

Surface waves

- Rayleigh and Love waves
 - Particle motion
 - Phase and group velocity
 - Dispersion
-
- Reading:
 - › Shearer, chapter 8
 - › Telford *et al.*, 4.2.4, 4.2.6

Mechanism

- Surface waves are always associated with a boundary.
- The (*e.g.*, horizontal) boundary disrupts vertical wave propagation *but* provides for special wave modes propagating along it.
- Because there are 2 or 4 boundary conditions to satisfy (*e.g.*, displacement and stress continuity), surface waves always consist of 2 or 4 interacting wave modes:
 - *P* and *SV* wave modes (*Rayleigh* or *Stoneley* waves);
 - Two *SH* modes (*Love* waves).
- Such waves are “tied” to the surface and exponentially decrease away from it.

Surface wave potentials

- General wave equations for potentials:

$$\nabla^2 \phi = \frac{1}{V_P^2} \frac{\partial^2 \phi}{\partial t^2}, \quad \text{P-wave}$$

$$\nabla^2 \psi_V = \frac{1}{V_S^2} \frac{\partial^2 \psi_V}{\partial t^2}, \quad \text{SV-wave}$$

$$\nabla^2 \psi_H = \frac{1}{V_S^2} \frac{\partial^2 \psi_H}{\partial t^2}. \quad \text{SH-wave}$$

- Surface waves are combinations of solutions with *complex* (e.g., pure imaginary) wavenumbers along z .
 - e.g., for Rayleigh wave:

$$\phi = A e^{-mz} e^{i(kx - \omega t)},$$

$$\psi_V = B e^{-nz} e^{i(kx - \omega t)}.$$

- Question: why are such solutions not allowed without a boundary?

Depth dependence

- To satisfy the wave equations for any k and ω , m and n must equal (*show this*):

$$m = \sqrt{k^2 - \frac{\omega^2}{V_P^2}},$$

P -wave component
in Rayleigh wave

$$n = \sqrt{k^2 - \frac{\omega^2}{V_S^2}}.$$

SV -wave component

- note that therefore, for any surface wave:

$$k > \frac{\omega}{V_S} > \frac{\omega}{V_P},$$

and so $V_{Surface} = \frac{\omega}{k} < V_S.$

- To further describe the solution, we need to:
 - 1) consider ω and A as free variables;
 - 2) determine B and $k(\omega)$ from the boundary conditions.

“Dispersion relation”

Rayleigh waves ("ground roll")

- *Rayleigh waves* propagate along the free surface

- The displacements are as usual:

$$\vec{u}_P(\vec{x}, \vec{z}) = \left(\frac{\partial \phi}{\partial x}, 0, \frac{\partial \phi}{\partial z} \right) \quad \text{P-wave}$$

$$\vec{u}_S(\vec{x}, \vec{z}) = \left(-\frac{\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x} \right) \quad \text{SV-wave}$$

- and traction:

$$\vec{F}_P(\vec{x}, \vec{z}) = \left(2\mu \frac{\partial^2 \phi}{\partial x \partial z}, 0, \lambda \nabla^2 \phi + 2\mu \frac{\partial^2 \phi}{\partial z^2} \right)$$

$$\vec{F}_S(\vec{x}, \vec{z}) = \left(\mu \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right), 0, 2\mu \frac{\partial^2 \psi}{\partial x \partial z} \right)$$

- For a free surface, the boundary conditions read: $\sigma_{xz} = \sigma_{zz} = 0$,

- Solution:

$$\phi = e^{-mz} e^{i(kx - \omega t)},$$

$$\psi_V = B e^{-nz} e^{i(kx - \omega t)}.$$

we can set $A = 1$ and seek B and $k(\omega)$

Rayleigh waves ("ground roll")

- Result (for $\sigma=0.25$): relative P - and S -wave amplitudes:

$$\phi = e^{-0.848kz} e^{i(kx - \omega t)}, \quad \text{P-wave}$$

$$\psi_V = 1.468i e^{-0.393z} e^{i(kx - \omega t)}. \quad \text{SV-wave}$$

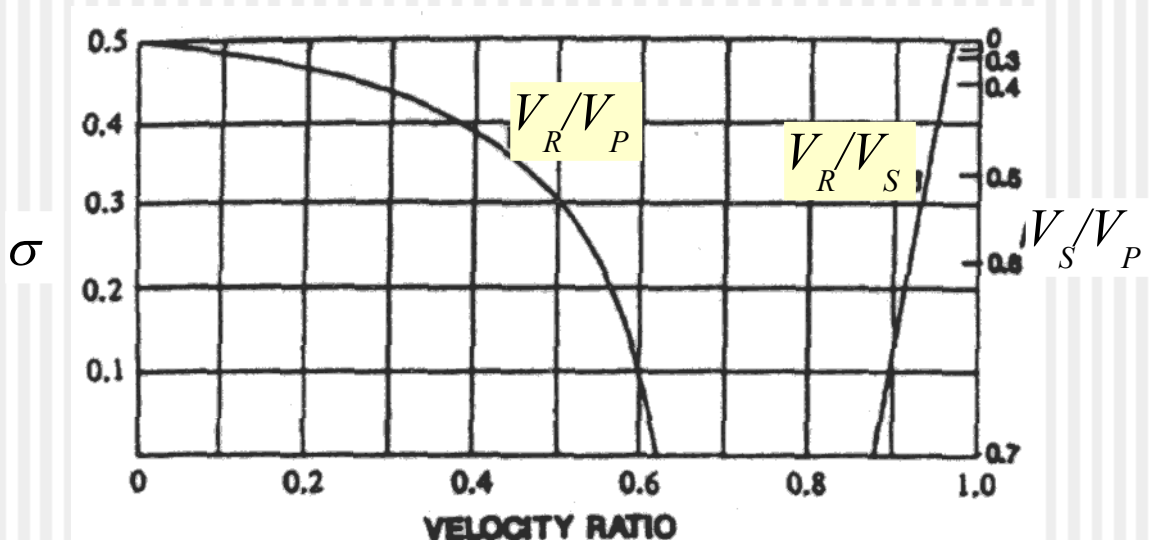
- ...and dispersion relation:

$$k = V_R \omega. \quad \text{(This means no dispersion!)}$$

- Rayleigh wave velocity:

$$V_R = 0.919 V_S$$

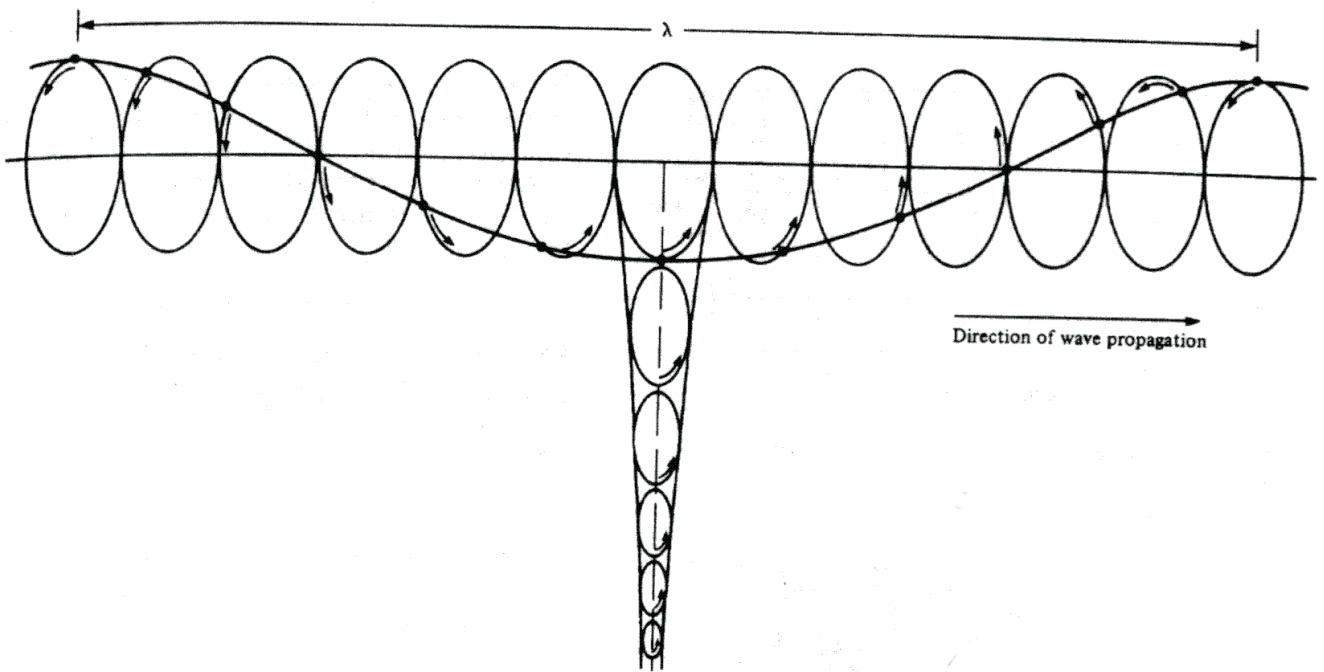
- For varying σ :



Rayleigh waves ("ground roll")

