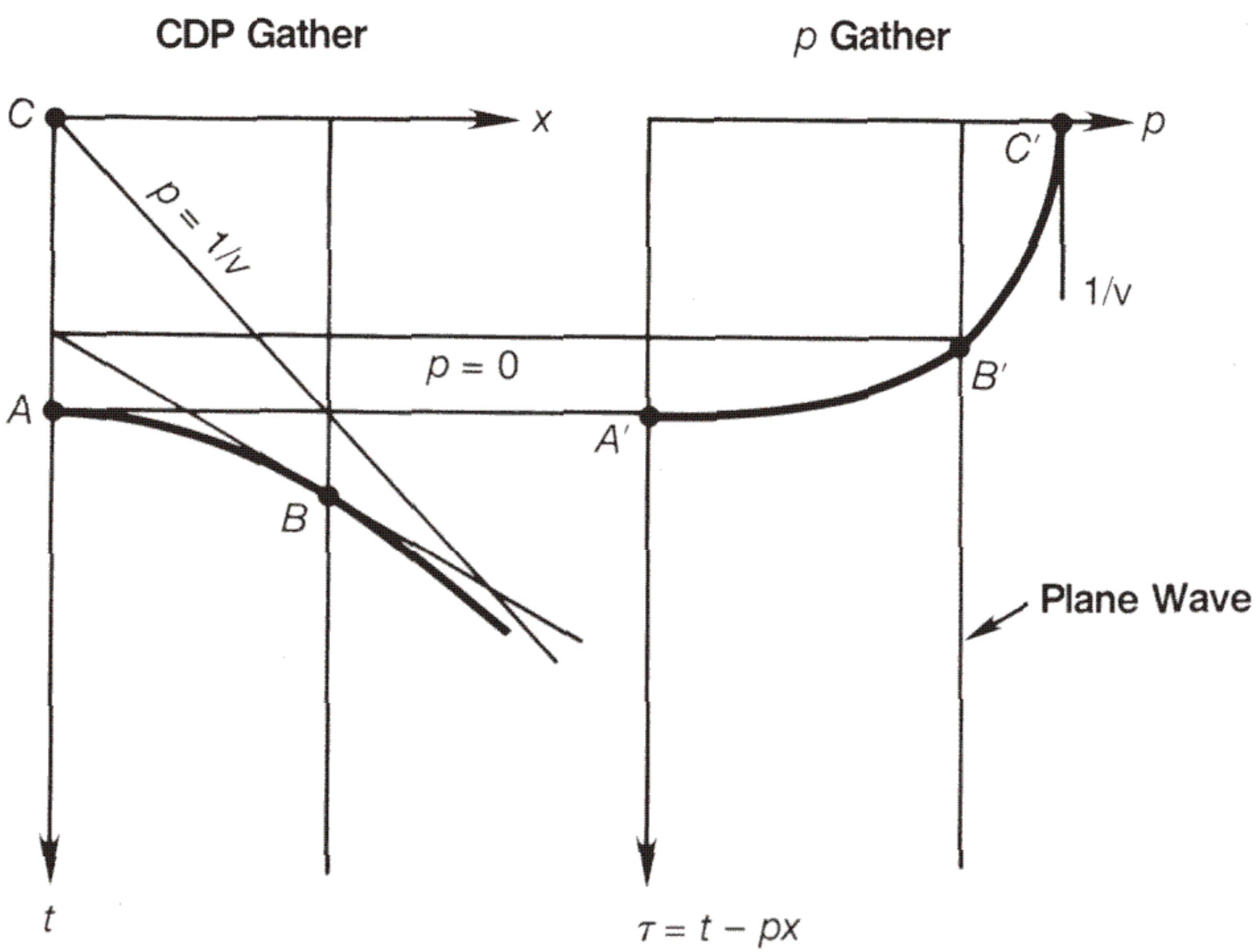
$$S(p,\tau)=\sum_{x} u(x,\tau+px)$$

$t=\tau+px$
describes the wavefront of a plane wave

- This is done for every τ (intercept time) and *p* (slowness), resulting in a (τ,p) section
- The difference from *f-k* is in using plane waves *localized in time* (pulses instead of harmonic functions),
  - ...and therefore filtering can be based on *moveouts* AND *times* of the events.

