Surface waves

- Mechanism (role of boundary)
- Particle motion and polarization
  - Rayleigh and Love waves
- Phase and group velocity
  - Velocity dispersion
- Multiple wave modes
  - Description from energy equipartitioning
- MASW

- Reading:
  - Shearer, chapter 8
  - Telford *et al.*, 4.2.4, 4.2.6

Mechanism

- Surface waves are always associated with a boundary
- The (*e.g.*, horizontal) boundary disrupts vertical wave propagation *but* provides means for special wave modes propagating along it
  - Instead of oscillatory (sin() or cos()) shapes typical for a body wave, depth dependence of amplitude in a surface wave is principally exponential: $exp(-z/\delta)$, where δ is the skin depth
  - Surface waves are "tied" to the surface and exponentially decrease away from it
- Because there are 2 or 4 boundary conditions to satisfy (e.g., displacement and stress continuity), surface waves always consist of 2 or 4 interacting wave modes:
  - *P* and *SV* wave modes (*Rayleigh* or *Stoneley* waves; polarized orthogonally to the boundary);
  - Two *SH* modes (*Love* waves; polarized parallel to the boundary).

Surface-wave potentials

- General wave equations for potentials:

$$\nabla^2\phi = \frac{1}{V_P^2}\frac{\partial^2\phi}{\partial t^2}$$ *P*-wave

$$\nabla^2\psi_V = \frac{1}{V_S^2}\frac{\partial^2\psi_V}{\partial t^2}$$ *SV*-wave

$$\nabla^2\psi_H = \frac{1}{V_S^2}\frac{\partial^2\psi_H}{\partial t^2}$$ *SH*-wave

- Surface waves are combinations of solutions with *complex* (*e.g.*, pure imaginary) wavenumbers along *z.*
  - *e.g.*, for Rayleigh wave:

$$\phi = Ae^{-mz}e^{i(kx-\omega t)}$$

$$\psi_V = Be^{-nz}e^{i(kx-\omega t)}$$

  - Question: why are such solutions not allowed without a boundary?

Skin depth

- You have seen similar exponential depth dependencies for oscillating electromagnetic (EM) waves penetrating a conductor. There, these dependencies were characterized by the thickness of "skin layer", or "skin depth" δ:

$$A(z) = A(0)e^{-z/\delta}$$

- At depth $z = \delta$, wave amplitude decreases by the factor $1/e \approx 0.37$ relative to the amplitude at the surface
- The same applies to seismic surface waves:
  - P- and S-wave amplitudes decrease by $1/e$ at skin depths:

$$\delta_P = \frac{1}{n} \quad \text{and} \quad \delta_S = \frac{1}{m}$$

General depth dependence and surface-wave velocity

- To satisfy the wave equations for any k and ω, m and n must equal (*show this*):

$$m = \sqrt{k^2 - \frac{\omega^2}{V_P^2}}$$

P-wave component in Rayleigh wave

$$n = \sqrt{k^2 - \frac{\omega^2}{V_S^2}}$$

SV-wave component

  - note that therefore, for *any* surface wave:

$$k > \frac{\omega}{V_S} > \frac{\omega}{V_P}$$

and so

$$V_{\text{Surface wave}} = \frac{\omega}{k} < V_S$$

- To further describe the solution, we need to:
  1) consider ω and A as free variables;
  2) determine B and $k(\omega)$ from the *boundary conditions*.

"Dispersion relation"

Example: Rayleigh wave
("ground roll",
vertically-polarized surface wave)

- *Rayleigh waves* propagate along the *free surface*
- The displacements are as usual:

$$\mathbf{u}_P(x,z)=\left(\frac{\partial\phi}{\partial x},\quad 0,\quad \frac{\partial\phi}{\partial z}\right)$$

P-wave

$$\mathbf{u}_S(x,z)=\left(\frac{-\partial\psi}{\partial z},\quad 0,\quad \frac{\partial\psi}{\partial x}\right)$$

SV-wave

- and traction:

$$\mathbf{F}_P(x,z)=\left(2\mu\frac{\partial^2\phi}{\partial x\partial z},\quad 0,\quad \lambda\nabla^2\phi+2\mu\frac{\partial^2\phi}{\partial z^2}\right)$$

$$\mathbf{F}_S(x,z)=\left(\mu\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right),\quad 0,\quad 2\mu\frac{\partial^2\psi}{\partial x\partial z}\right)$$

- For a free surface, the boundary conditions read: $\sigma_{xz}=\sigma_{zz}=0$,
- Let us look for solution in the form:

$$\phi=e^{-mz}e^{i(kx-\omega t)}$$

$$\psi_V=Be^{-nz}e^{i(kx-\omega t)}$$

we can set $A = 1$ and seek B and $k(\omega)$

Rayleigh wave
("ground roll" on top of a uniform half space)

