Geometrical Seismics
Refraction

In this and the following lectures, we briefly review some material from Geol335, some key points from the preceding lectures, and add some additional points related to

acquisition and interpretation of seismic records

The new material is highlighted like this: New!

Outline of this lecture:

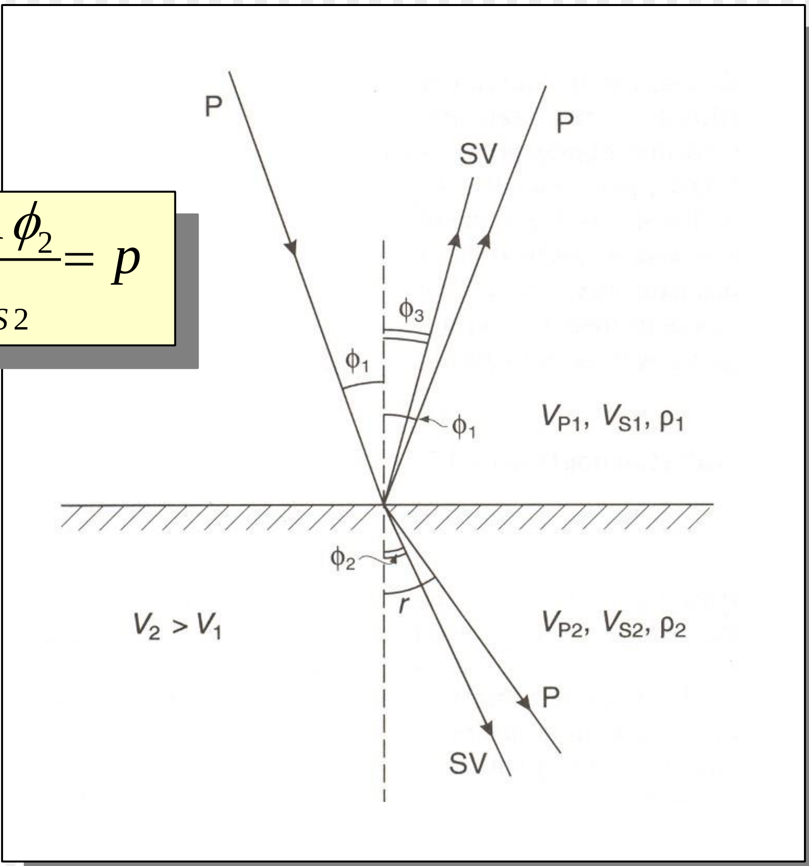
- Refraction basics
  - Head waves
  - Diving waves
- Effects of vertical velocity gradients

- Reading:
  - Sheriff and Geldart, Chapter 4.2 - 4.3.

Snell's Law of Refraction

- When waves (rays) penetrate a medium with different velocity, they *refract*, i.e. bend toward or away from the normal to the velocity boundary.
- The *Snell's Law of refraction* relates the angles of incidence and emergence of waves refracted on a velocity contrast:

$$\frac{\sin\phi_1}{V_{P1}}=\frac{\sin\phi_3}{V_{S1}}=\frac{\sin r}{V_{P2}}=\frac{\sin\phi_2}{V_{S2}}=p$$



- The constant p is called *ray parameter* (projection of the slowness vector on the boundary)
- Note that refraction angles depend on the velocities alone!

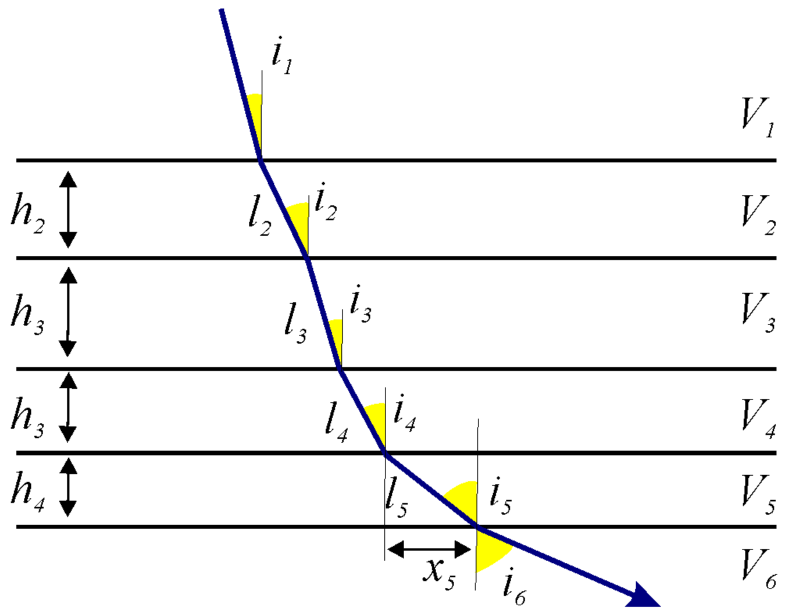
Refraction in a stack of horizontal layers

Ray parameter, *p*, *uniquely specifies* the entire ray.

