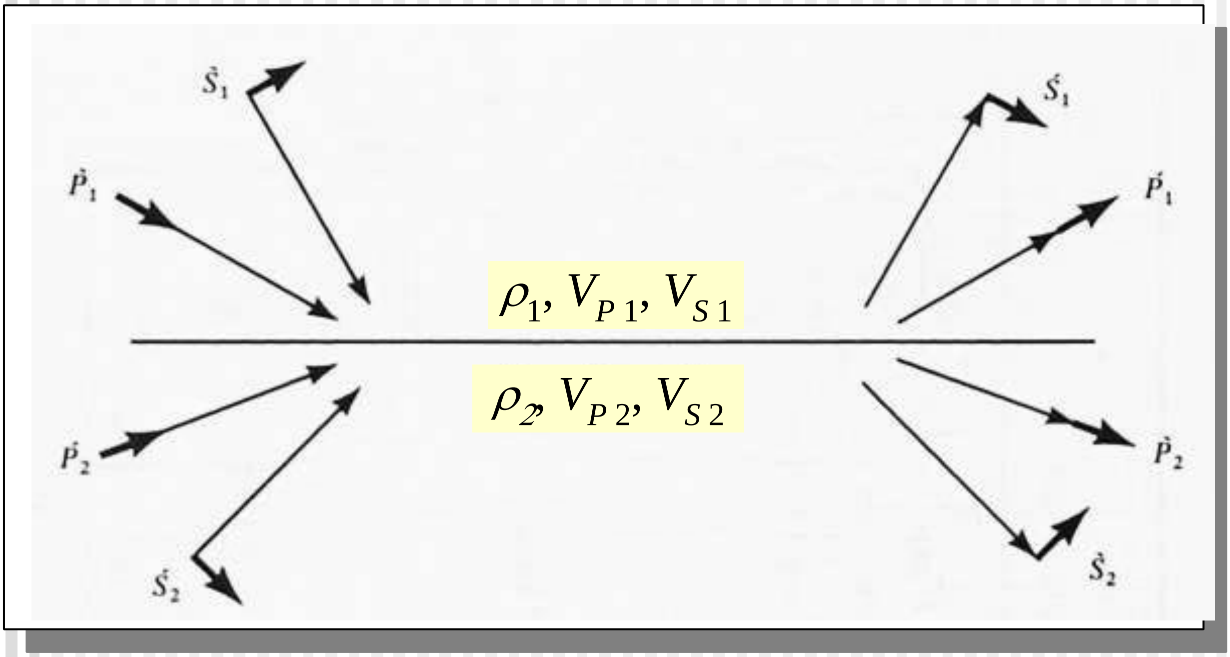
Reflection coefficients and AVO

- Reflection and P/SV mode conversion of plane waves
- Snell's law
- Scattering matrix
  - Zoeppritz and Knott’s equations for reflection coefficients
- Amplitude vs. Angle and Offset relations
- AVO cross-plotting

- Reading:
  - Telford et al., Section 4.2.
  - Shearer, 6.3, 6.5
  - Sheriff and Geldart, Chapter 3

Surface reflection transmission, and conversion

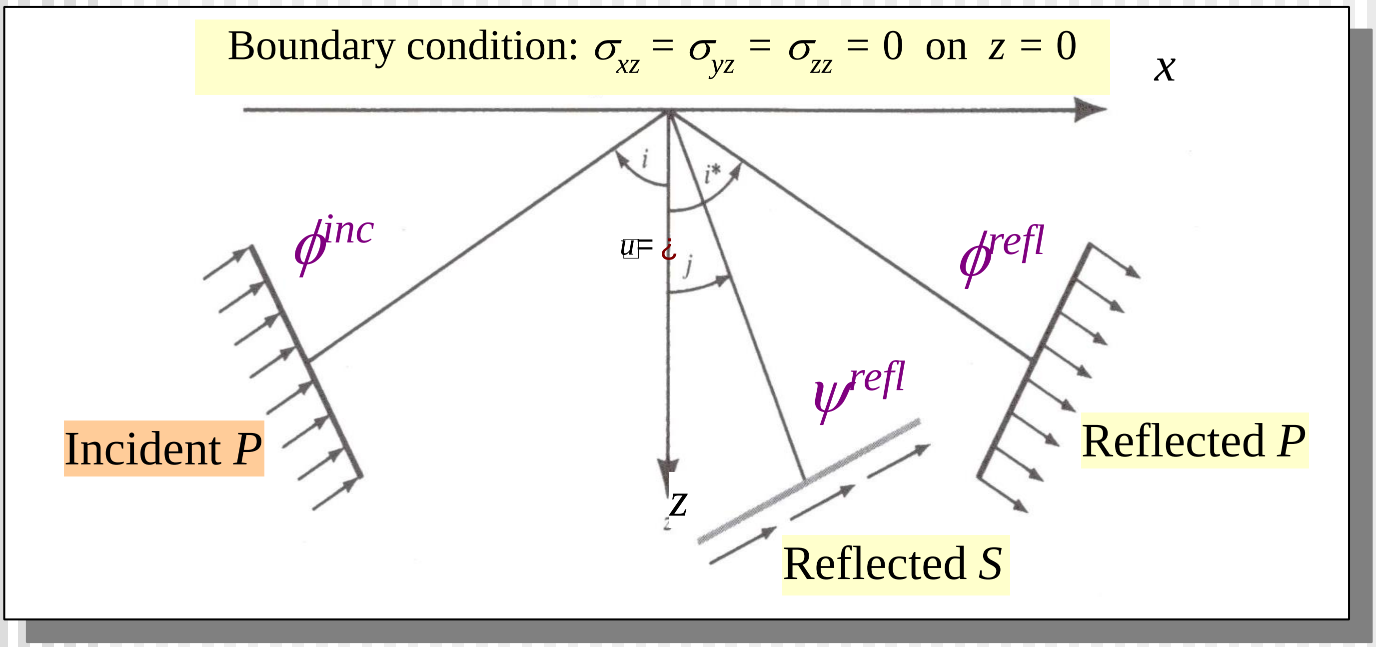
- Consider waves incident on a welded horizontal interface of two uniform half-spaces:
  - Because of their vertical motion, *P* and *SV* waves *couple* to each other on the interface,
  - therefore, there are 8 possible waves interacting with each other at the boundary.



What about *SH* waves?
Without lateral contrasts in material properties, SH waves do not couple with P and SV. SH wave can be considered separately, as a single scalar field

Free-surface reflection and conversion

- Consider a *P* wave incident on a free surface:



- Each of the *P*- or *S*-waves is described by potentials:

$$\mathbf{u}_P(x,z) = \left(\frac{\partial \phi}{\partial x}, \quad 0, \quad \frac{\partial \phi}{\partial z} \right) \qquad \phi = \phi^{inc} + \phi^{refl} \qquad P\text{-waves}$$

$$\mathbf{u}_S(x,z) = \left(\frac{-\partial \psi}{\partial z}, \quad 0, \quad \frac{\partial \psi}{\partial x} \right) \qquad \psi = \psi^{refl} \qquad SV\text{-wave}$$

Above, ψ is the component of vector potential ψ oriented along axis Y (orthogonally to the plane of propagation). To describe SH waves, vector ψ should be oriented along axis X or Z.

Free-surface reflection and conversion (2)

- Traction (force acting on the surface):

$$\mathbf{F}_P\left(x,z\right)=\left(2\mu\frac{\partial^2\phi}{\partial x\partial z},\quad 0,\quad \lambda\nabla^2\phi+2\mu\frac{\partial^2\phi}{\partial z^2}\right)$$ *P*-wave

$$\mathbf{F}_S\left(x,z\right)=\left(\mu\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right),\quad 0,\quad 2\mu\frac{\partial^2\psi}{\partial x\partial z}\right)$$ *SV*-wave

- Consider *plane harmonic* waves:

$$\phi^{inc}=A_P^{inc}\exp\left[i\omega\left(\frac{\mathbf{x}\cdot\mathbf{n}_{incP}}{V_P}-t\right)\right]$$ incident *P*

$$\phi^{refl}=A_P^{refl}\exp\left[i\omega\left(\frac{\mathbf{x}\cdot\mathbf{n}_{reflP}}{V_P}-t\right)\right]$$ reflected *P*

$$\psi^{refl}=A_S^{refl}\exp\left[i\omega\left(\frac{\mathbf{x}\cdot\mathbf{n}_{reflS}}{V_S}-t\right)\right]$$ reflected *SV*

- *Q*: What are the dependencies of ϕ and ψ above on coordinate x?