- Result (for Poisson's ratio $\nu = 0.25$): relative *P*- and *S*-wave amplitudes:

$$\phi = e^{-0.848kz} e^{i(kx-\omega t)}$$ *P*-wave

$$\psi_V = 1.468ie^{-0.393z} e^{i(kx-\omega t)}$$ *SV*-wave

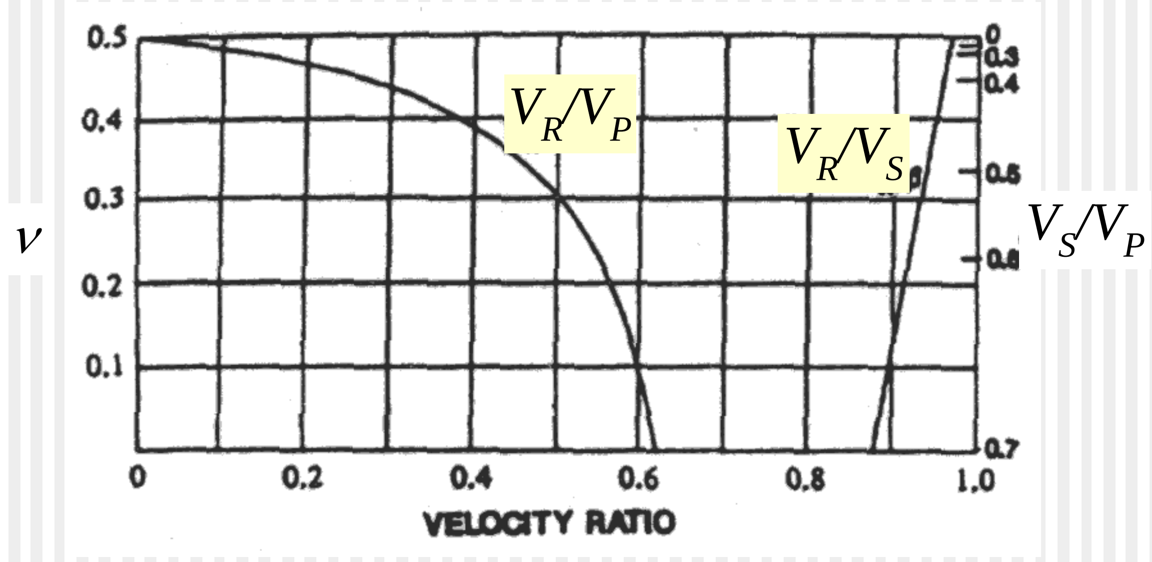
- ...and "dispersion relation" $k(\omega)$:

$$k = V_R \omega$$

- Rayleigh wave velocity is frequency-independent:

$$V_R = 0.919 V_S$$ **(This means *no* dispersion!)**

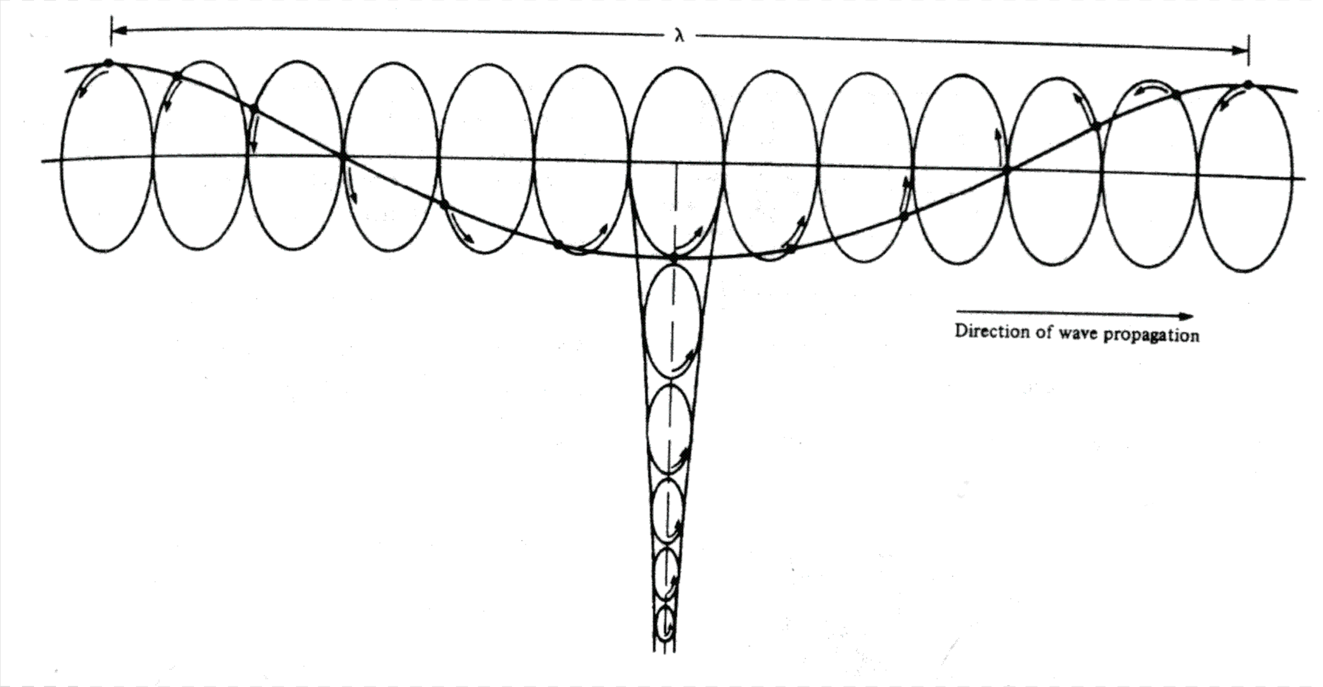
- For varying σ, scaled Rayleigh-wave velocities and V_S/V_P look like this:



Rayleigh wave
("ground roll")

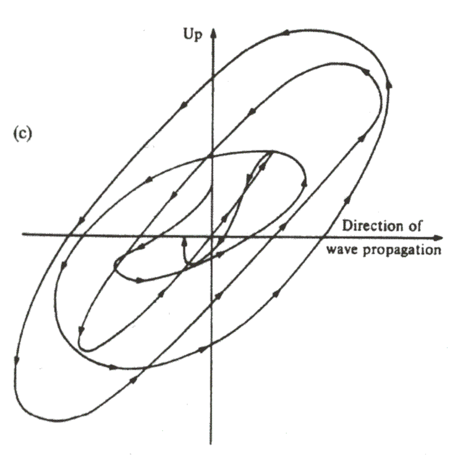
How does it follow from the equations for potentials and displacements?