It does not depend on layer thicknesses or velocities.

Travel times and distances accumulate along the ray to yield the total $T(x)$

$$T_n = \sum_{k=1}^{n} t_k \qquad X_n = \sum_{k=1}^{n} x_k$$



For any layer: $\sin i_k = pV_k$

$$l_k = \frac{h_k}{\cos i_k} = \frac{h_k}{\sqrt{1-(pV_k)^2}}$$

$$t_k = \frac{l_k}{V_k} = \frac{h_k}{V_k\sqrt{1-(pV_k)^2}}$$

$$x_k = l_k \sin i_k = \frac{h_k(pV_k)}{\sqrt{1-(pV_k)^2}}$$

Critical Angle of Refraction

- Consider a faster medium overlain with a lower-velocity layer (this is a typical case).
- *Critical angle* of incidence in the slower layer is such that the refracted waves (rays) travel horizontally in the faster layer ($\sin r = 1$)
- The critical angles thus are:

$$i_C = \sin^{-1} \frac{V_{P_1}}{V_{P_2}} \quad \textit{for P-waves,}$$

$$i_C = \sin^{-1} \frac{V_{S_1}}{V_{S_2}} \quad \textit{for S-waves.}$$

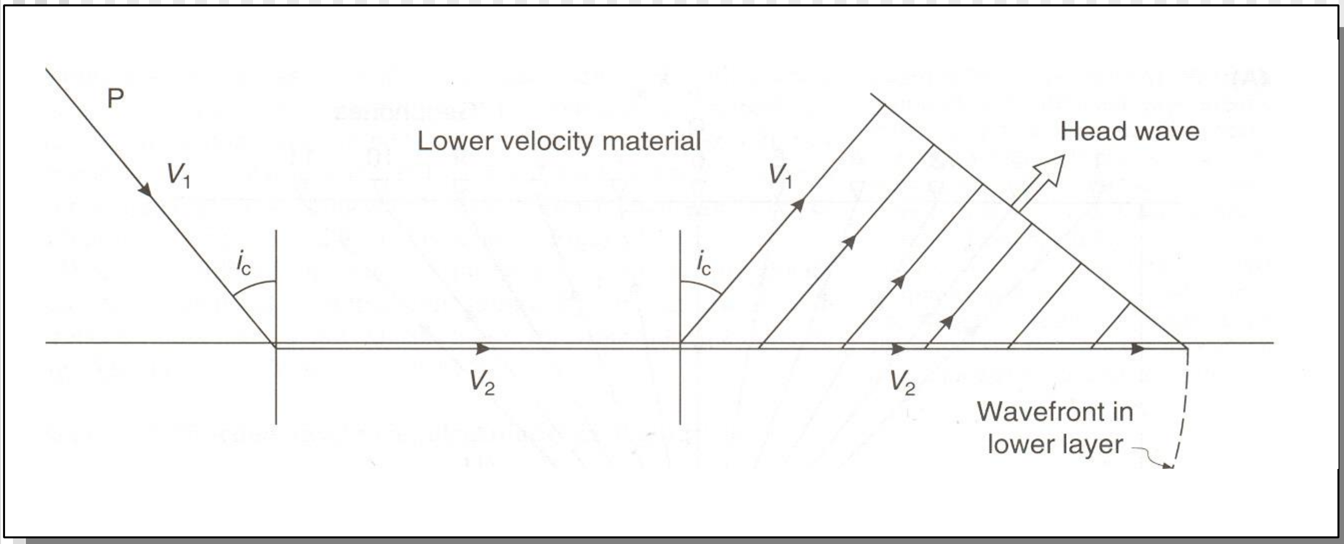
- Critical *ray parameter*: $p^{critical} = \dfrac{1}{V_{refractor}}$
- If the incident wave strikes the interface at an angle exceeding the critical angle, *no refracted or head wave is generated*
- Note that i_C should better be viewed as a *property of the interface*, not of a particular ray.