Free-surface reflection and conversion (3)

- The boundary condition is: ***Traction Force***$(x,t) = 0$
- Note that functional dependencies of ϕ and ψ on (x,t) are:

$$\exp\left[i\omega\left(\frac{\sin i}{V_P}x - t\right)\right]$$

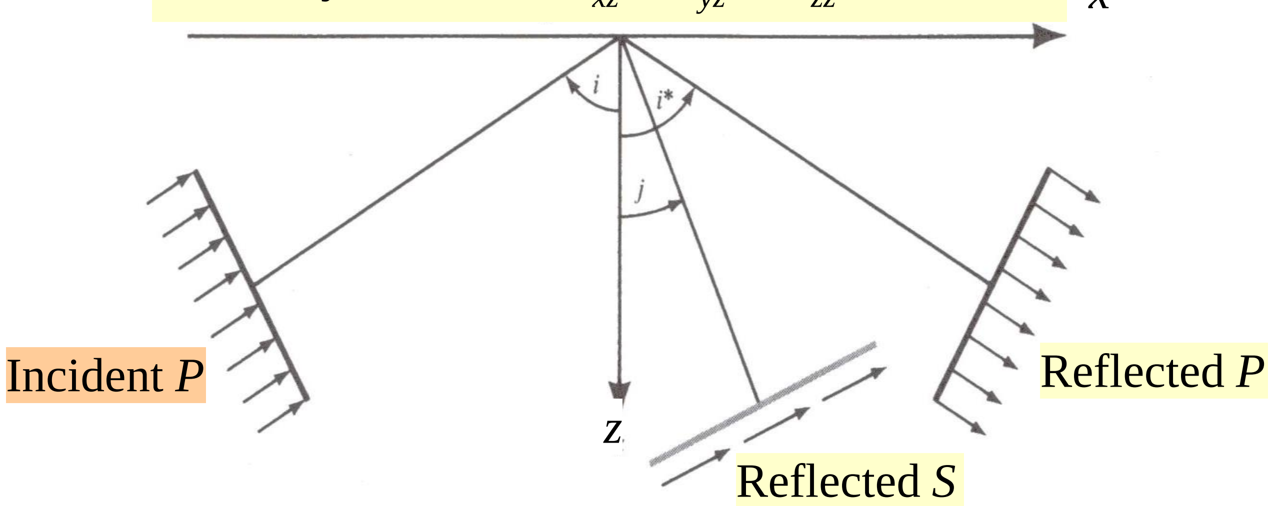
$$\exp\left[i\omega\left(\frac{\sin i^*}{V_P}x - t\right)\right]$$

$$\exp\left[i\omega\left(\frac{\sin j}{V_S}x - t\right)\right]$$

- These relations must be satisfied for <u>any x</u> ; consequently, you have the *Snell's law*:

$$\frac{\sin i}{V_P} = \frac{\sin i^*}{V_P} = \frac{\sin j}{V_S} = p$$

Boundary condition: $\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$ on $z=0$



Free-surface reflection and conversion (4)

- Displacement in plane waves is thus:

$$\mathbf{u}_P\left(x,z\right)=\left(i\omega p\phi,\quad 0,\quad \pm i\omega\frac{\cos j}{V_P}\phi\right)$$

P-waves

$$\mathbf{u}_S\left(x,z\right)=\left(\mp i\omega\frac{\cos j}{V_P}\psi,\quad 0,\quad i\omega p\psi\right)$$

SV-wave

- ...and traction:

$$\mathbf{F}_P\left(x,z\right)=\left(-2\rho V_S^2 p\phi,\quad 0,\quad -\rho\left(1-2V^2p^2\right)i\omega^2 V_S\phi\right)$$

$$\mathbf{F}_S\left(x,z\right)=\left(\rho\left(1-2V^2p^2\right)i\omega^2 V_S\psi,\quad 0,\quad 2\rho V_S^2 p\psi\right)$$

Free-surface reflection and conversion (5)

- Traction vector at the surface must vanish:

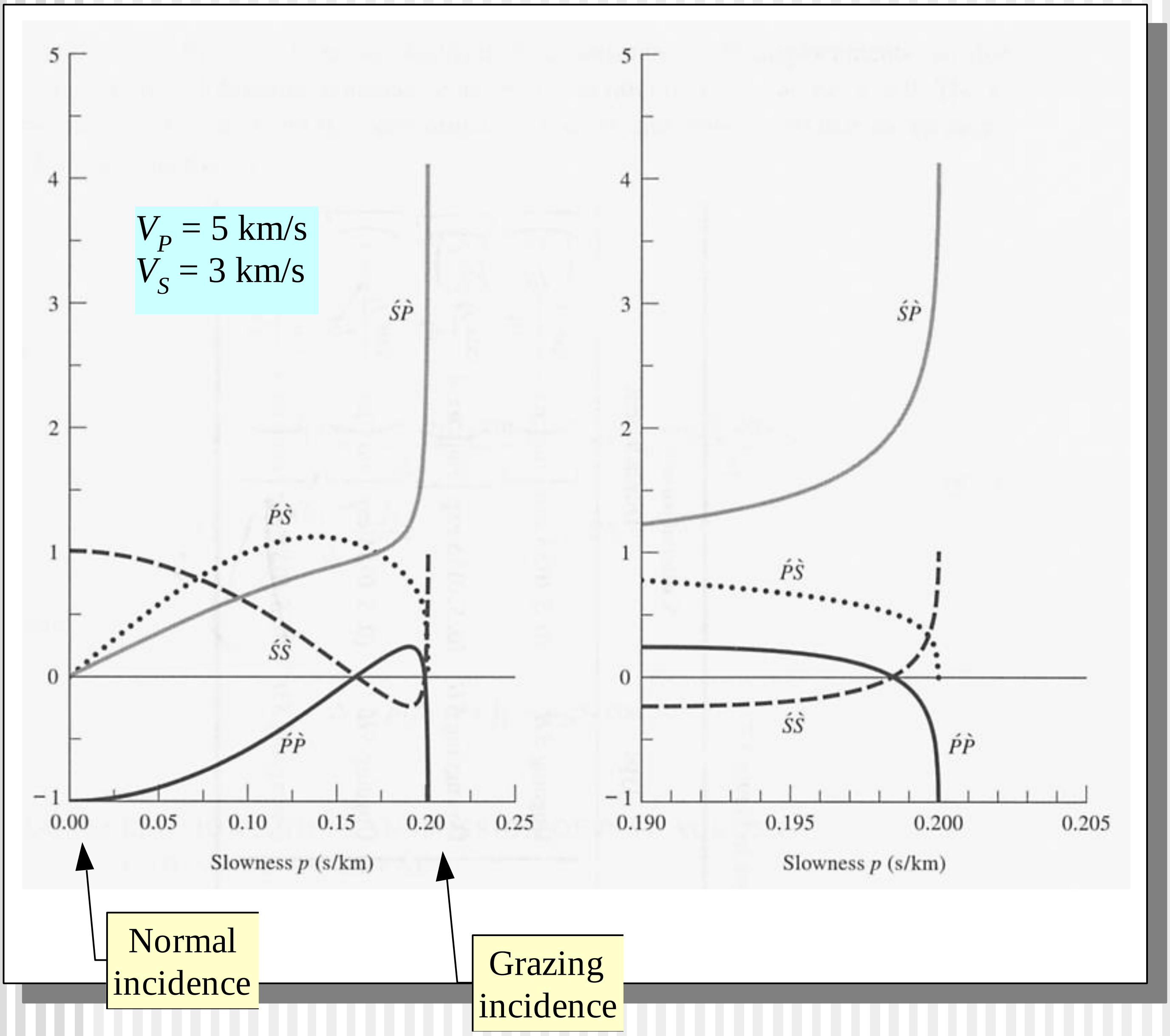
  $F_x = F_z = 0$

  - Therefore, we have two equations to constrain the amplitudes of the two reflected waves;
  - Their solution:

$$\frac{A_P^{refl}}{A_P^{inc}} = \frac{4V_S^4 p^2 \dfrac{\cos i}{V_P}\dfrac{\cos j}{V_S} - \left(1-2V_S^2 p^2\right)^2}{4V_S^4 p \dfrac{\cos i}{V_P}\dfrac{\cos j}{V_S} + \left(1-2V_S^2 p^2\right)^2}$$

$$\frac{A_S^{refl}}{A_P^{inc}} = \frac{-4V_S^2 p \dfrac{\cos i}{V_P}\left(1-2V_S^2 p^2\right)}{4V_S^4 p \dfrac{\cos i}{V_P}\dfrac{\cos j}{V_S} + \left(1-2V_S^2 p^2\right)^2}$$