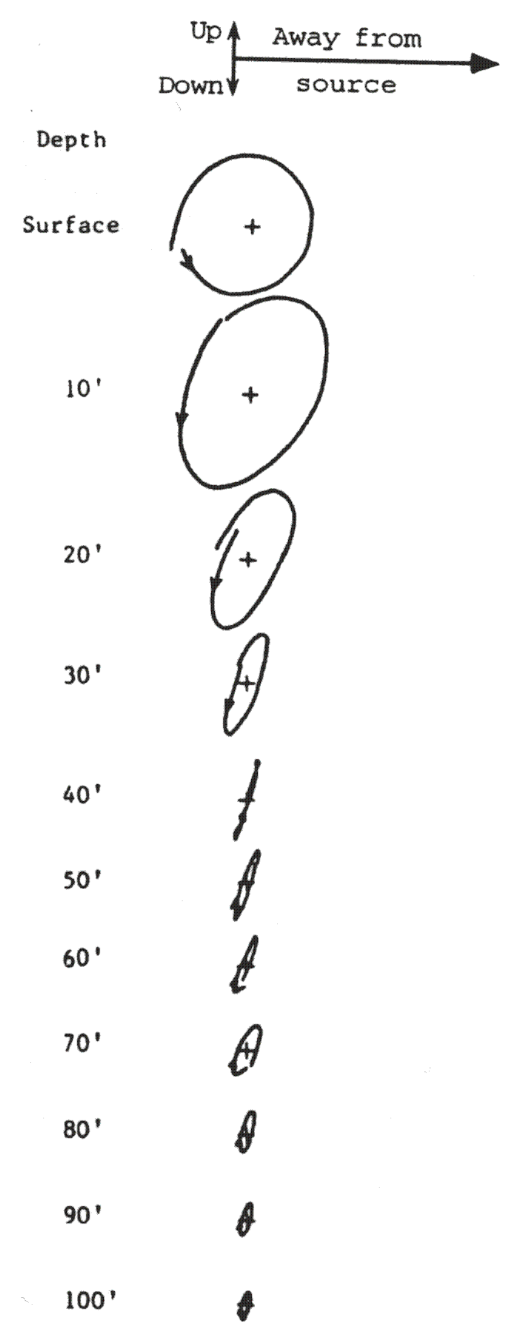
- Particle motion is elliptical and *retrograde* (counter-clockwise when the wave is moving left to right, like a wheel of a vehicle spinning backward):



Rayleigh waves
(real "ground roll")

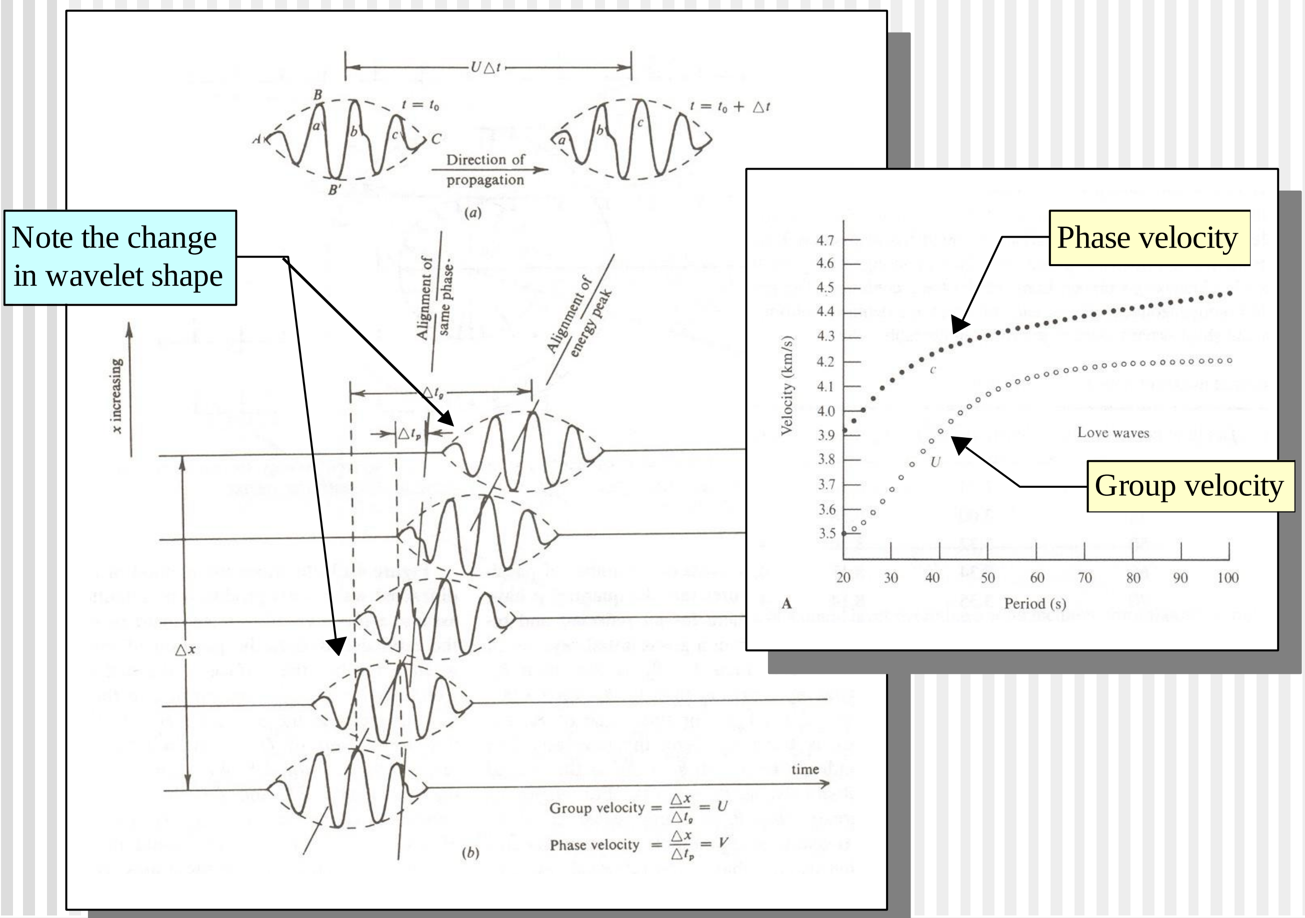
- Real Earth is never a *uniform* half-space, and thus in Rayleigh waves:
  - Particle motion paths are tilted and complex;
  - Retrograde motion may change into prograde at some depth;
  - Normal dispersion *is* present.