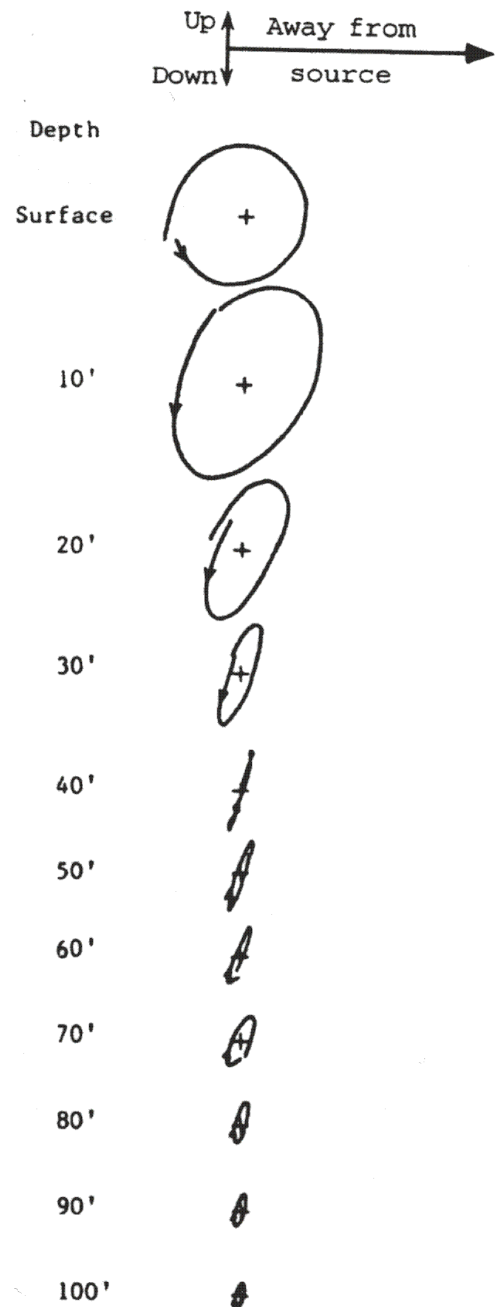
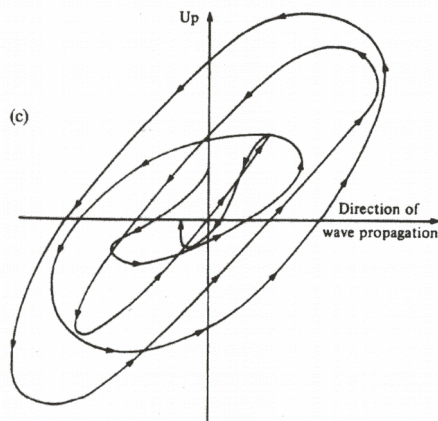
How does it follow from the equations for potentials and displacements?

- Particle motion is elliptical and *retrograde* (counter-clockwise when the wave is moving left to right):



Rayleigh waves ("ground roll")

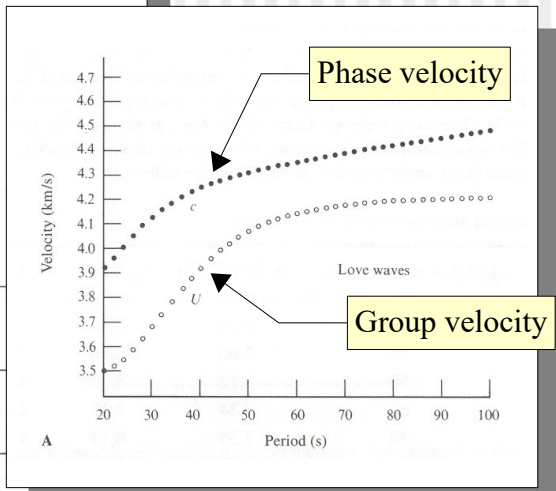
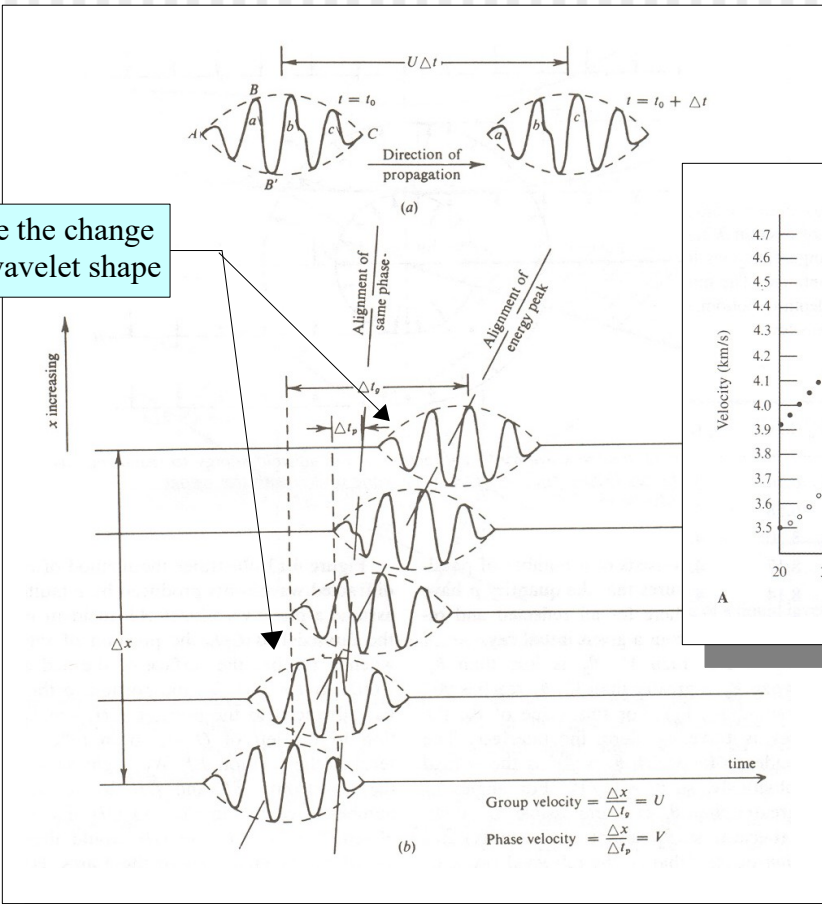
- Real Earth is never a *uniform* half-space, and thus in Rayleigh waves:
 - Particle motion paths are tilted and complex;
 - Retrograde motion may change into prograde at some depth;
 - Normal dispersion *is* present.