Free-surface reflection and conversion (5)

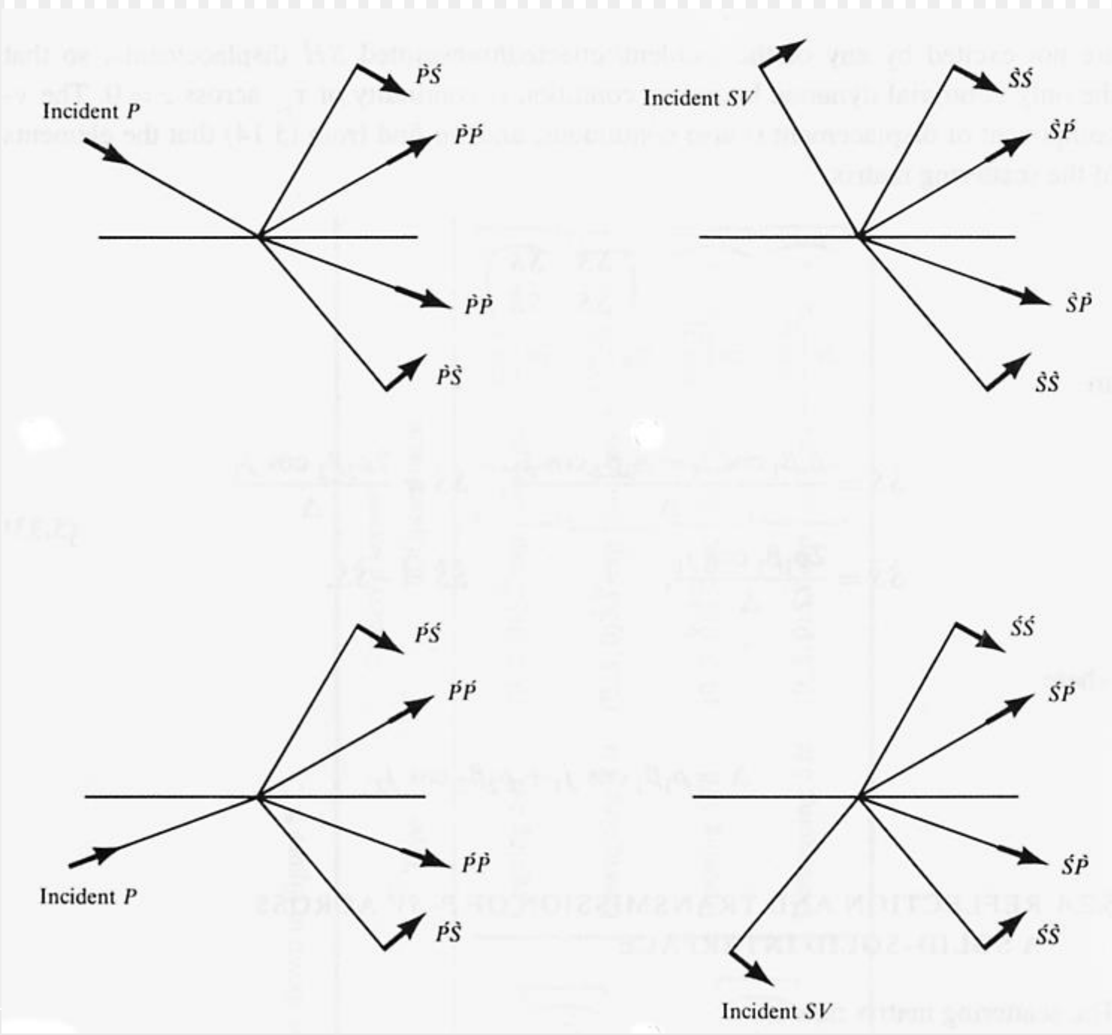


Amplification by the free surface

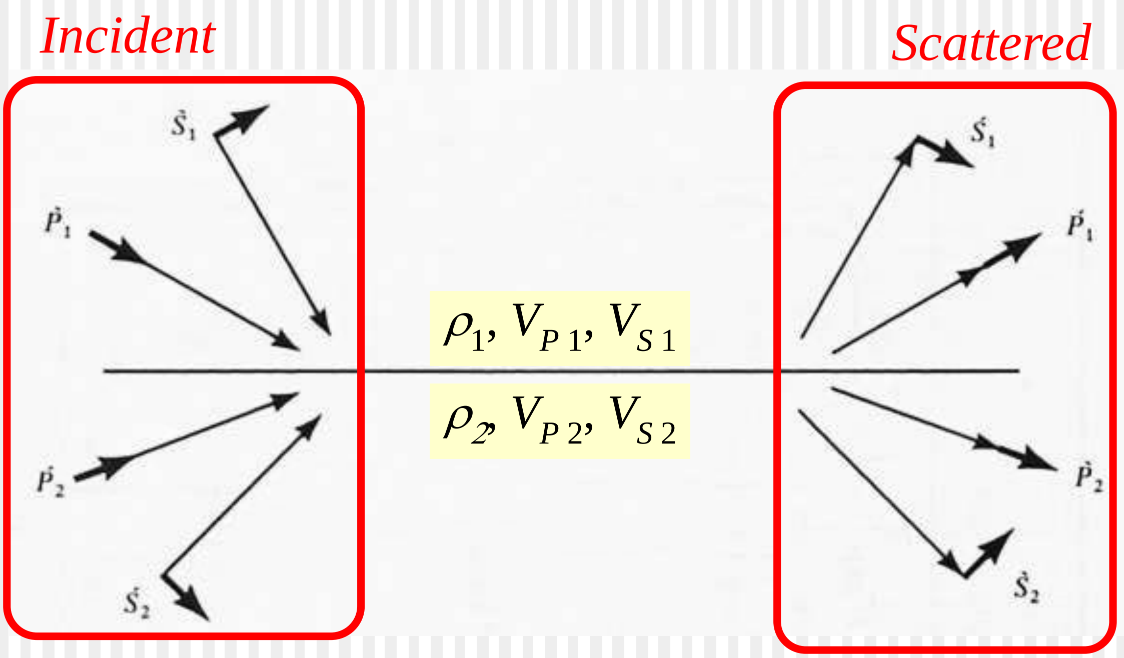
- Consider this question: If we have recorded ground velocity of, for example, 1 μm/s at the surface, does it equal the particle velocity within the incident wave in the subsurface?
- The answer is "no". At the surface, we record the interfering incident, reflected, and converted waves
  - At vertical incidence, the amplitude of the reflected wave equals the incident-wave amplitude (and there is no converted wave). Therefore, **the amplitude recorded at the free surface equals twice the amplitude of the incident wave**
  - This effect of doubling the amplitude is known as "amplification" by the free surface. There also exist other types of amplification related to the effects of the low-velocity weathering zone
  - Note that this amplification effect corresponds to the both free-surface reflection coefficients $R = -1$ for P waves and $R = 1$ for S waves (preceding slide)
    - For P waves, the direction of positive polarity is downward in the reflected wave, and so a reflection with $R = -1$ amplifies the upward ground motion in the incident wave
    - In an S wave, $R = 1$ has the same effect of boosting of the rightward ground motion

Complete reflection/transmission problem

- There are 16 possible reflection/transmission coefficients on a welded contact of two half-spaces
- These coefficients are shown by polarization arrows of outgoing waves, for four incident waves in the diagrams below