Rayleigh-wave dispersion

- Ideal Rayleigh wave (in a uniform half-space) is non-dispersive (wave velocity is the same at all frequencies, and therefore wave shape remains the same during propagation)
- However, all <u>*real*</u> surface waves exhibit dispersion
  - It is because the subsurface is always layered

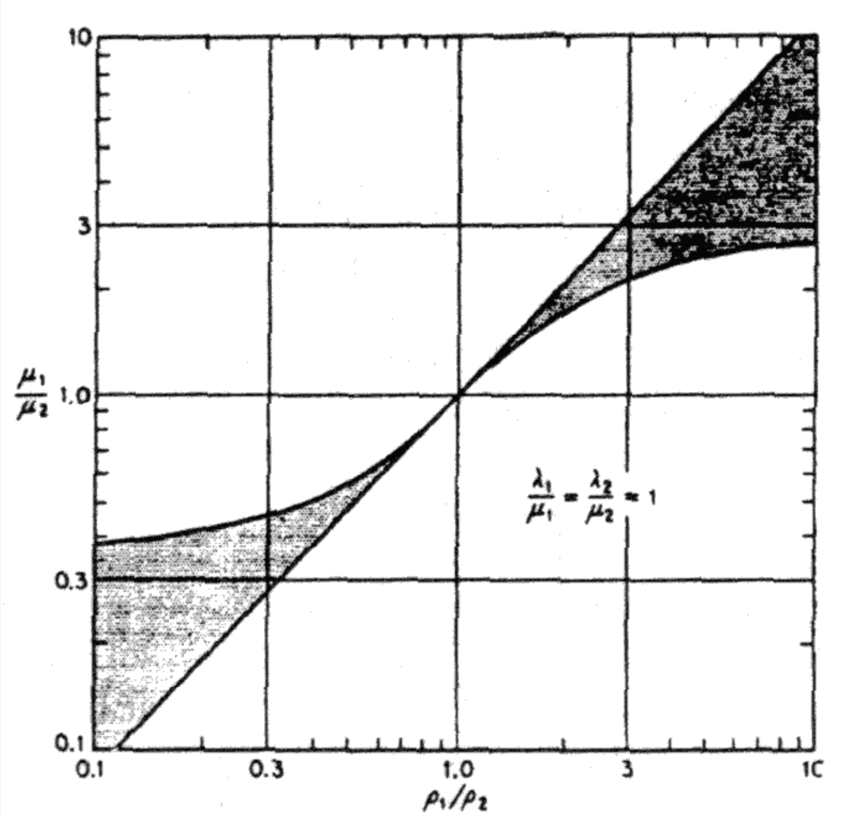


Stoneley waves

- These waves propagate along the contact of *two* semi-infinite media
  - They are *P*/*SV* in nature, like Rayleigh waves;
  - They always exist when one of the media is a fluid;
    - An important example is the *tube wave* propagating along a fluid-filled borehole
    - The surface of the borehole serves as the free surface for the Rayleigh wave above. Wave amplitude exponentially decreases radially
  - If both media are solids, Stoneley waves exist only when $V_{S1} \approx V_{S2}$ and ρ and μ lie within narrow limits *(plot on next page)*

Stoneley waves

- Parameter combinations (gray shading) for which the Stoneley waves exist near the boundary of two solids