Rayleigh-wave dispersion

- Ideal Rayleigh wave (in a uniform half-space) is non-dispersive
- However, all real surface waves exhibit dispersion
 - It is because the subsurface is always layered

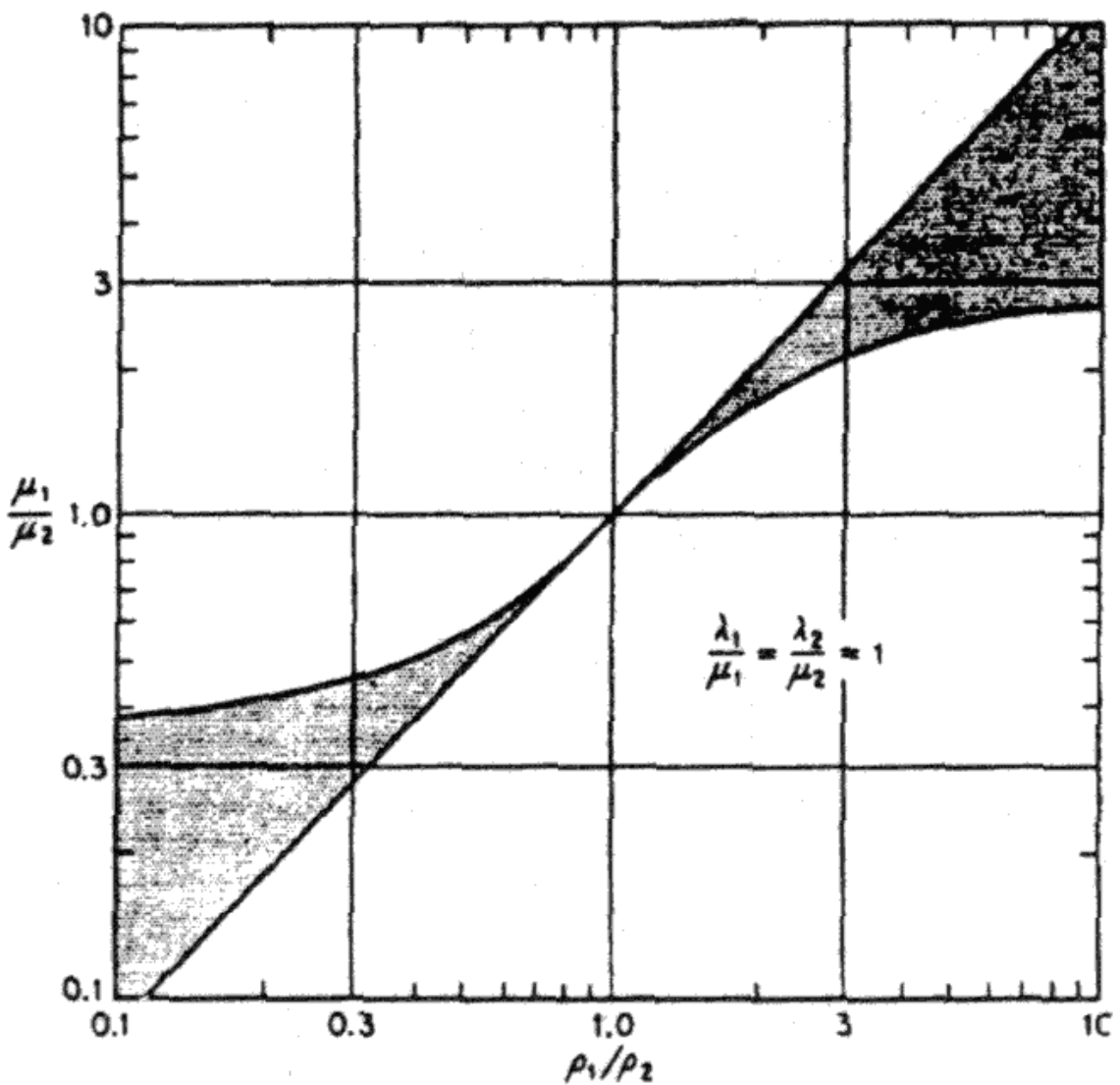
Note the change in wavelet shape



Stoneley waves

- These waves propagate along the contact of *two* semi-infinite media
 - They are *P/SV* in nature, like Rayleigh waves;
 - They always exist when one of the media is a fluid;
 - An important example is *tube wave* propagating along a fluid-filled borehole.
 - If both media are solids, Stoneley waves exist only when $V_{S1} \approx V_{S2}$ and ρ and μ lie within narrow limits (*plot on next page*)