Scattering matrix



- All 16 possible reflection coefficients can be summarized in the *scattering matrix*:

$$\mathbf{S} = \begin{bmatrix} \overset{\searrow}{P}\overset{\nearrow}{P} & \overset{\searrow}{S}\overset{\nearrow}{P} & \overset{\nearrow}{P}\overset{\nearrow}{P} & \overset{\nearrow}{S}\overset{\nearrow}{P} \\ \overset{\searrow}{P}\overset{\nearrow}{S} & \overset{\searrow}{S}\overset{\nearrow}{S} & \overset{\nearrow}{P}\overset{\nearrow}{S} & \overset{\nearrow}{S}\overset{\nearrow}{S} \\ \overset{\searrow}{P}\overset{\searrow}{P} & \overset{\searrow}{S}\overset{\searrow}{P} & \overset{\nearrow}{P}\overset{\searrow}{P} & \overset{\nearrow}{S}\overset{\searrow}{P} \\ \overset{\searrow}{P}\overset{\searrow}{S} & \overset{\searrow}{S}\overset{\searrow}{S} & \overset{\nearrow}{P}\overset{\searrow}{S} & \overset{\nearrow}{S}\overset{\searrow}{S} \end{bmatrix}$$

so that all reflected and transmitted wave amplitudes can be obtained from input ones by matrix multiplication:

$$\begin{pmatrix} \overset{\nearrow}{P}_1 \\ \overset{\nearrow}{S}_1 \\ \overset{\searrow}{P}_2 \\ \overset{\searrow}{S}_2 \end{pmatrix} = \mathbf{S} \begin{pmatrix} \overset{\searrow}{P}_1 \\ \overset{\searrow}{S}_1 \\ \overset{\nearrow}{P}_2 \\ \overset{\nearrow}{S}_2 \end{pmatrix}$$

Reflection and transmission at a welded boundary

- The scattering matrix can be used to easily derive all possible reflection and refraction amplitudes at once:
  - Consider matrices **M** and **N** expressing the differences of the displacement and traction across the boundary through the incident and scattered fields:
    - Because displacements and tractions are linear combinations of the incident and scattered fields, they can be expressed by a matrix product
    - The differences of displacements and tractions must be zero on the welded boundary, and therefore:

$$\begin{pmatrix} u_x \\ u_z \\ \sigma_{xz} \\ \sigma_{zz} \end{pmatrix}_{\substack{\text{above} \\ \text{boundary}}} - \begin{pmatrix} u_x \\ u_z \\ \sigma_{xz} \\ \sigma_{zz} \end{pmatrix}_{\substack{\text{below} \\ \text{boundary}}} = \mathbf{M} \begin{pmatrix} \overset{\nearrow}{P}_1 \\ \overset{\nearrow}{S}_1 \\ \overset{\searrow}{P}_2 \\ \overset{\searrow}{S}_2 \end{pmatrix} - \mathbf{N} \begin{pmatrix} \overset{\searrow}{P}_1 \\ \overset{\searrow}{S}_1 \\ \overset{\nearrow}{P}_2 \\ \overset{\nearrow}{S}_2 \end{pmatrix} = \mathbf{0}$$

    - Thus, the scattered-wave amplitudes are related to the incident-wave amplitudes by:

$$\mathbf{M} \begin{pmatrix} \overset{\nearrow}{P}_1 \\ \overset{\nearrow}{S}_1 \\ \overset{\searrow}{P}_2 \\ \overset{\searrow}{S}_2 \end{pmatrix} = \mathbf{N} \begin{pmatrix} \overset{\searrow}{P}_1 \\ \overset{\searrow}{S}_1 \\ \overset{\nearrow}{P}_2 \\ \overset{\nearrow}{S}_2 \end{pmatrix}$$

Reflection and transmission amplitudes at a boundary (cont.)

- From the equation in the preceding slide, the general solution for the scattering matrix $\mathbf{S}$ is:

$$\mathbf{S} = \mathbf{M}^{-1}\mathbf{N}$$

  - From known M and N (next slide), this matrix product can be easily calculated in Matlab or other engineering software
- The scattering matrix $\mathbf{S}$ can be used to easily derive all possible reflection and refraction amplitudes at once:
  - This matrix contains all *Zoeppritz's* (at normal incidence) *and Knott's equations* (at oblique incidence) giving all reflected, transmitted, and converted wave amplitudes, for waves incident from either side of the boundary and their combinations:

$$\begin{pmatrix} \overset{\nearrow}{P}_1 \\ \overset{\nearrow}{S}_1 \\ \overset{\searrow}{P}_2 \\ \overset{\searrow}{S}_2 \end{pmatrix} = \mathbf{S} \begin{pmatrix} \overset{\searrow}{P}_1 \\ \overset{\searrow}{S}_1 \\ \overset{\nearrow}{P}_2 \\ \overset{\nearrow}{S}_2 \end{pmatrix}$$

Boundary-condition matrices **M** and **N**

- Matrices **M** and **N** consist of the coefficients of plane-wave amplitudes and tractions for *P*- and *SV*-waves:

$$\mathbf{M}=\begin{bmatrix} -V_{P1}p & -\cos j_1 & V_{P2}p & \cos j_2 \\ \cos i_1 & -V_{S1}p & \cos i_2 & -V_{S2}p \\ 2\rho_1 V_{S1}^2 p\cos i_1 & \rho_1 V_{S1}\left(1-2V_{S1}^2p^2\right) & 2\rho_2 V_{S2}^2 p\cos i_2 & \rho_2 V_{S2}\left(1-2V_{S2}^2p^2\right) \\ -\rho_1 V_{P1}\left(1-2V_{S1}^2p^2\right) & 2\rho_1 V_{S1}^2 p\cos j_1 & \rho_2 V_{P2}\left(1-2V_{S2}^2p^2\right) & -2\rho_2 V_{S1}^2 p\cos j_2 \end{bmatrix}$$

$$\mathbf{N}=\begin{bmatrix} V_{P1}p & \cos j_1 & -V_{P2}p & -\cos j_2 \\ \cos i_1 & -V_{S1}p & \cos i_2 & -V_{S2}p \\ 2\rho_1 V_{S1}^2 p\cos i_1 & \rho_1 V_{S1}\left(1-2V_{S1}^2p^2\right) & 2\rho_2 V_{S2}^2 p\cos i_2 & \rho_2 V_{S2}\left(1-2V_{S2}^2p^2\right) \\ \rho_1 V_{P1}\left(1-2V_{S1}^2p^2\right) & -2\rho_1 V_{S1}^2 p\cos j_1 & -\rho_2 V_{P2}\left(1-2V_{S2}^2p^2\right) & 2\rho_2 V_{S1}^2 p\cos j_2 \end{bmatrix}$$

Normal-incidence case

- At normal incidence, $i_1=i_2=j_1=j_2=0$, and $p=0$:

$$
\mathbf{M}=\begin{array}{cccc} P & S & P & S \\ \left[\begin{matrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & \rho_1 V_{S1} & 0 & \rho_2 V_{S2} \\ -\rho_1 V_{P1} & 0 & \rho_2 V_{P2} & 0 \end{matrix}\right] \end{array}
\qquad
\mathbf{N}=\begin{array}{cccc} P & S & P & S \\ \left[\begin{matrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & \rho_1 V_{S1} & 0 & \rho_2 V_{S2} \\ \rho_1 V_{P1} & 0 & -\rho_2 V_{P2} & 0 \end{matrix}\right] \end{array}
$$

- The *P- and S-waves do not interact at normal incidence*, and so we can look, e.g., at *P*-waves only (extract the odd-numbered columns):

$$
\mathbf{M}=\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ -\rho_1 V_{P1} & \rho_2 V_{P2} \end{bmatrix}
\qquad
\mathbf{N}=\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ \rho_1 V_{P1} & -\rho_2 V_{P2} \end{bmatrix}
$$

Note that these two constraints are satisfied automatically

Impedance, $\rho V=Z$

- Drop the two trivial equations (#1 and 3) and obtain:

$$
\begin{bmatrix} \overset{\searrow}{P}\overset{\nearrow}{P} & \overset{\nearrow}{P}\overset{\nearrow}{P} \\ \overset{\searrow}{P}\overset{\searrow}{P} & \overset{\nearrow}{P}\overset{\searrow}{P} \end{bmatrix}
= M^{-1}N = \begin{bmatrix} 1 & 1 \\ -Z_1 & Z_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ Z_1 & -Z_2 \end{bmatrix}
= \frac{1}{Z_1+Z_2} \begin{bmatrix} Z_2-Z_1 & 2Z_2 \\ 2Z_1 & Z_1-Z_2 \end{bmatrix}
$$