Love waves

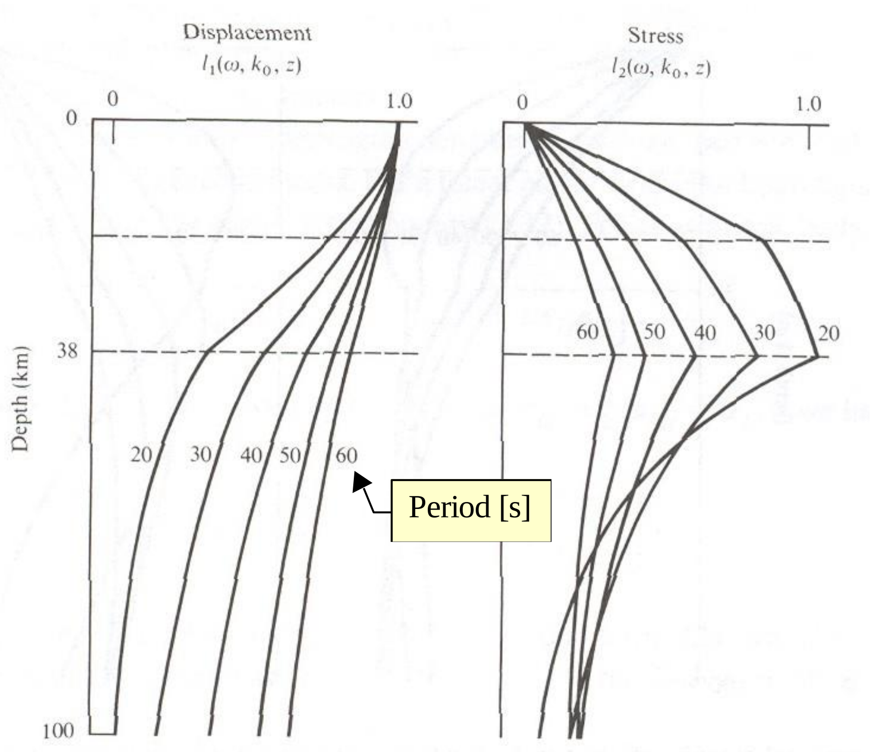
- These are *SH*-type waves propagating along the free surface
  - particle motion is transverse and parallel to the surface;
- Because there is just one *SH* potential, *two* modes are required to satisfy the two boundary conditions ($\sigma_{xz} = \sigma_{zz} = 0$ on the free surface)
- Thus, Love waves exist when the semi-infinite medium is overlain with a layer with different elastic properties.
  - … this situation is quite common.

Love-wave dispersion

- Love waves are dispersive:
- At high frequencies, its velocity approaches the S-wave velocity in the surface layer
- At low frequencies, velocity is close to that of the lower layer

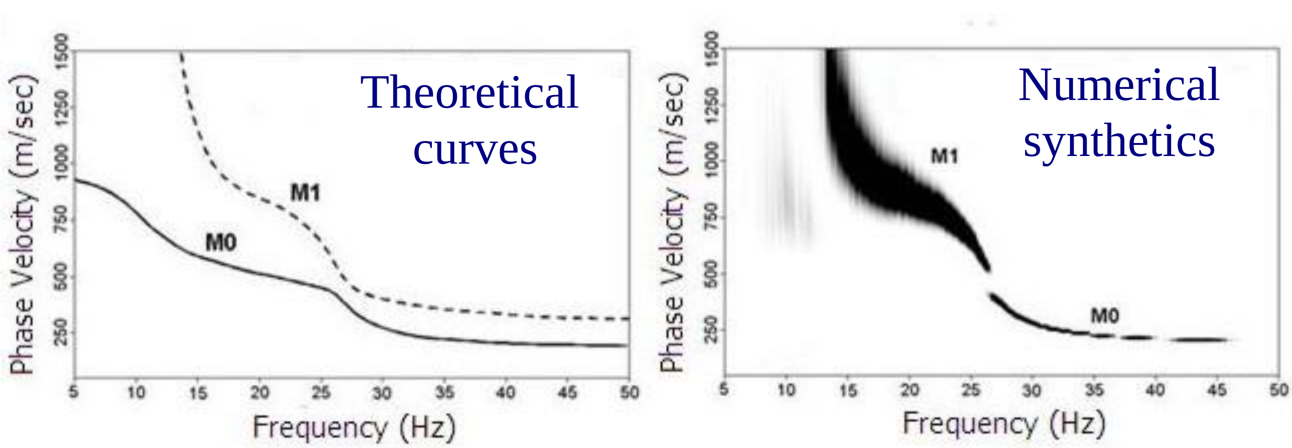
Depth of sampling increases with period. This is common to all surface waves.

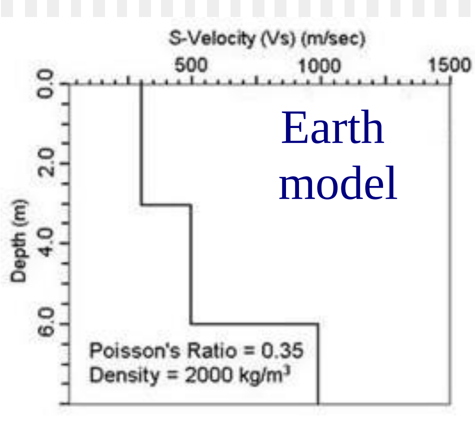
Displacement and stress in a Love wave



Surface-wave modes ("branches")

- In layered structures, multiple surface-wave branches, or "modes" exist for the same frequency
- The branch with the lowest phase velocity (longest wavelength) is called the fundamental mode