Stoneley waves



Love waves


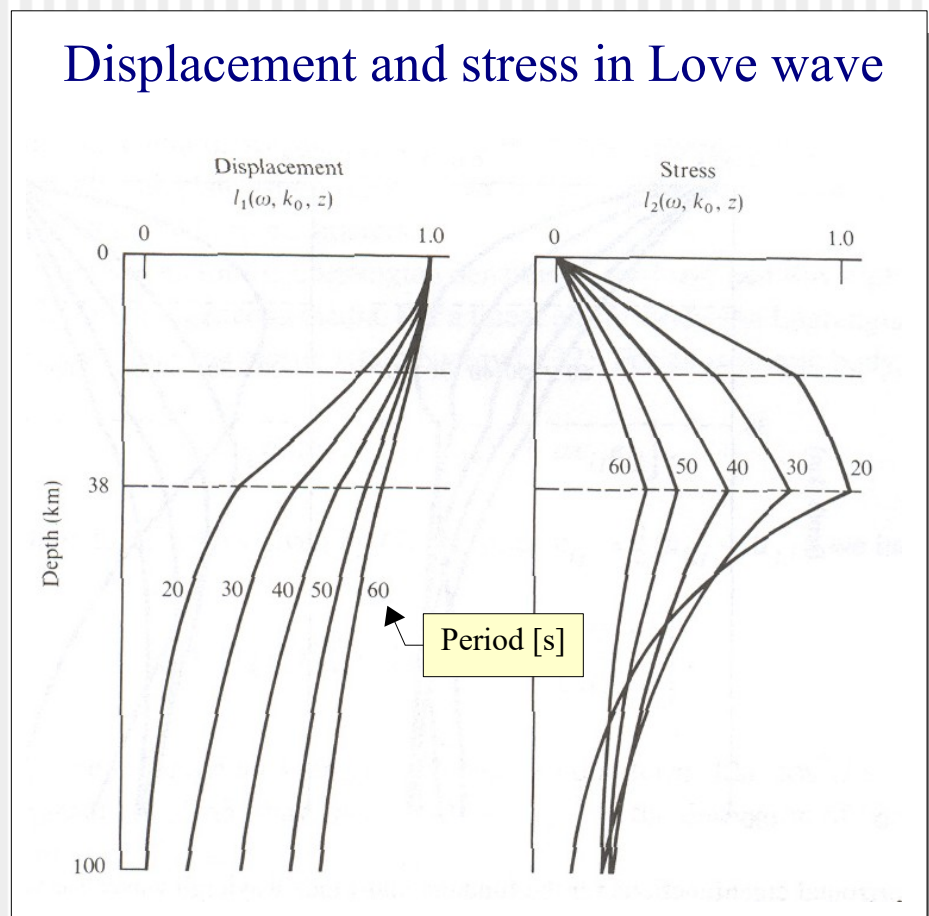
- These are *SH*-type waves propagating along the free surface
 - particle motion is transverse and parallel to the surface;
- Because there is just one *SH* potential, two modes are required to satisfy the two boundary conditions ($\sigma_{xz} = \sigma_{zz} = 0$ on the free surface)
- Thus, Love waves exist when the semi-infinite medium is overlain with a layer with different elastic properties.
 - ... this situation is quite common.

Love-wave dispersion

- Love waves are dispersive:
- At high frequencies, its velocity approaches the S-wave velocity in the surface layer
- At low frequencies, velocity is close to that of the lower layer.

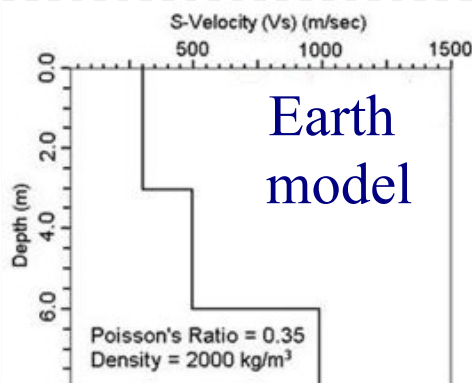
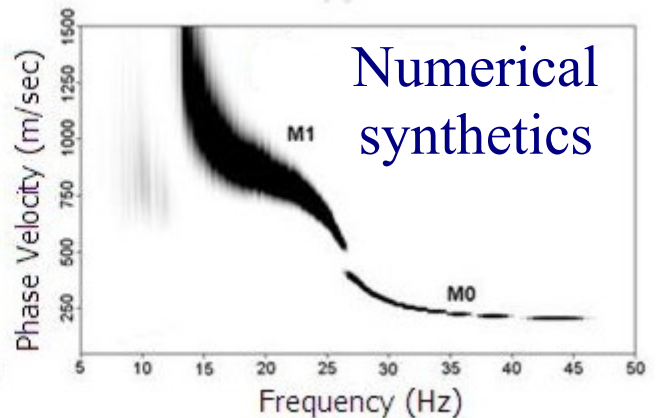
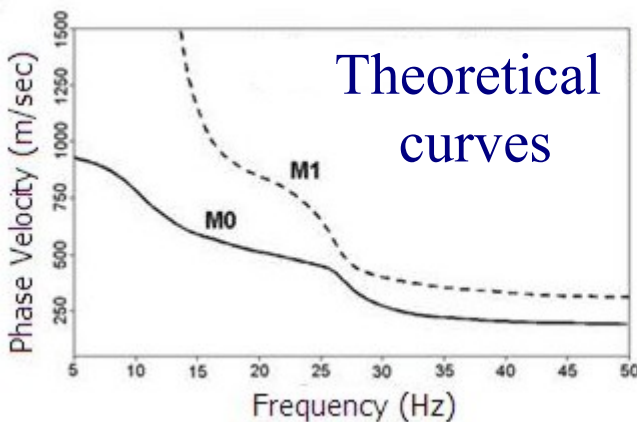
Displacement and stress in Love wave

Depth of sampling increases with period. This is common to all surface waves.