Reflection and transmission coefficients

Reflection and Transmission at normal incidence

- Thus, at normal incidence (in practice, for angles up to ~15°)
  - Reflection coefficient:

$$R = \frac{Z_2 - Z_1}{Z_1 + Z_2} \approx \frac{\Delta Z}{2Z} \approx \frac{1}{2}\Delta\left(\ln Z\right) \approx \frac{1}{2}\left(\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right)$$

  - Transmission coefficient:

$$T = \frac{2Z_1}{Z_1 + Z_2}$$

  - Energy reflection coefficient (for energy flux):

$$E_R = R^2$$

  - Energy transmission coefficient:

$$E_T = 1 - E_R = \frac{2Z_1 Z_2}{Z_1 + Z_2}$$

    - Note that the energy coefficients do not depend on the direction of wave propagation, but *R* changes its sign
    - $R < 0$ is seen as to *polarity reversal* in reflection records

Typical impedance contrasts and reflectivities

Table 3.1 *Energy reflected at interface between two media*

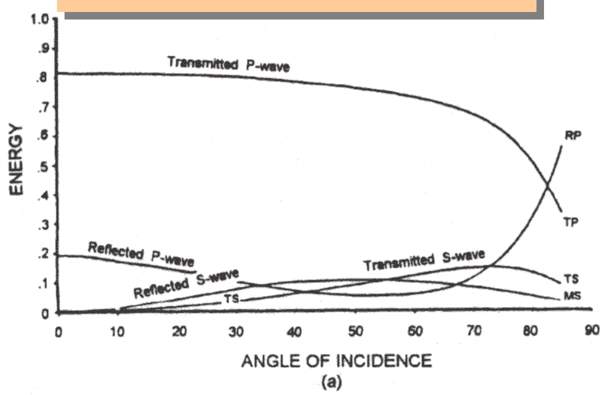
Interface	First medium Velocity	First medium Density	Second medium Velocity	Second medium Density	Z_1/Z_2	R	E_R
Sandstone on limestone	2.0	2.4	3.0	2.4	0.67	0.2	0.040
Limestone on sandstone	3.0	2.4	2.0	2.4	1.5	−0.2	0.040
Shallow interface	2.1	2.4	2.3	2.4	0.93	0.045	0.0021
Deep interface	4.3	2.4	4.5	2.4	0.97	0.022	0.0005
"Soft" ocean bottom	1.5	1.0	1.5	2.0	0.50	0.33	0.11
"Hard" ocean botom	1.5	1.0	3.0	2.5	0.20	0.67	0.44
Surface of ocean (from below)	1.5	1.0	0.36	0.0012	3800	−0.9994	0.9988
Base of weathering	0.5	1.5	2.0	2.0	0.19	0.68	0.47
Shale over water sand	2.4	2.3	2.5	2.3	0.96	0.02	0.0004
Shale over gas sand	2.4	2.3	2.2	1.8	1.39	−0.16	0.027
Gas sand over water sand	2.2	1.8	2.5	2.3	0.69	0.18	0.034

All velocities in km/s, densities in g/cm^3; the minus signs indicate 180° phase reversal.

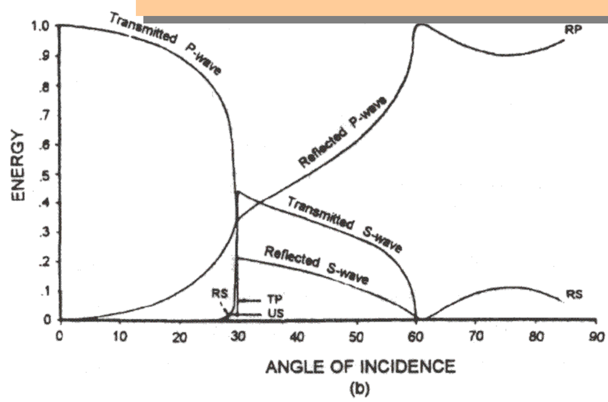
Oblique incidence
Amplitude Variation with Angle (AVA)

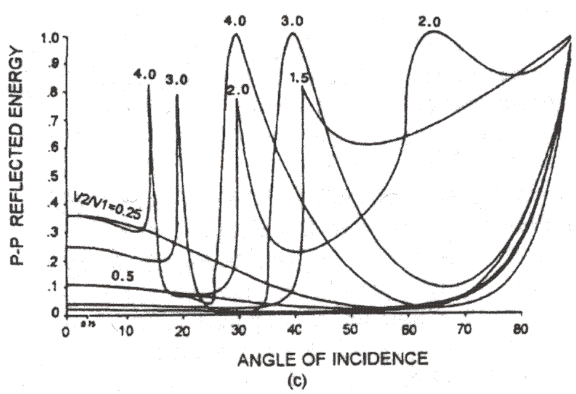
- At oblique incidence, we have to use the full $\mathbf{M}^{-1}\mathbf{N}$ expression for $\mathbf{S}$
  - *Amplitudes* and *polarities* of the reflections vary with incidence angles.

Fast to slow:
$V_{P2}/V_{P1} = 0.5$, $\rho_2/\rho_1 = 0.8$; $\nu_2 = 0.25$

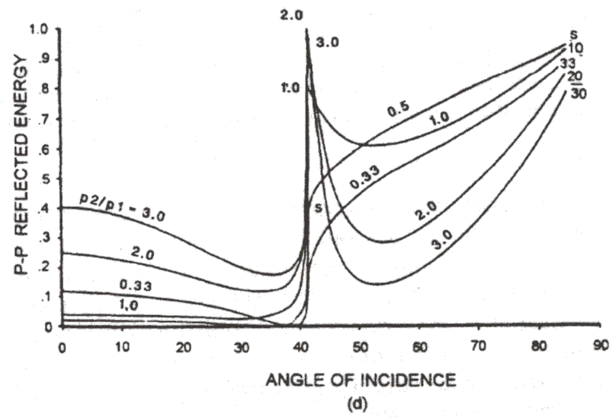


Slow to Fast:
$V_{P2}/V_{P1} = 2.0$, $\rho_2/\rho_1 = 0.5$; $\nu_2 = 0.3$





Fraction of *P*-wave reflection energy,
for various V_{P2}/V_{P1}
$\rho_2/\rho_1 = 1.0$; $\nu_1 = \nu_2 = 0.25$



Fraction of *P*-wave reflection energy,
for various ρ_2/ρ_1
$V_{P2}/V_{P1} = 1.5$; $\nu_1 = \nu_2 = 0.25$

Oblique incidence
Small-contrast AVA approximation

- Consider the case of small variations of V_P, V_S, ρ, and therefore ray-angle across the boundary
  - Shuey's (1985) formula gives the variation of R from the case on normal incidence in terms of ΔV_P and $\Delta \nu$ (Poisson's ratio):

Important at >~30°

$$\frac{R(\theta)}{R(0)} \approx 1 + P\sin^2\theta + Q\left(\tan^2\theta - \sin^2\theta\right)$$

Important at typical reflection angles

where:

$$R(0) \approx \frac{1}{2}\left(\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right)$$

$$P = \left[Q - \frac{2(1+\nu)(1-2\nu)}{1-\nu}\right] + \frac{\Delta\nu}{R(0)(1-\nu)^2}$$

$$Q = \frac{\frac{\Delta V_P}{V_P}}{\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}} = \frac{1}{1 + \frac{\Delta\rho/\rho}{\Delta V_P/V_P}}$$

P is related to "AVO gradient" (next slide). Note that it contains the effects of Poisson's ratio (ν and $\Delta\nu$)

Amplitude Variation with Offset (AVO)

- AVO is a group of interpretation techniques designed to detect reflection AVA effects:
  - Records processed with *true amplitudes* (preserving proportionality to the actual recorded amplitudes);
  - Source-receiver offsets converted to the incidence angles;
  - From pre-stack (variable-offset) data gathers, parameters *R*(0), *P* and *Q* are estimated by fitting the following dependencies on θ:

$$R(\theta) \approx R(0)\left[1 + P\sin^2\theta + Q\left(\tan^2\theta - \sin^2\theta\right)\right]$$