Head wave

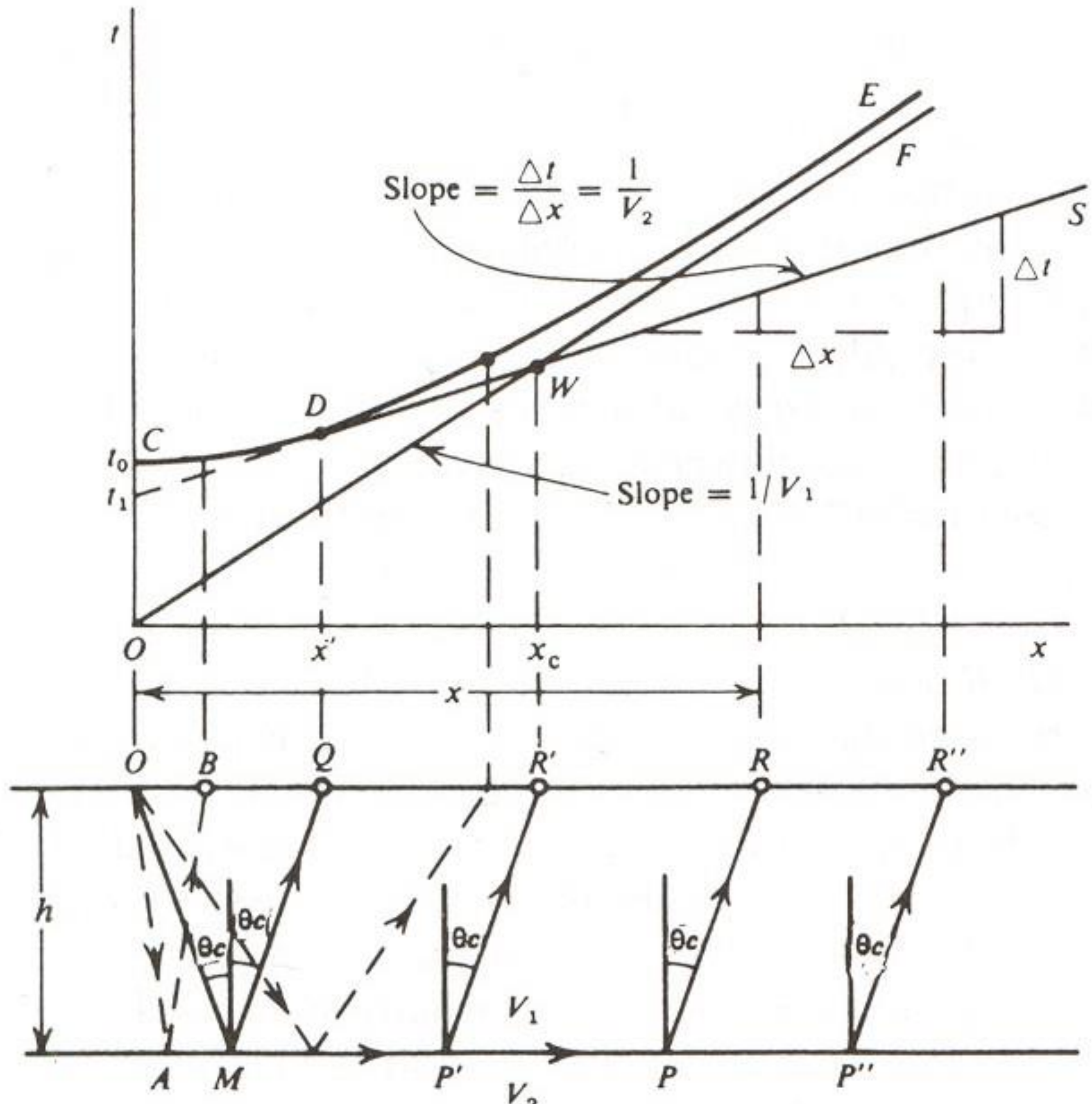
- At critical incidence in the upper medium, a *head wave* is generated in the lower one.
- Although head waves carry very little energy, they are useful approximation for interpreting seismic wave propagation in the presence of strong velocity contrasts.
- Head waves are characterized by planar wavefronts inclined at the critical angle in respect to the velocity boundary. Their travel-time curves are straight lines:

$$t = t_0 + \frac{x}{V_{app}}$$

Here, t_0 is the *intercept time*, and V_{app} is the *apparent velocity*.



