Elastic properties of rocks

Additional topics not used in exams or homework but useful for interpreting seismic data, rock-physics experiments, and reservoir engineering:

- Empirical relations between mechanical properties or rocks
- Effects of porosity and fluids
- Fluid substitution

Elastic moduli from ρ and seismic velocities

- Mechanical properties of the earth's subsurface are usually estimated by analysing the velocities of seismic waves in it
- From expressions for wave velocities (geol335 and next lectures), we can estimate the elastic moduli:

$$\mu = \rho V_S^2$$

$$\lambda = \rho\left(V_P^2 - 2V_S^2\right)$$

$$K = \lambda + \frac{2}{3}\mu = \rho\left(V_P^2 - \frac{4}{3}V_S^2\right)$$

Empirical wave-velocity density relations

- It is usually difficult (or impossible) to determine all three properties (λ, μ, and ρ), and so some of them have to be estimated from the others by using empirical relations observed for the given rock type
- In the next slides, we show some of the commonly used relations between ρ, V_P, and V_S

Empirical relations: *density* from V_P

(See the plot is in the following slide)

- Nafe-Drake curve for a wide variety of sedimentary and crystalline rocks (Ludwig, 1970):

$$\rho\left[\mathrm{g/cm^3}\right] = 1.6612 \cdot V_P - 0.4721 \cdot V_P^2 + 0.0671 \cdot V_P^3 - 0.0043 \cdot V_P^4 + 0.000106 \cdot V_P^5$$

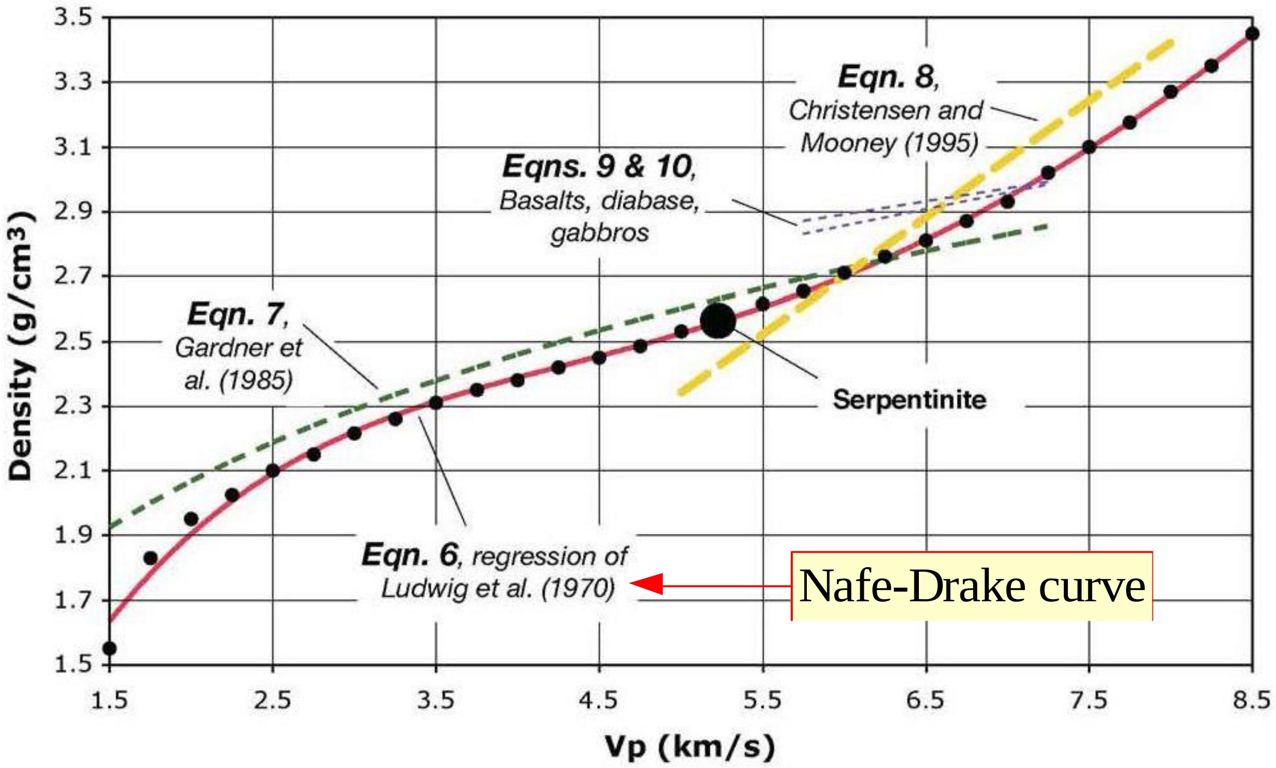
- Gardner's rule for sedimentary rocks and 1.5 < V_P < 6.1 km/s (Gardner et al., 1984):