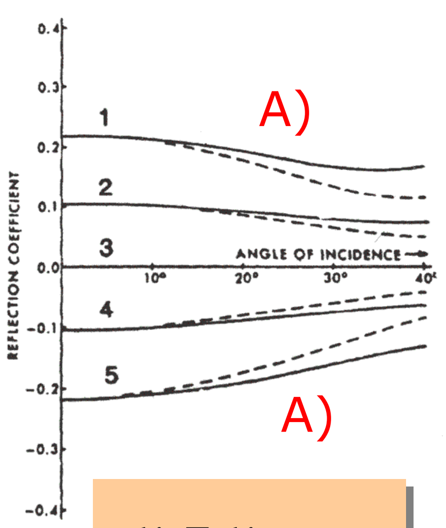
  - Thus, additional attributes are extracted to distinguish between materials with different ν
- In a two-term (linear) approximation, the above expression is often written as

$$R(\theta) \approx I + G\sin^2\theta$$

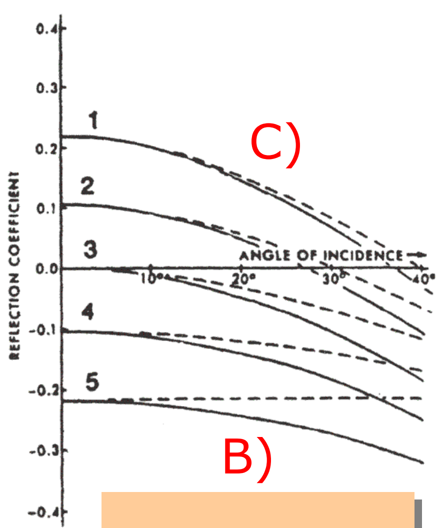
where $I = R(0)$ is called "AVO intercept" and $G = P \cdot R(0)$ is the "AVO gradient"

AVA patterns

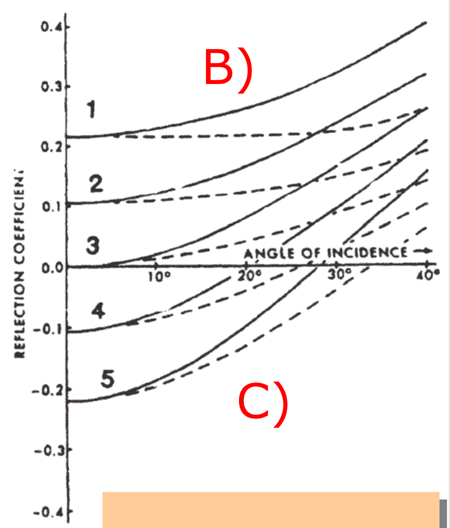
- There are three typical AVA behaviours:
  - A) Amplitude decreases with angle without crossing 0;
  - B) Amplitude increases;
  - C) Amplitude decreases and crosses 0 (reflection polarity changes).



$\nu_2 = \nu_1$
= 0.3 (solid lines)
= 0.2 (dashed)



$\nu_2 < \nu_1$
: 0.4 to 0.1 (solid)
: 0.3 to 0.1 (dashed)



$\nu_2 > \nu_1$
: 0.1 to 0.4 (solid)
: 0.1 to 0.2 (dashed)

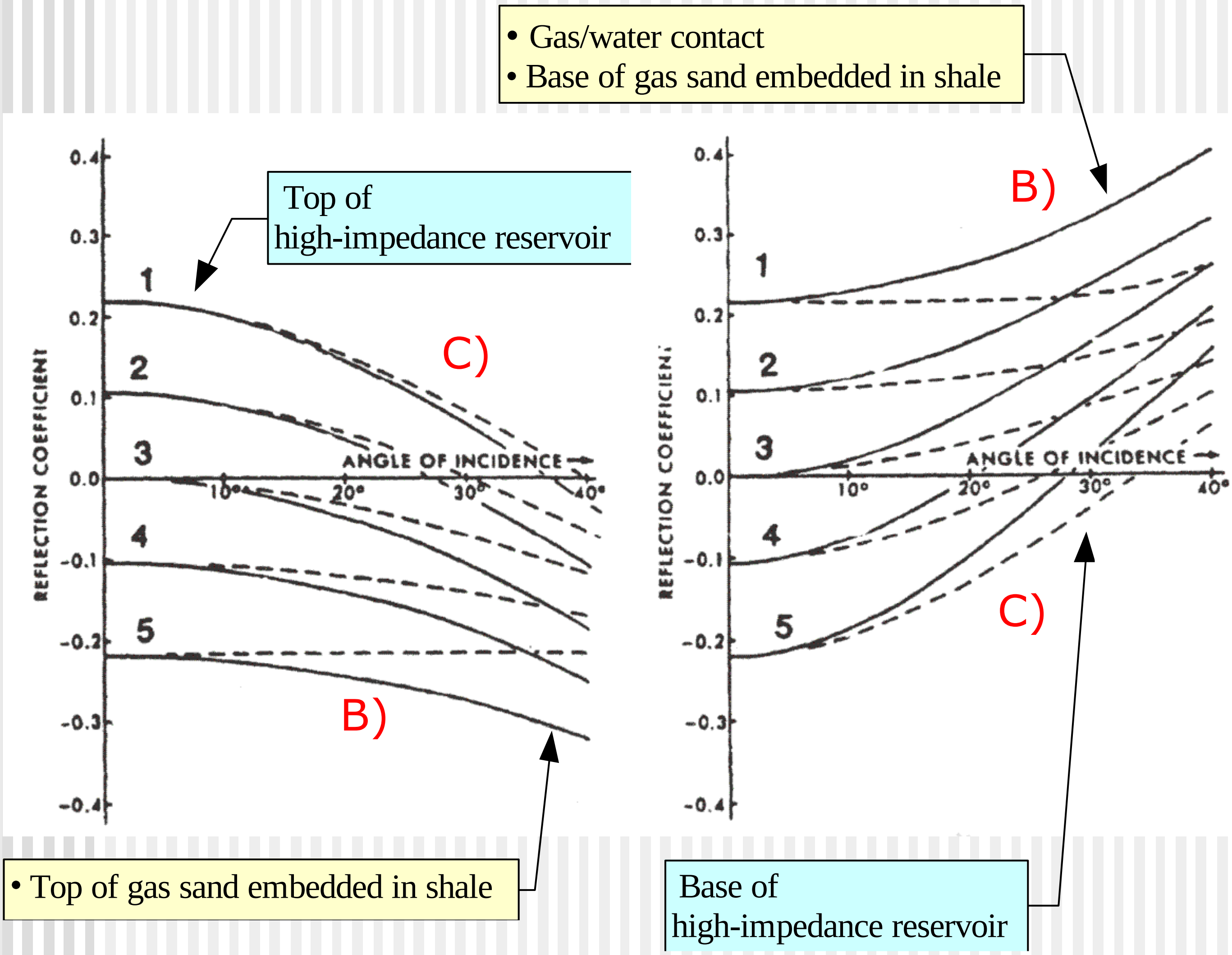
This is considered as a "Normal" case

Such behaviours are considered "AVO (AVA) anomalies"

(Above: $V_{P2}/V_{P1} = \rho_2/\rho_1$ =1.25; 1.11; 1.0; 0.9, and 0.8)