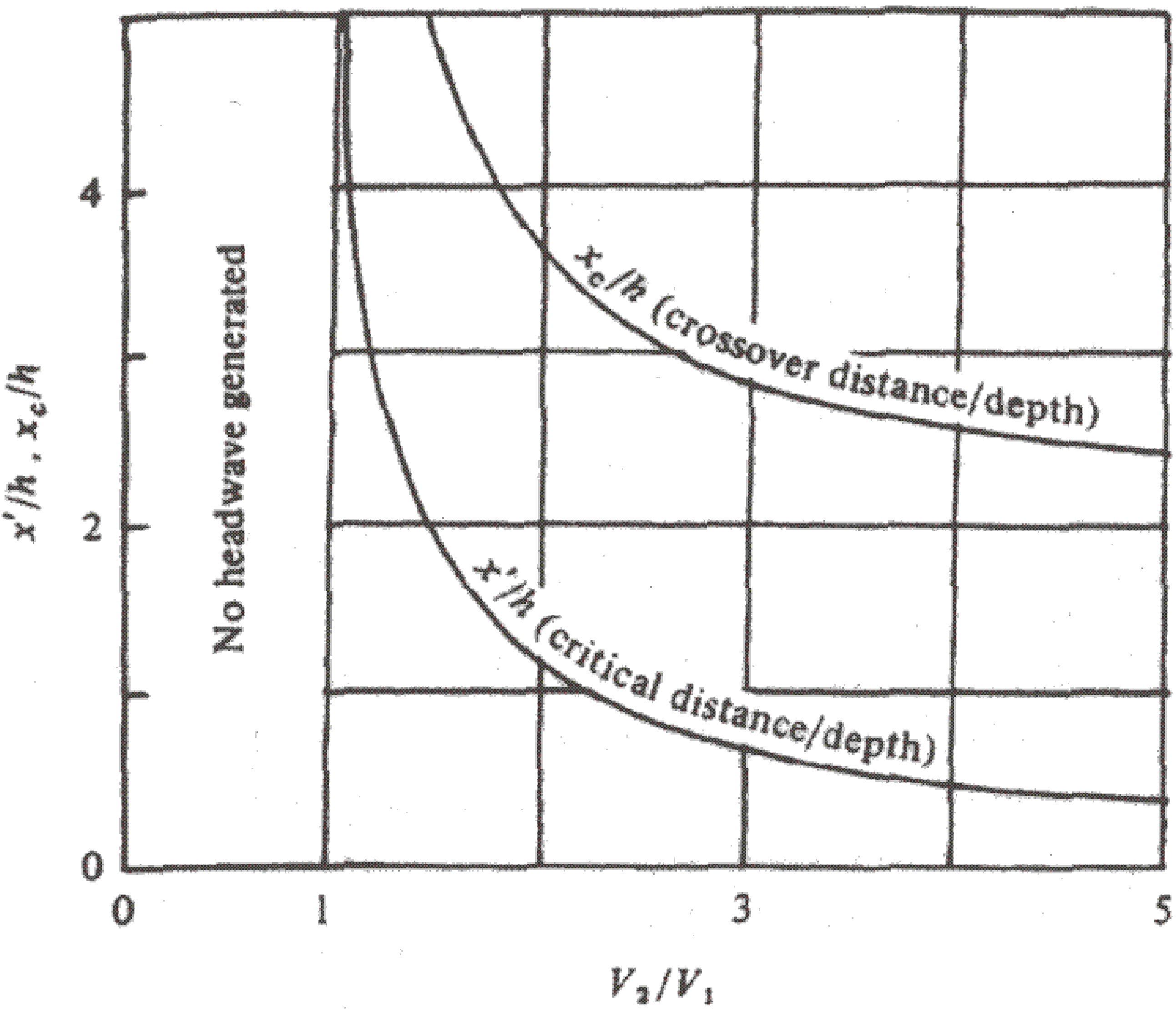
Relation between reflection- and refraction travel-times



From Sheriff and Geldart, 1995

New!