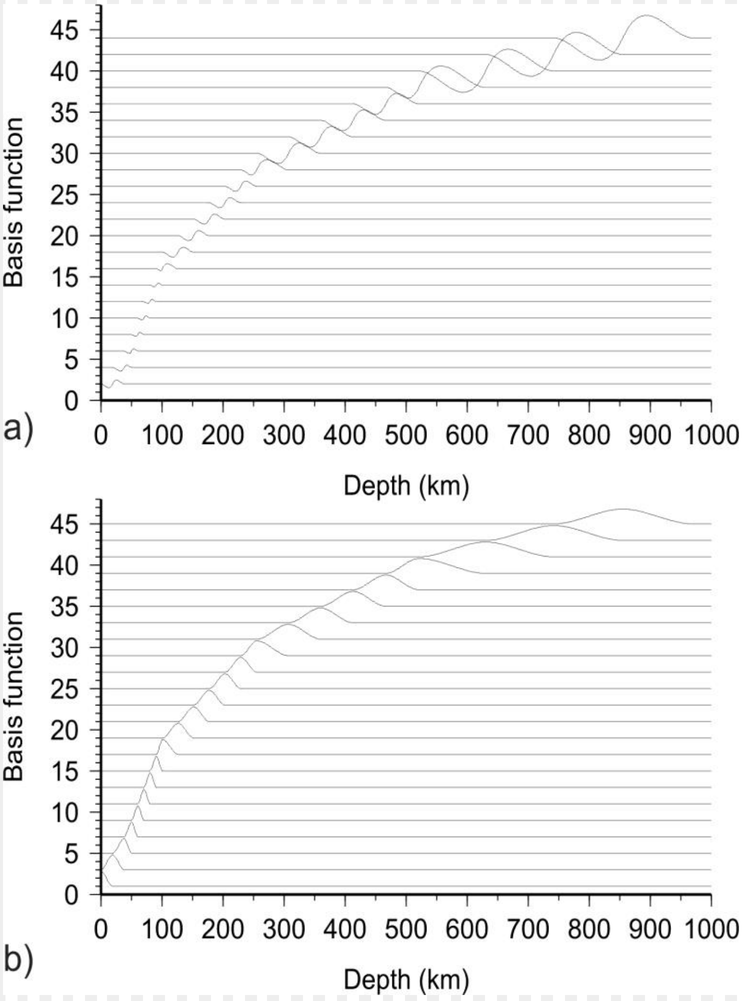
Surface-wave modes (theory)

- This is a brief explanation of what is done in the surface-wave code of your Lab 3
- To see that multiple wave modes exist and determine their parameters, the following matrix method can be used:

1) Chose a set of N basis functions in depth, $f_i(z)$, and express the potential (or displacement) through them:

$$\psi\left(x,z,t\right)=e^{i\left(\omega t-kx\right)}\sum_{i=1}^{N}c_i f_i\left(z\right)$$

Example of basis functions for Love waves within whole Earth



Surface-wave modes
(theory, cont.)

2) For a given k, express the total kinetic and potential energies; they will be quadratic matrix products of c_i:

$$E_{kin} = \int \left(\frac{\rho}{2} \dot{u}_i \dot{u}_i \right) dz = \omega^2 c_i A_{ij} c_j$$

Note that E_{kin} is always proportional to ω^2

$$E_{el} = \int \left(\frac{\lambda}{2} \varepsilon_{ii} \varepsilon_{jj} + \mu \varepsilon_{ij} \varepsilon_{ij} \right) dz = c_i B_{ij} c_j$$

2) Recall that in a wave, $E_{kin} = E_{el}$ (this is one of manifestations of "energy equipartitioning") and therefore:

$$\omega^2 c_i A_{ij} c_j = c_i B_{ij} c_j$$

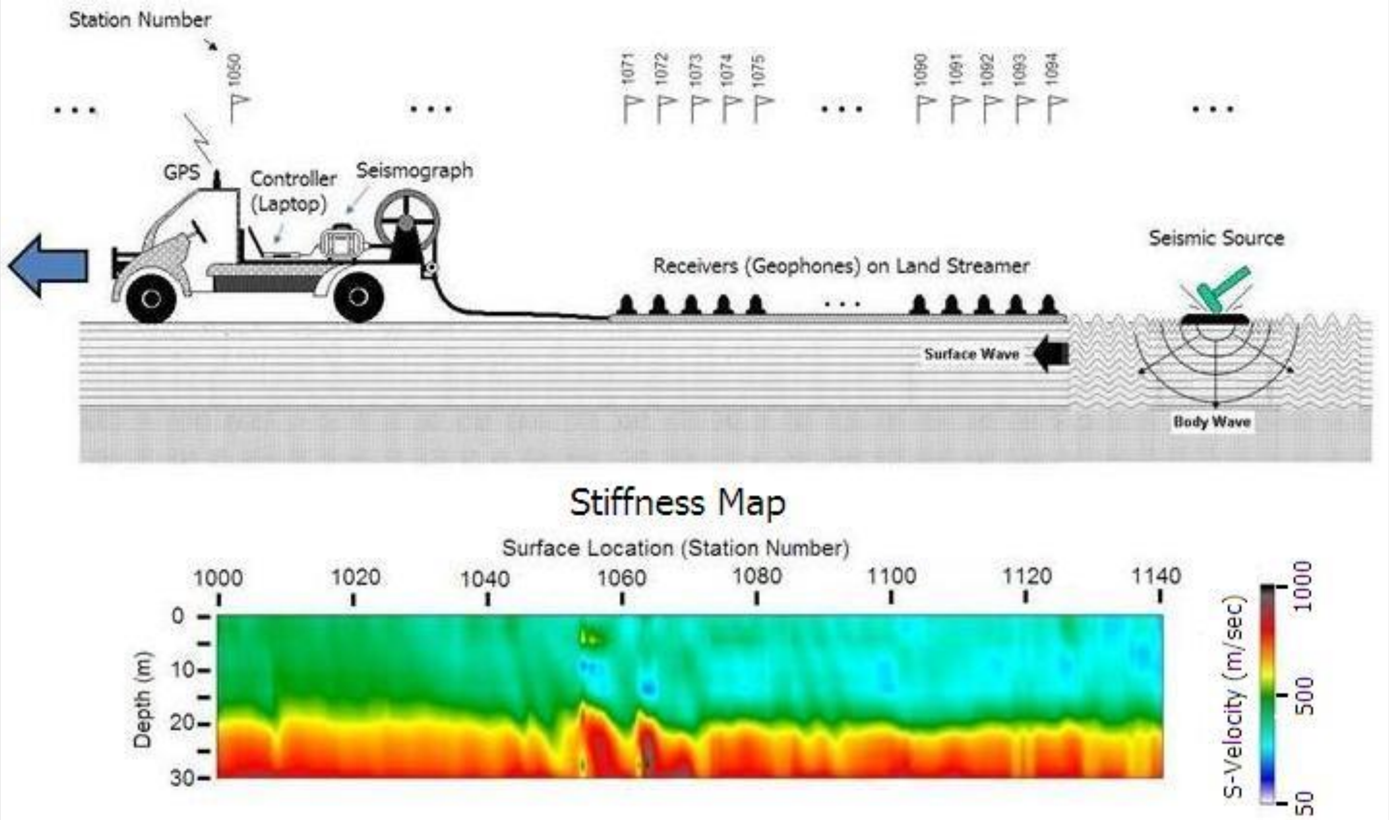
2) This means that c_i is an eigenvector of matrix $\mathbf{A}^{-1}\mathbf{B}$, and ω^2 is the corresponding eigenvalue:

$$\left(\mathbf{A}^{-1}\mathbf{B} - \omega^2 \mathbf{I} \right) \mathbf{c} = 0$$

There may be up to N positive eigenvalues. They give frequencies of up to N modes.

MASW (SASW) method

- Multichannel (or Spectral) Analysis of Surface Waves
- Uses dispersion $V(f)$ measurements to invert for $V_S(z)$ and $\mu(z)$
  - Geotechnical applications



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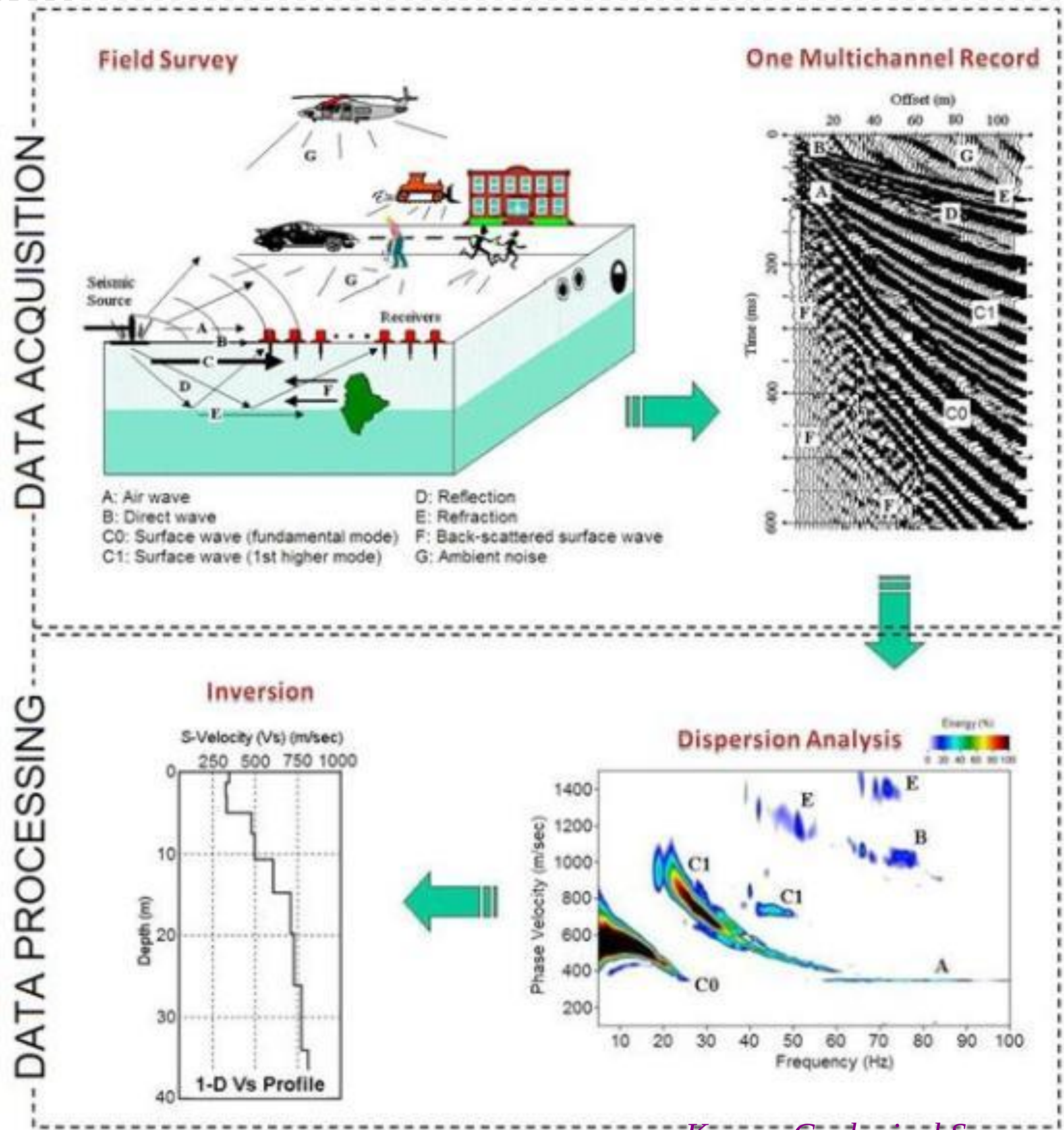
MASW method