From Ostrander, 1984

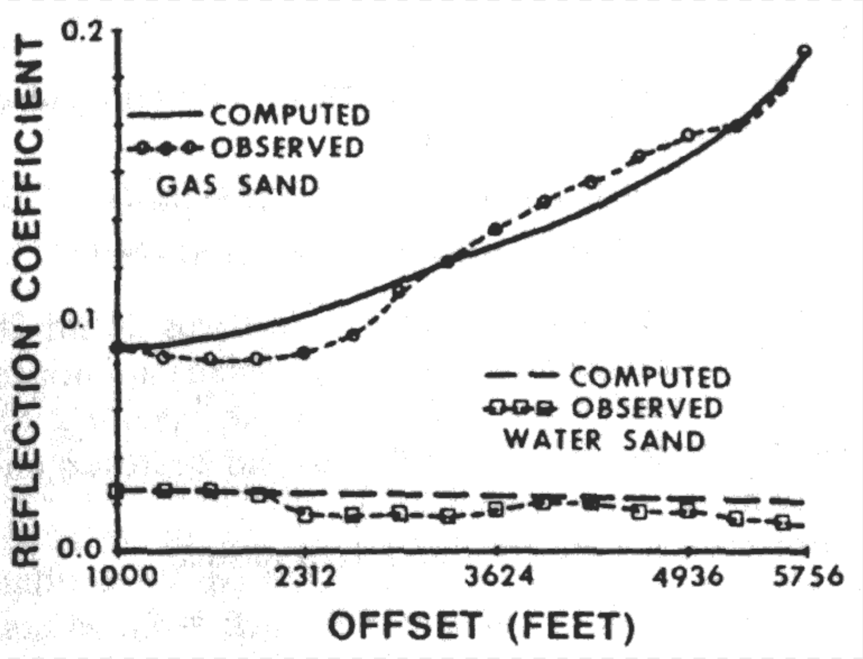
AVA (AVO) anomalies



From Ostrander, 1984

Amplitude Variation with Offset (AVO)
Gas sand vs. wet sand

- Gas-filled pores tend to reduce V_P more than V_S, and as a result, the Poisson's ratio (ν) is reduced.
- Thus, negative ΔV_P *and* $\Delta\nu$ cause negative-polarity bright reflection ("bright spot") *and* an AVO effect (increase in reflection amplitude with offset). These effects are regarded as indicators of hydrocarbons
  - However, not every AVO anomaly is related to a commercial reservoir...



From Yu, 1985

AVO cross-plotting

- Different combinations of reflection intercept (parameter *R*(0)) with gradient (sign and value of parameter *P*) are summarized by four "AVO classes". These classes give a common framework for interpreting AVO effects.

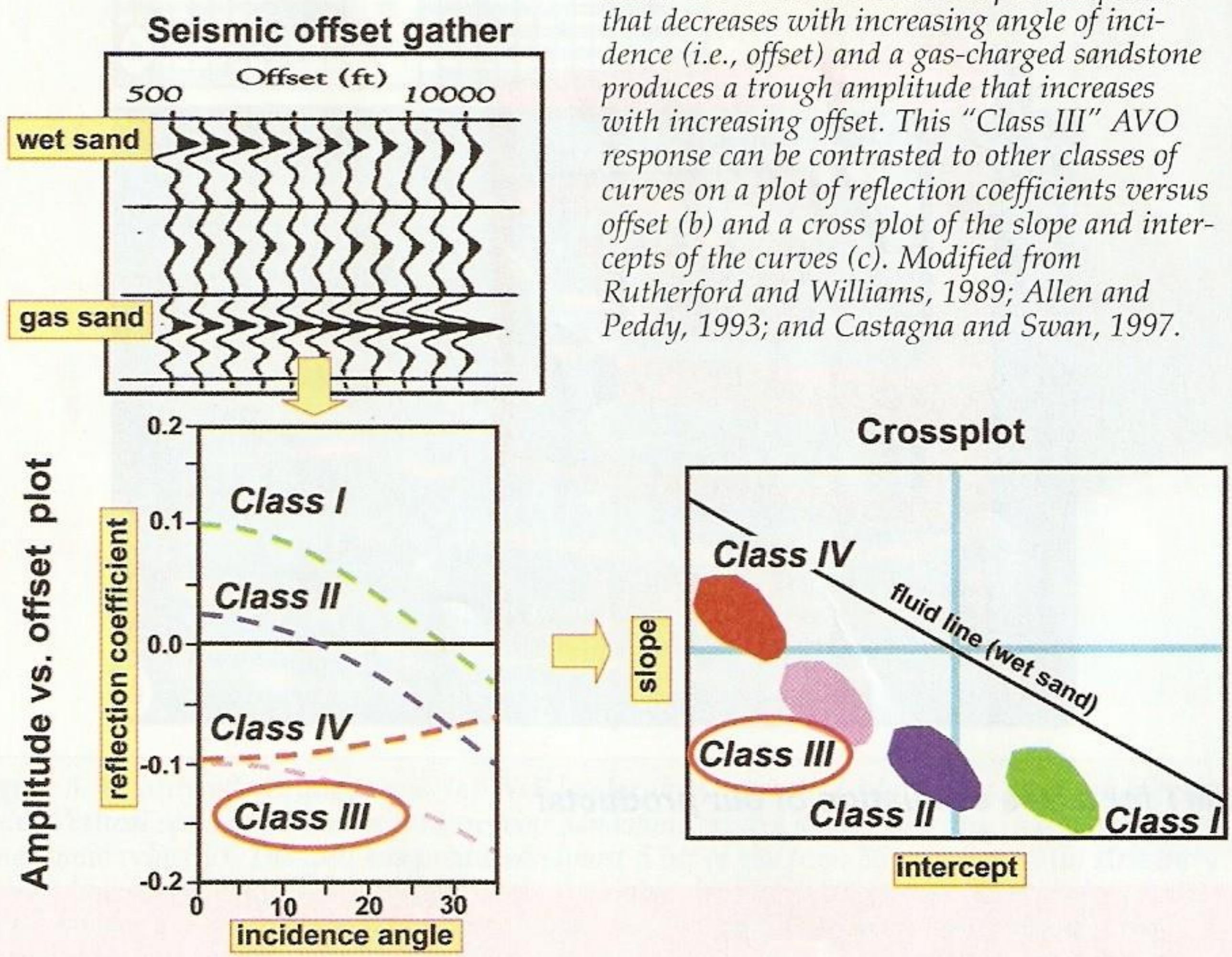
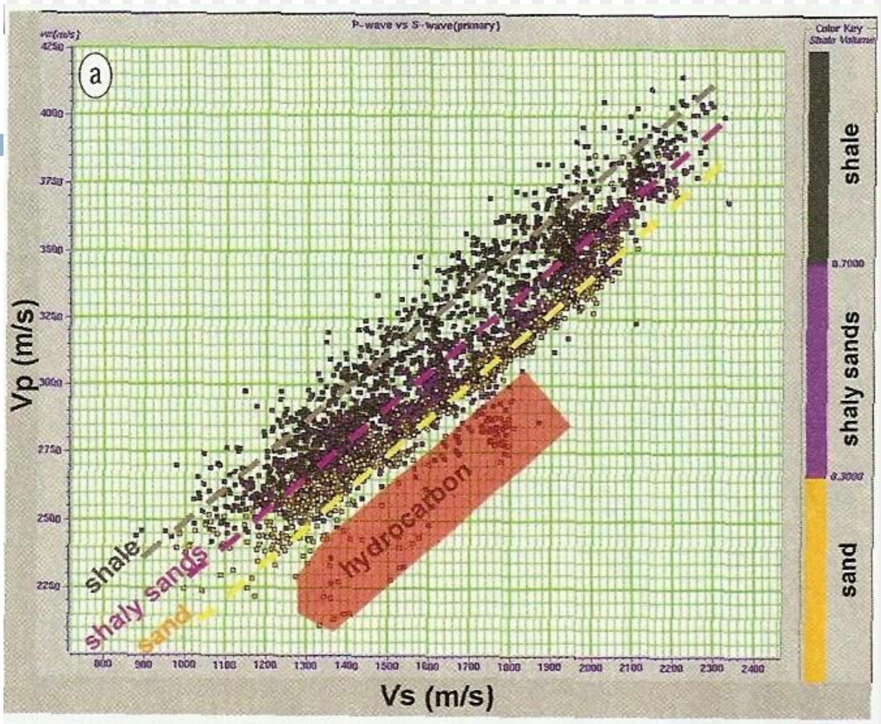