Critical and Cross-over distances vs. velocity contrast



- Note that the distances are *proportional* to the depth and *decrease* with increasing velocity contrast across the interface

Travel times
(Horizontal refractor)

- Direct wave:

$$t(x) = \frac{x}{V_1}$$

- Head wave:

$$p = \frac{1}{V_2}$$

$$\sin i = pV_1 \qquad \cos i = \sqrt{1-(pV_1)^2}$$

$$t = 2\frac{h_1}{V_1 \cos i} + p(x - 2h_1 \tan i) = \frac{2h_1}{V_1 \cos i}(1 - pV_1 \sin i) + px = \frac{2h_1}{V_1}\cos i + px$$

$$t_0 = \frac{2h_1}{V_1}\cos i = \frac{2h_1}{V_1}\sqrt{1-(pV_1)^2}$$

this is also sin i

intercept time, t_0

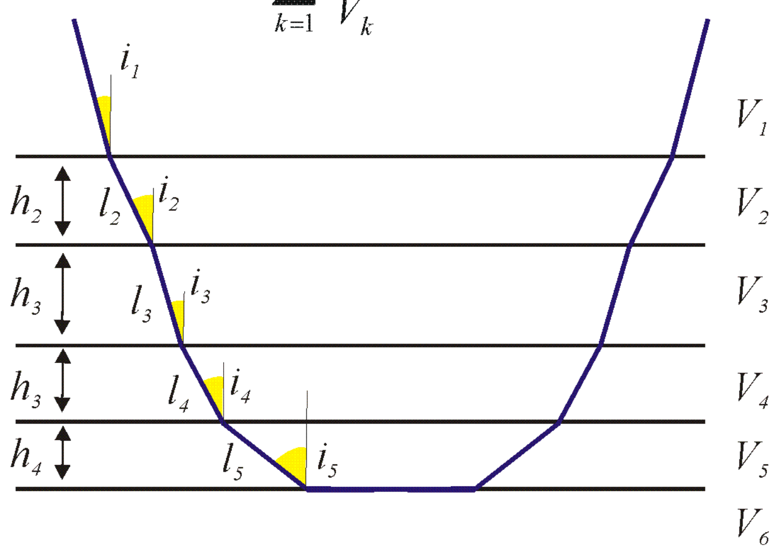
Travel times

(Multiple horizontal layers)

- Ray parameter p is the same for all layers and equals the *critical ray parameter* for the bottom (refracting) boundary;
- Intercept time t_0 is accumulated across all layers above the refraction level:

$$p = \frac{1}{V_{refractor}}$$

$$t = \sum_{k=1}^{n} \frac{2h_k}{V_k} \cos i_k + px$$



For any layer: $\sin i_k = pV_k$

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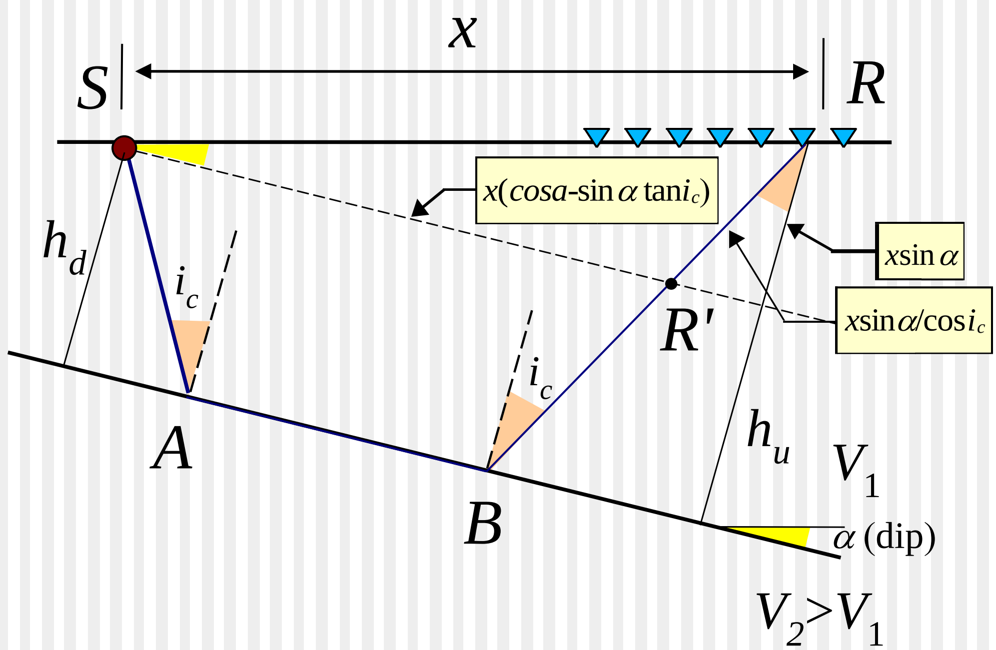
Travel times