Refractions and reflections in τ-p domain

- Reflections (straight lines in (x,t) become points,
- ...and reflections (hyperbolas in (x,t)) - ellipses



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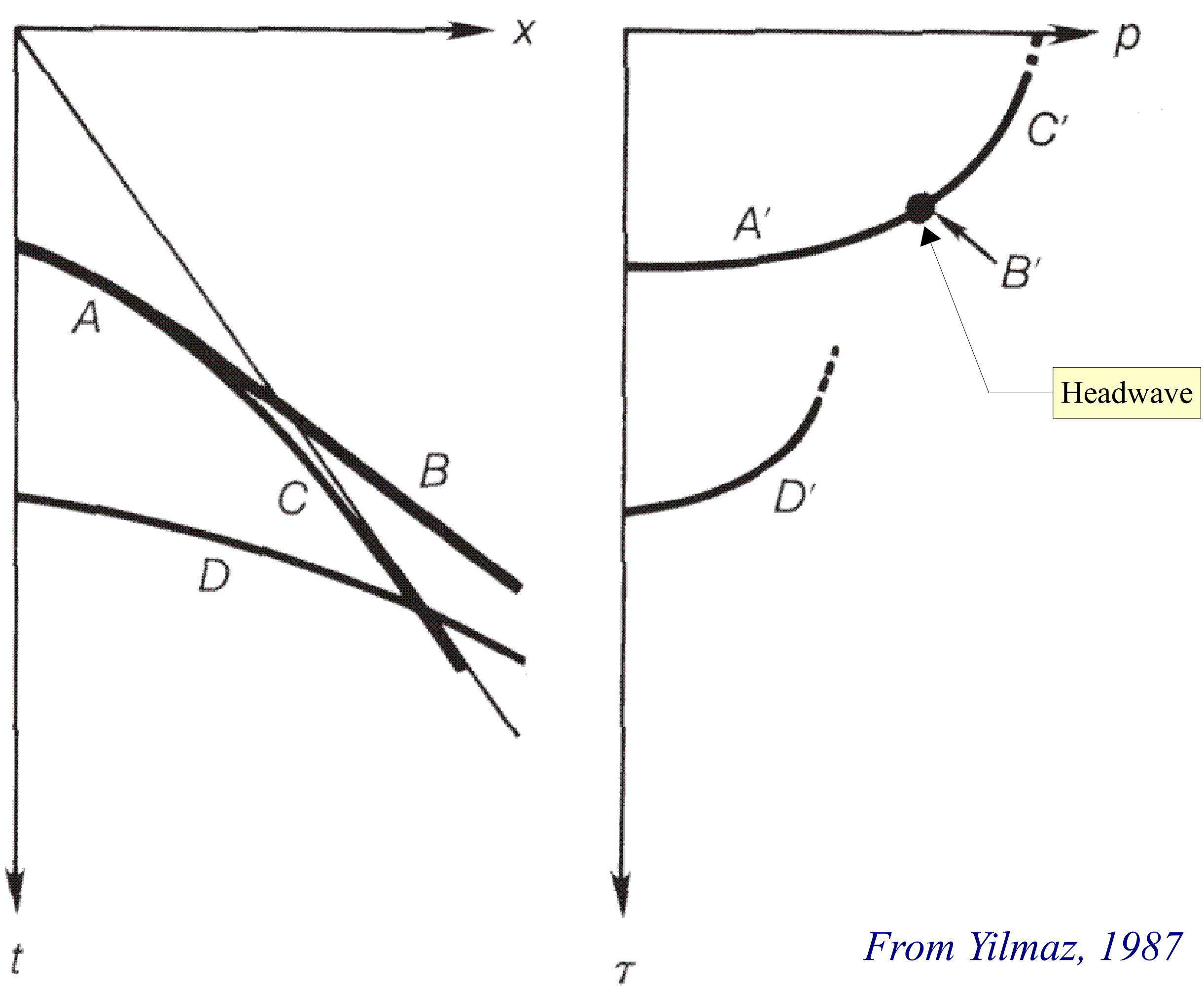
Several reflections in τ-p domain