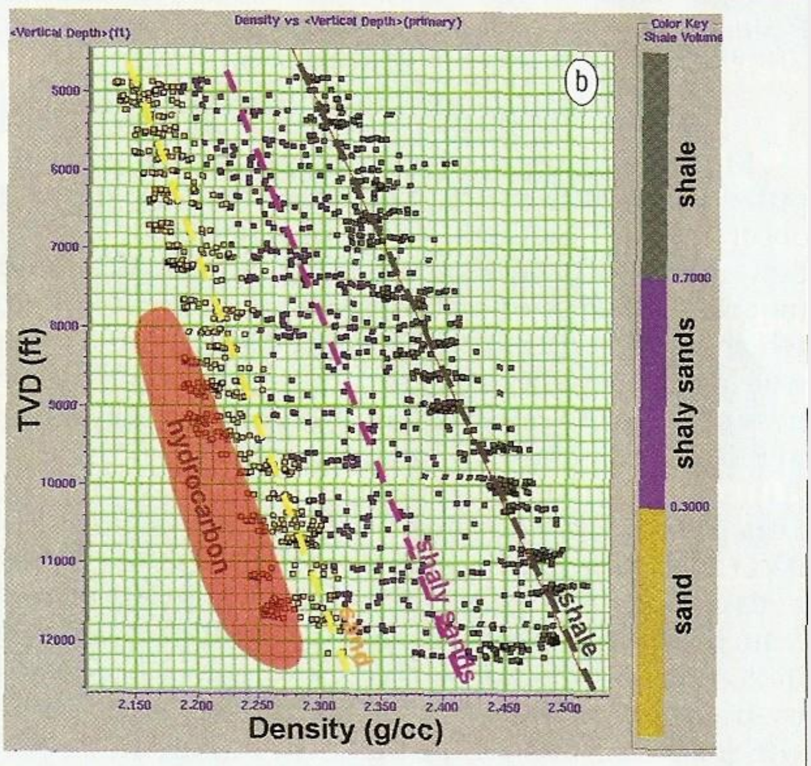
Figure 7. Principles of amplitude-versus-offset (AVO) analysis. (a) Example where the top of a water-wet sandstone creates a peak amplitude that decreases with increasing angle of incidence (i.e., offset) and a gas-charged sandstone produces a trough amplitude that increases with increasing offset. This "Class III" AVO response can be contrasted to other classes of curves on a plot of reflection coefficients versus offset (b) and a cross plot of the slope and intercepts of the curves (c). Modified from Rutherford and Williams, 1989; Allen and Peddy, 1993; and Castagna and Swan, 1997.

From Young et al, 2007

Cross-plotting

- When interpreting geophysical data, it is important to analyze **combinations of multiple physical and geological properties**
- Cross-plotting different pairs of parameters is a useful tool to such analysis



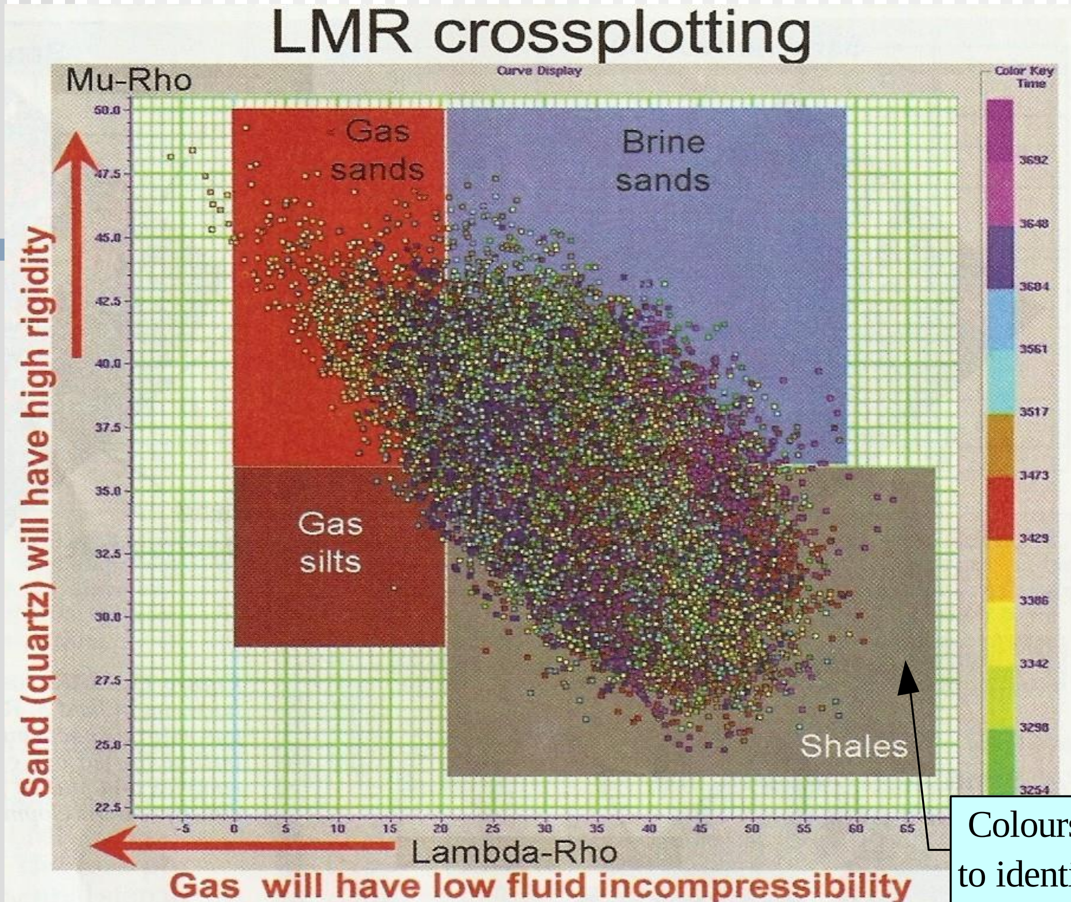


From Young et al, 2007

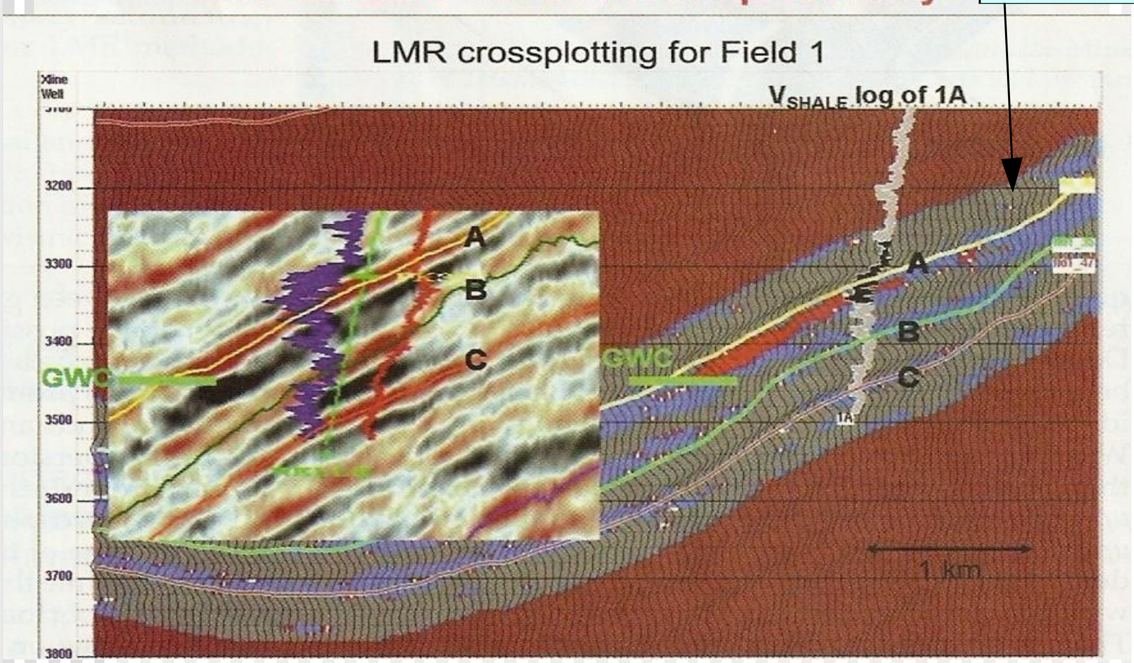
Rock-physics Indicators

- Rock-physics parameters can be derived from the shapes of AVO (AVA) responses:
  - λ ("incompressibility") is considered as the most sensitive pore-fluid indicator
  - μ (rigidity) is insensitive to fluid but sensitive to the matrix
    - μ increases with increasing quartz content (*e.g.*, in sand *vs*. clay).
  - ρ is sensitive to gas content.

λ-μ-ρ cross-plotting



Colours correspond
to identified ($\lambda\rho$,$\mu\rho$) zones



From Young et al, 2007