(Dipping refractor)



$$t = \frac{2h_d}{V_1}\cos i_c + \frac{1}{V_2}x(\cos\alpha - \sin\alpha \tan i_c) + \frac{1}{V_1}\frac{x\sin\alpha}{\cos i_c}$$

$\sin i_c$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1 \cos i_c}\left[\frac{V_1}{V_2}(\cos\alpha \cos i_c - \sin\alpha \sin i_c) + \sin\alpha\right]$$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1 \cos i_c}\left[\cos\alpha \cos i_c \sin i_c + \sin\alpha \cos^2 i_c\right]$$

$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\left[\cos\alpha \sin i_c + \sin\alpha \cos i_c\right]$$

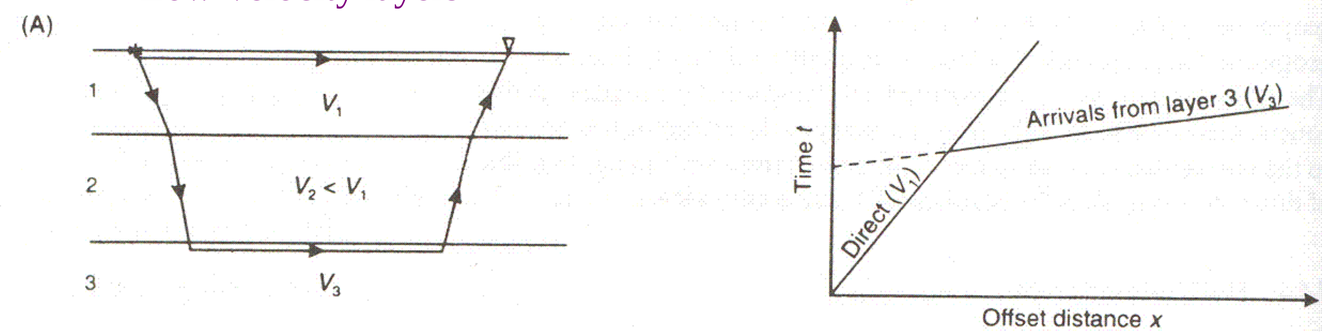
$$= \frac{2h_d}{V_1}\cos i_c + \frac{x}{V_1}\sin(i_c + \alpha)$$

would change to '-' for up-dip shooting

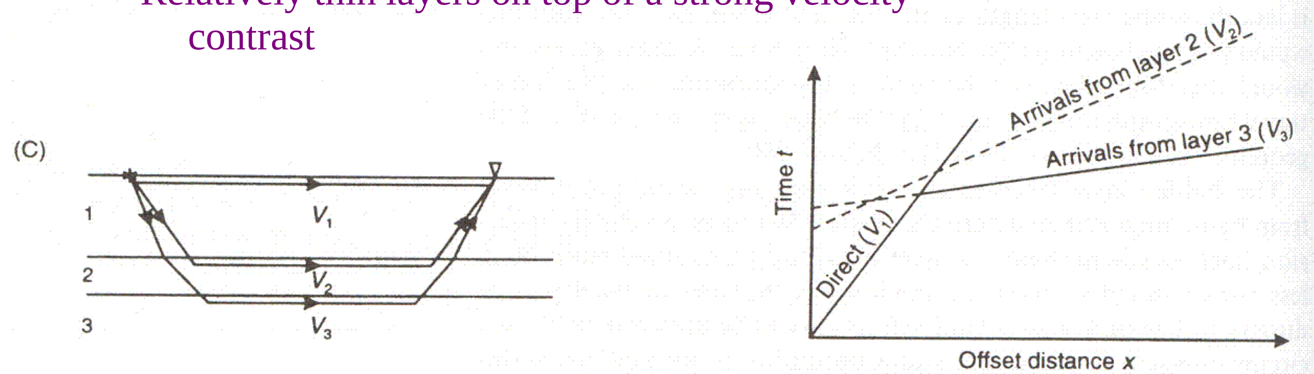
Hidden-Layer Problem

- Velocity contrasts *may not manifest themselves* in refraction (first-arrival) travel times. Recall the three typical cases:

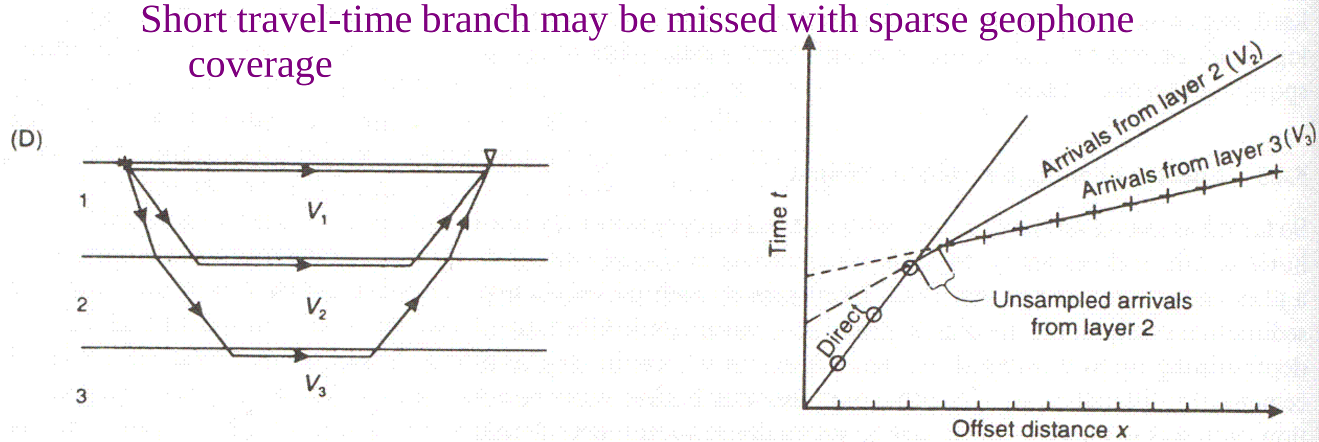
Low-velocity layers



Relatively thin layers on top of a strong velocity contrast

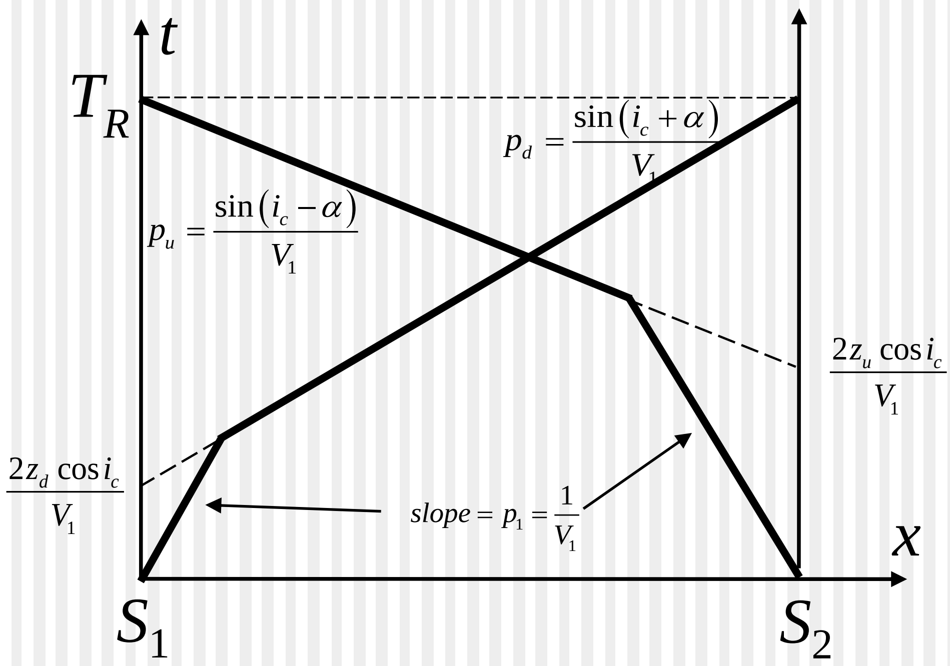


Short travel-time branch may be missed with sparse geophone coverage



Reversed travel times

- One needs *reversed* recording (in opposite directions) for resolution of dips.
- The *reciprocal times*, T_R, must be equal for reversed shots.
- Dipping refractor is indicated by:
  - Different *apparent velocities* ($=1/p$, TTC slopes) in the two directions;
    - determine V_2 and α (refractor velocity and dip).
  - Different *intercept times*.
    - determine h_d and h_u (interface depths).