$$\rho\left[\mathrm{g/cm^3}\right] = 1.74 \cdot V_P^{0.25}$$

- For crystalline rocks with $5.5 < V_P < 7.5$ km/s (Christensen and Mooney, 1995):

$$\rho\left[\mathrm{g/cm^3}\right] = 0.541 + 0.3601 \cdot V_P$$

Empirical relations: *density* from V_P



From T. Brocher, USGS Open File Report 05-1317, 2005

Empirical relations:
V_S from V_P

(See the plot is in the following slide)

- "Mudline" for clay-rich sedimentary rocks (Castagna et al., 1985)

$$V_S\left[\text{km/s}\right] = \left(V_P - 1.36\right)/1.16$$

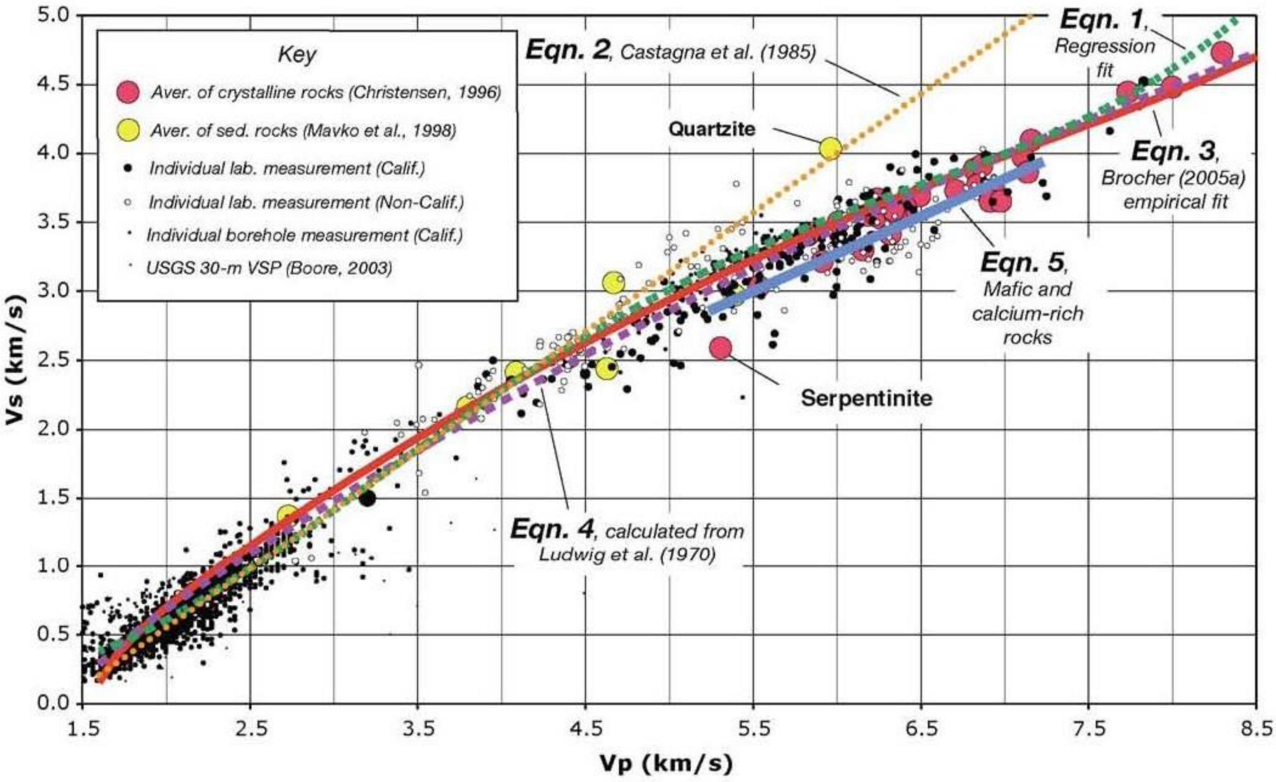
- Extension for higher-velocity crustal rocks (1.5 < V_P < 8 km/s; California, Brocher, 2005):

$$V_S\left[\text{km/s}\right] = 0.7858 - 1.2344 \cdot V_P + 0.7949 \cdot V_P^2 - 0.1238 \cdot V_P^3 + 0.0064 \cdot V_P^4$$