Surface-wave branches

- In layered structures, **multiple surface-wave branches**, or “**modes**” exist for the same frequency
- The branch with the lowest phase velocity (longest wavelength) is called the **fundamental mode**

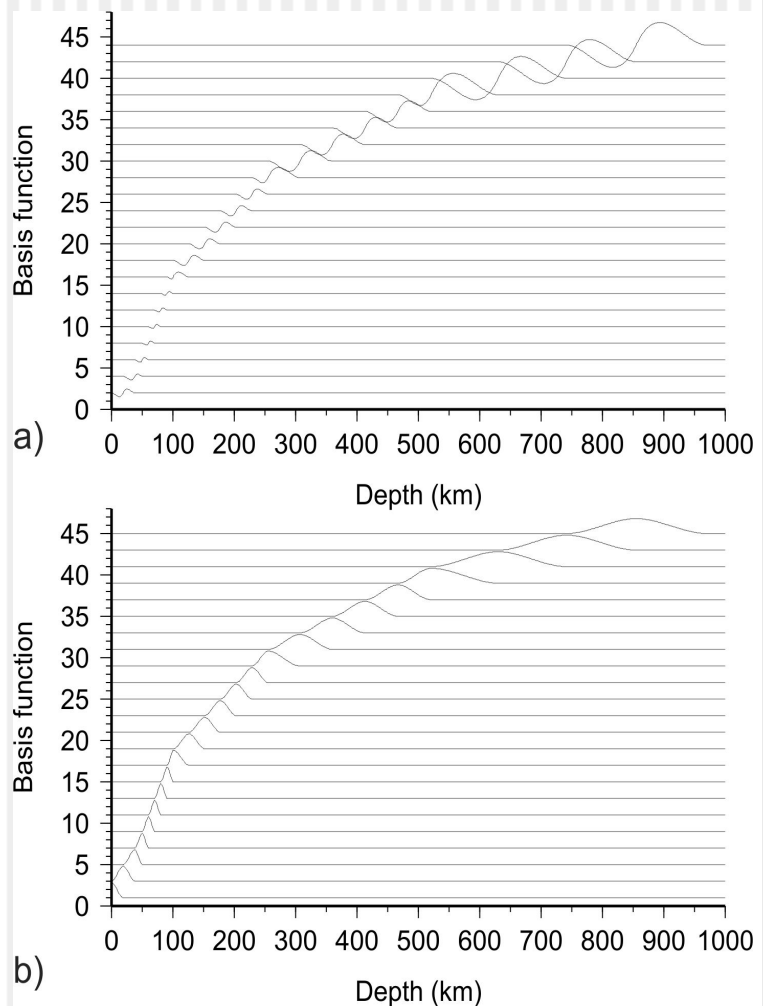


Surface-wave branches (theory)

- To see that multiple branches exist and determine their parameters, the following matrix method can be used:

- 1) Chose a set of N basis functions in depth, $f_i(z)$, and express the potential (or displacement) through them:

Example of basis functions for global Love waves



$$\psi(x, z, t) = e^{i(\omega t - kx)} \sum_{i=1}^N c_i f_i(z)$$

Surface-wave branches (theory, cont.)

- 2) For a given k , express the total kinetic and potential energies; they will be quadratic matrix products of c_i :

$$E_{kin} = \int \left(\frac{\rho}{2} \dot{u}_i \dot{u}_i \right) dz = \omega^2 c_i A_{ij} c_j$$

Note that E_{kin} is always proportional to ω^2

$$E_{el} = \int \left(\frac{\lambda}{2} \varepsilon_{ii} \varepsilon_{jj} + \mu \varepsilon_{ij} \varepsilon_{ij} \right) dz = c_i B_{ij} c_j$$

- 3) Note that in a wave, $E_{kin} = E_{el}$, and therefore:

$$\omega^2 c_i A_{ij} c_j = c_i B_{ij} c_j$$

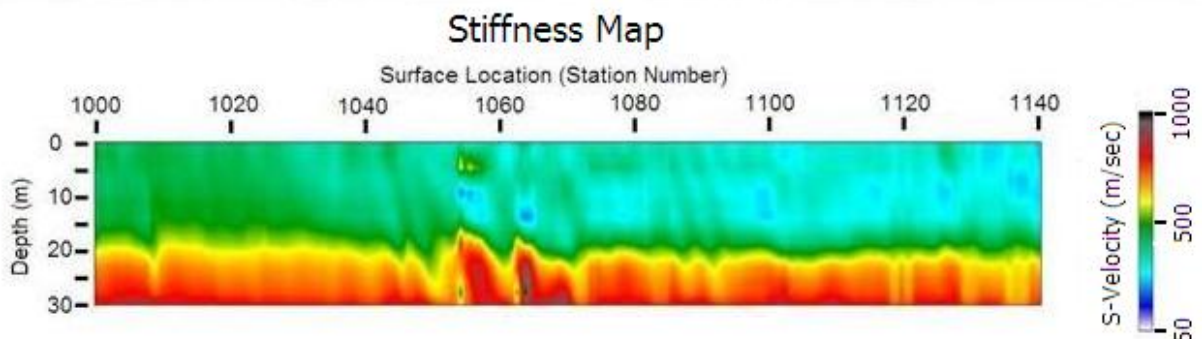
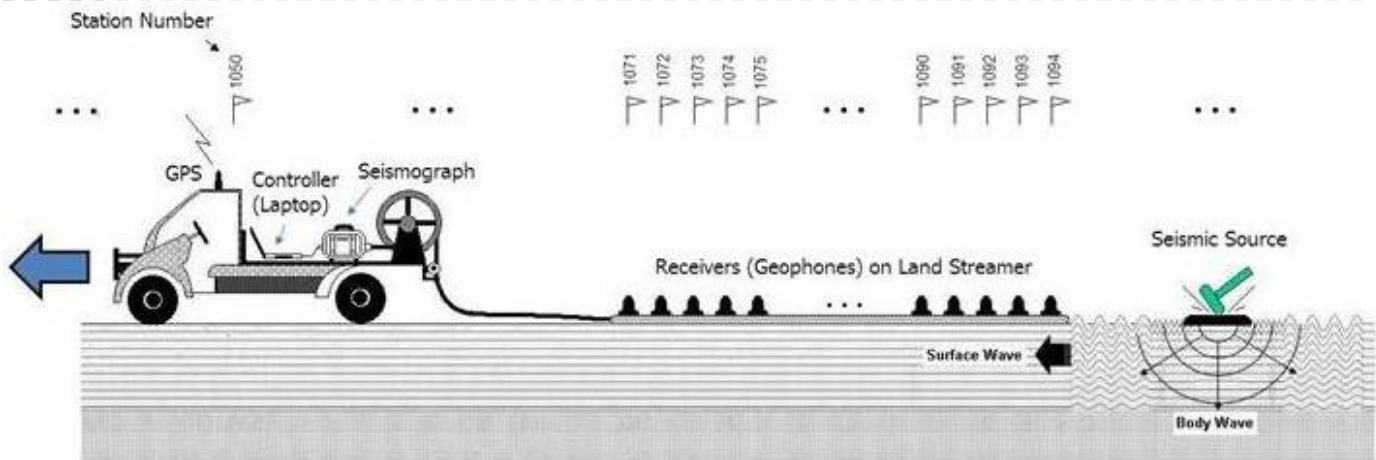
- 4) This means that c_i is an eigenvector of matrix $\mathbf{A}^{-1}\mathbf{B}$, and ω^2 is the corresponding eigenvalue:

$$\left(\mathbf{A}^{-1} \mathbf{B} - \omega^2 \mathbf{I} \right) \mathbf{c} = 0$$

There may be up to N positive eigenvalues. They give frequencies of up to N modes.

MASW (SASW) method

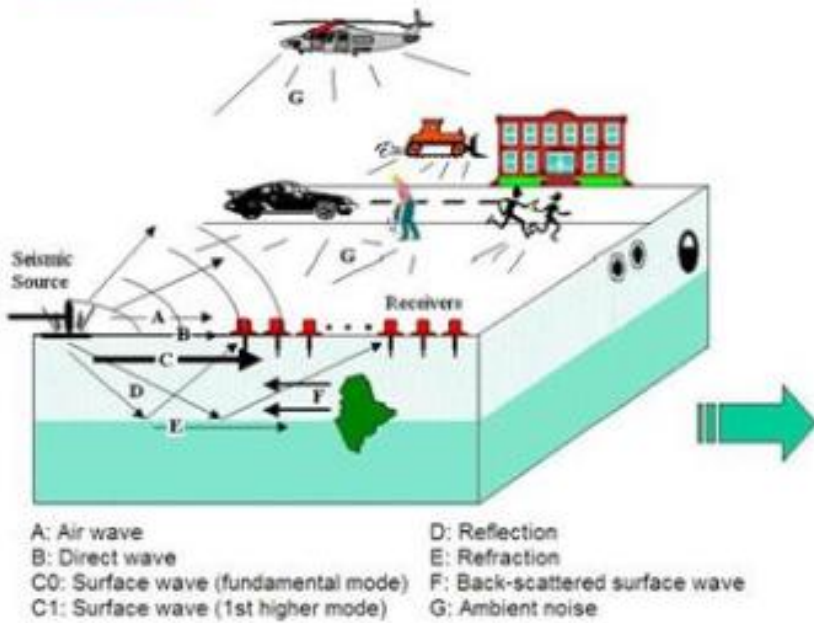
- Multichannel (or Spectral) Analysis of Surface Waves
- Uses dispersion $V(f)$ measurements to invert for $V_s(z)$ and $\mu(z)$
 - Geotechnical applications