Determination of refractor velocity and dip

- *Apparent velocity* is $V_{\text{app}} = 1/p$, where p is the *ray parameter* (*i.e.*, slope of the travel-time curve).
  - Apparent velocities are measured directly from the observed TTCs;
  - $V_{\text{app}} = V_{\text{refractor}}$ only for horizontal layering.
  - For a dipping refractor:
    - Down dip: $V_d = \dfrac{V_1}{\sin(i_c + \alpha)}$ (*slower than* V_1);
    - Up-dip: $V_u = \dfrac{V_1}{\sin(i_c - \alpha)}$ (*faster*).
- From the two reversed apparent velocities, i_c and α are determined:

$$i_c + \alpha = \sin^{-1}\frac{V_1}{V_d}, \qquad i_c - \alpha = \sin^{-1}\frac{V_1}{V_u}. \Rightarrow$$

$$i_c = \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} + \sin^{-1}\frac{V_1}{V_u}\right)$$
$$\alpha = \frac{1}{2}\left(\sin^{-1}\frac{V_1}{V_d} - \sin^{-1}\frac{V_1}{V_u}\right)$$

- From i_c, the refractor velocity is:

$$V_2 = \frac{V_1}{\sin i_c}$$

New!

Approximation of small refractor dip

- If refractor dip is small:

$$\frac{V_1}{V_d} = \sin(i_c + \alpha) \approx \sin i_c + \alpha \cos i_c$$

$$\frac{V_1}{V_u} = \sin(i_c - \alpha) \approx \sin i_c - \alpha \cos i_c$$

and therefore:

$$\sin i_c \approx \frac{V_1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right)$$

- and:

$$\frac{1}{V_2} \approx \frac{1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right)$$

- Thus, the *slowness of the refractor* is approximately the mean of the up-dip and down-dip *apparent slownesses*.

New!

Diving waves

- Consider velocity gradually increasing with depth: $V(z)$.
- Rays will bend upward at any point and eventually will return to the surface
  - Such waves are called *diving waves*.
- An *implicit* solution for the travel-time curve (x,t) can be obtained from the multiple-layer refraction formulas. In this implicit form, both x and p are given as functions of parameter p of the diving ray:

$$x(p) = 2\int_0^{h_{\max}} \frac{pV(z)\,dz}{\sqrt{1-\left(pV(z)\right)^2}},$$

$$t(p) = 2\int_0^{h_{\max}} \frac{dz}{V(z)\sqrt{1-\left(pV(z)\right)^2}}$$

where h_{max} is the depth at which $pV(h_{max}) = 1$. At this depth, the ray at ray parameter p becomes horizontal and turns back to the surface.

New!

Diving waves

Linear increase of velocity with depth

- Consider the case of: $V(z) = V_0 + az$

  a is generally between 0.3-1.3 1/s.
- Hence, denoting $u = pV = \sin i$:

Parametric representation of the (x,z,t) through u

$$x(u) = \int_{z_0}^{z} \frac{pVdz}{\sqrt{1-(pV)^2}} = \frac{1}{pa}\int_{u_0}^{u} \frac{udu}{\sqrt{1-u^2}} =$$

$$= \frac{1}{pa}\left(\sqrt{1-u^2} - \sqrt{1-u_0^2}\right) \equiv \frac{1}{pa}\sqrt{1-u^2} + x_c$$

$$z(u) = \frac{1}{pa}(u-u_0) = \frac{1}{pa}u + z_c$$

Denote the constants (centre of the circular ray path)

- The raypath is a circular *arc*:

$$(x-x_c)^2 + (z-z_c)^2 = \left(\frac{1}{pa}\right)^2$$

- and its time:

$$t(p) = \int_{z_0}^{z} \frac{dz}{V\sqrt{1-(pV)^2}} = \frac{1}{a}\int_{0}^{h_{max}} \frac{du}{u\sqrt{1-u^2}} =$$

$$= \frac{1}{a}\ln\left[\frac{u}{1-\sqrt{1-u^2}}\right]$$

New!

Diving waves

Layers with low velocities and high velocity gradients create complex travel-time curves