- Reflections can be separated by their intercept times
- Phases retain their waveforms – this simplifies interpretation and facilitates waveform shaping (e.g., deconvolution)



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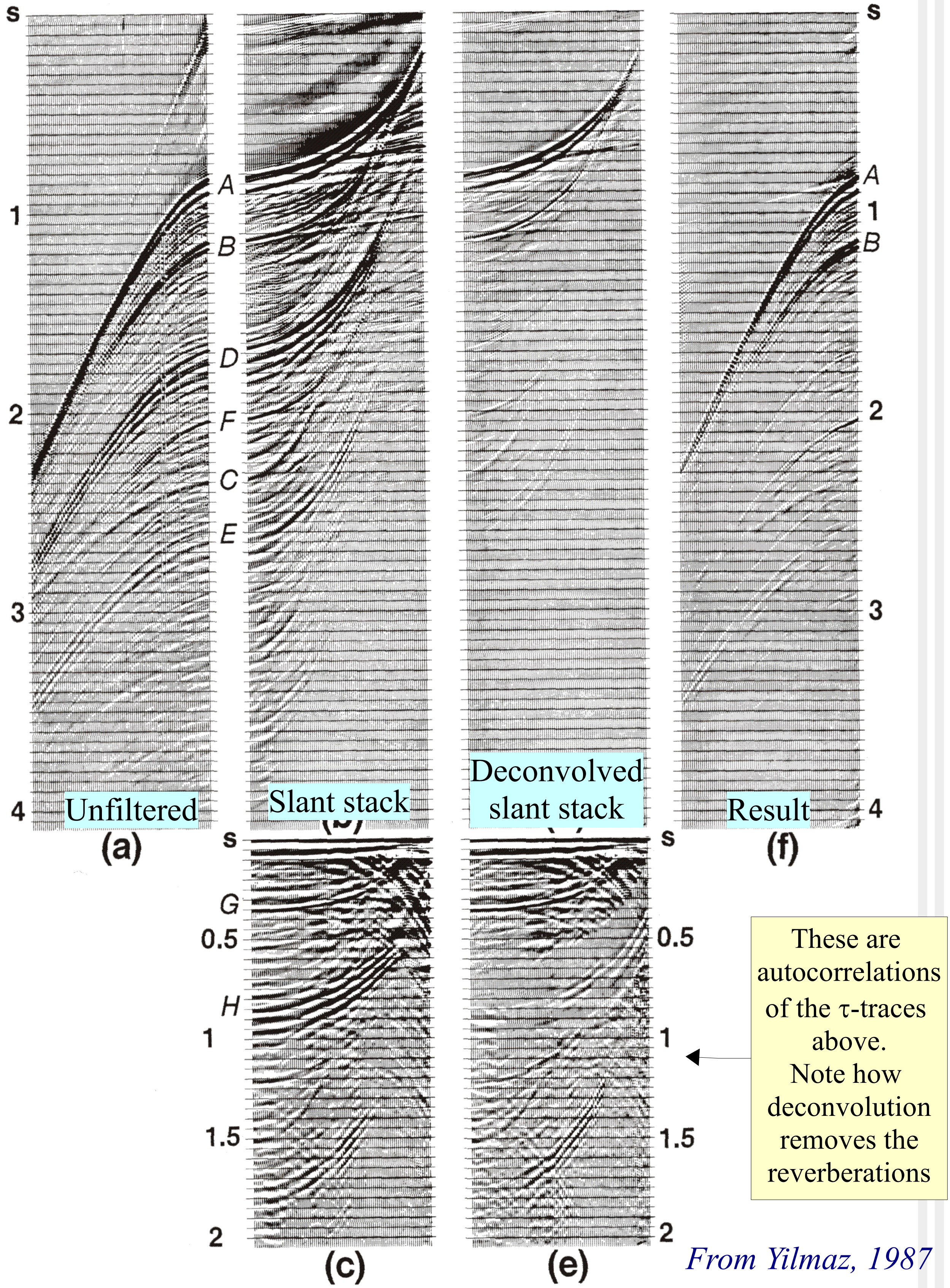
Multiple suppression using τ-p



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