- "Mafic line" for calcium-rich rocks (dolomites), mafic rocks, and gabbros (Brocher, 2005):

$$V_S\left[\text{km/s}\right] = 2.88 + 0.52\left(V_P - 5.25\right)$$

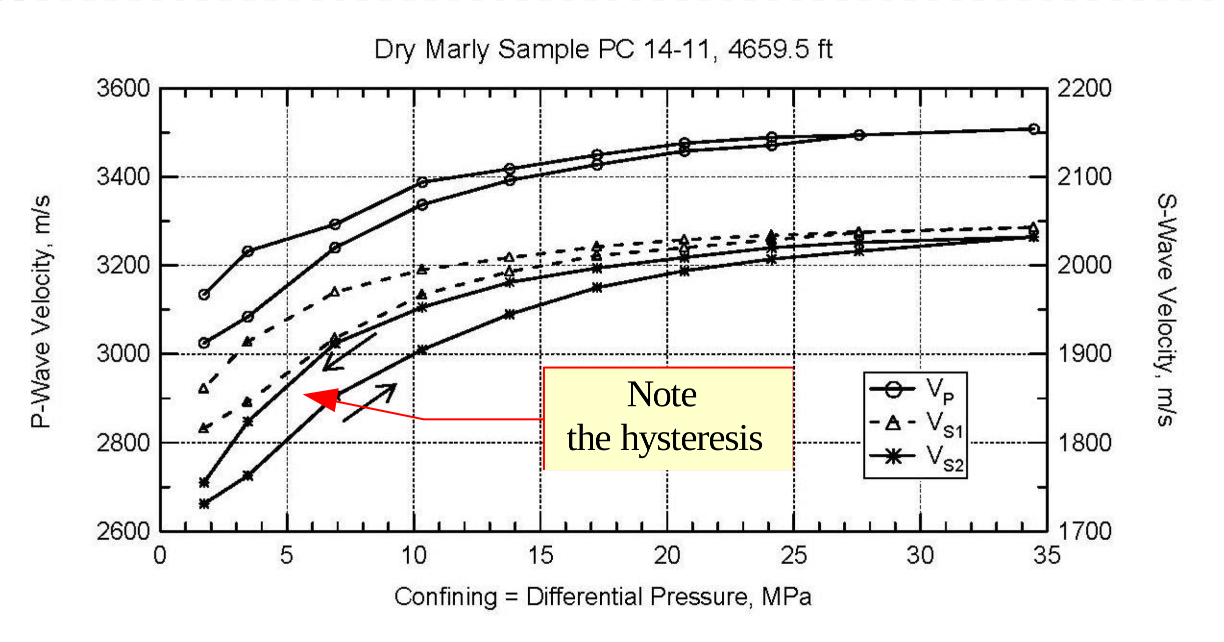
Empirical relations: V_S from V_P



From T. Brocher, USGS Open File Report 05-1317, 2005

Effect of pressure

- Differential pressure is the difference of confining pressure (pressure on the whole rock) and pore pressures (pressure of fluids within rock pores)
- Differential pressure closes cracks and generally increases the moduli, especially the bulk modulus *K*:



Dry carbonate sample from Weyburn reservoir

Effects of porosity

- Pores in rock
  - Reduce average density, ρ
  - Reduce the total elastic energy stored, and thus reduce the *K* and μ
- Fractures have similar effects, but these effects are always anisotropic

Effects of pore fluids

- Fluid in rock-matrix pores increases the bulk modulus
  - Gassmann's model (next page)
  - It has no effect on shear modulus
- Relative to rock-matrix ρ, density effectively decreases:

$$\rho \rightarrow \rho + \rho_f (1 - a)$$

  where ρ_f is fluid density, and a > 1 is the "tortuosity" of the pores
- In consequence, V_P increases, V_S decreases, and the Poisson's ratio and V_P/V_S increase

Gassmann's equation
("fluid substitution")

- Gassmann's model is the primary tool for interpreting fluid-related effects in oil/gas/water reservoirs
- Relates the elastic moduli of fluid-saturated rock (K_s, μ_s) to those of dry porous rock(K_d, μ_d):

$$K_s = K_d + \frac{K_0\left(1 - K_d / K_0\right)^2}{1 - K_d / K_0 - \phi\left(1 - K_0 / K_f\right)}$$

$$\mu_s = \mu_d$$

where K_f is the fluid bulk modulus, and K_0 is the bulk modulus of the matrix

  - Note relations: $K_f < K_d \le K_s \le K_0$

- Assumptions/approximations of this model:
  - Isotropic, homogeneous, elastic, monomineralic medium;
  - Pore space is well-connected and in pressure equilibrium;
  - Closed system with no fluid movement across boundaries;
  - No chemical reactions.