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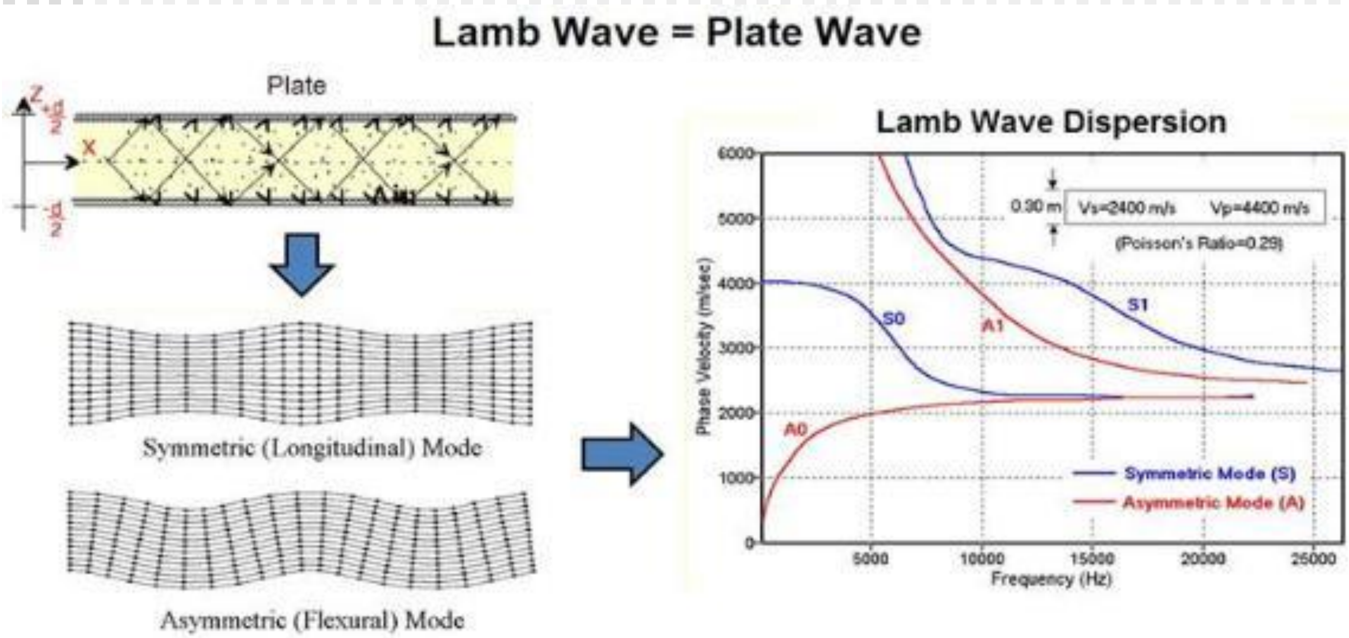
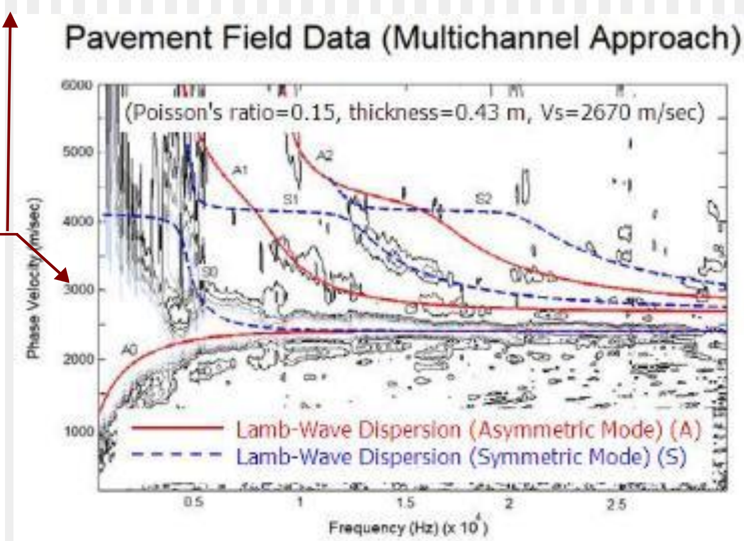
Lamb (plate) wave
(*e.g.*, in road pavement)

- The case of surface waves in a thin elastic plate is called the *Lamb's* problem



Note the difference
In dispersion curves
for symmetric
and asymmetric
deformations
of the plate



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