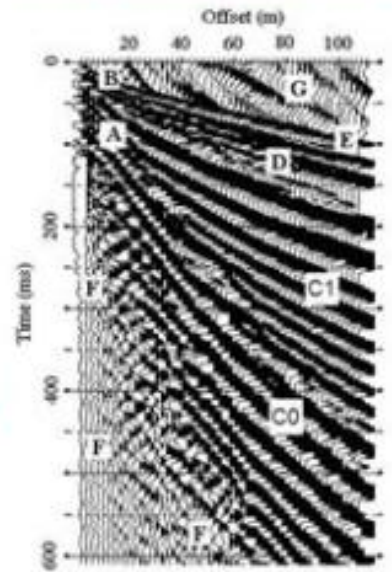
MASW method

DATA ACQUISITION

Field Survey

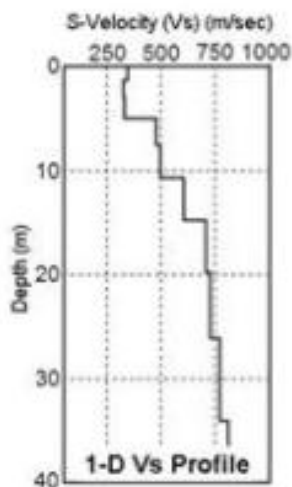


One Multichannel Record

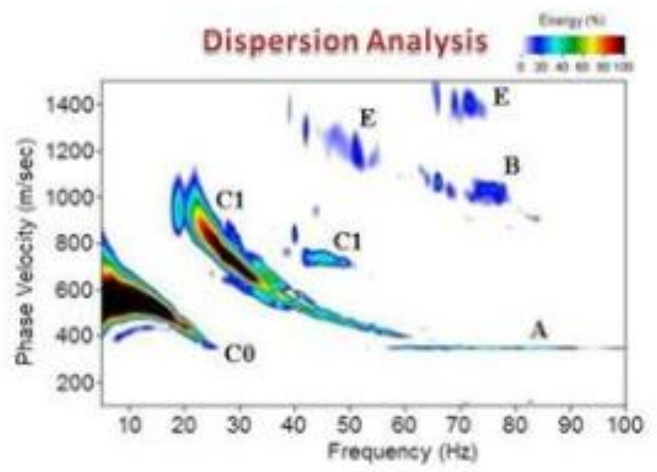


DATA PROCESSING

Inversion



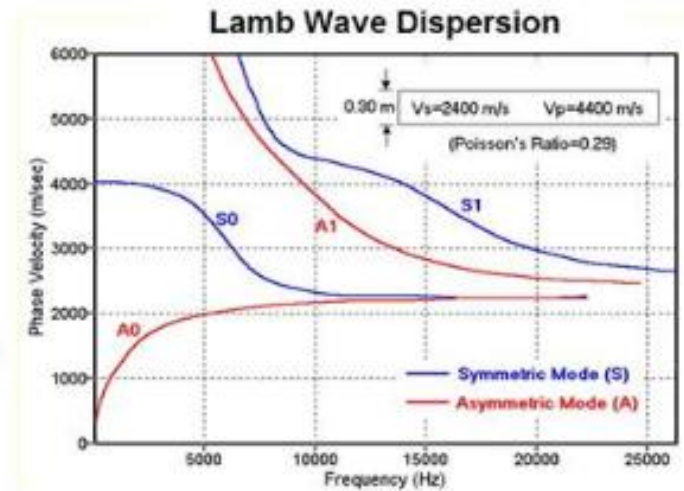
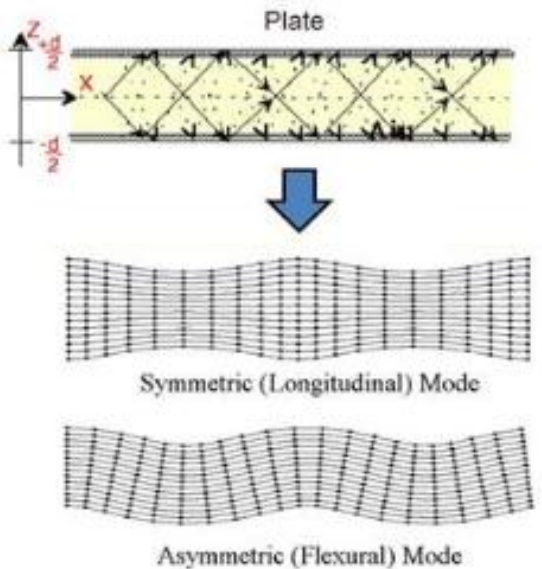
Dispersion Analysis



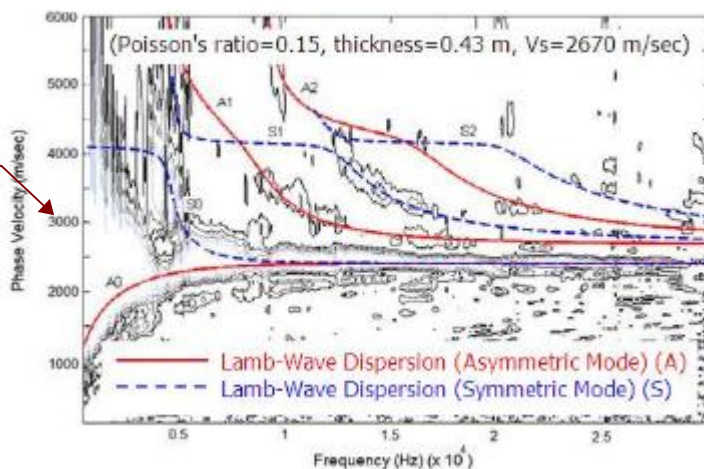
Lamb (plate) wave (e.g., in road pavement)

- The case of surface waves in a thin elastic plate is called the *Lamb's problem*

Lamb Wave = Plate Wave



Pavement Field Data (Multichannel Approach)



Note the difference
In dispersion curves
for symmetric
and asymmetric
deformations
of the plate