Wave Attenuation and Dispersion

- Mechanisms:
  - *Absorption* (anelastic)
  - *Scattering* (elastic)
- *P*- and *S*-wave, bulk and shear attenuation
- Mathematical descriptions
- Measurement
- Frequency dependence
- Velocity dispersion, its relation to attenuation

- Reading:
  - Shearer, 6.2, 6.6
  - Sheriff and Geldart, Sections 2.7, 6.5

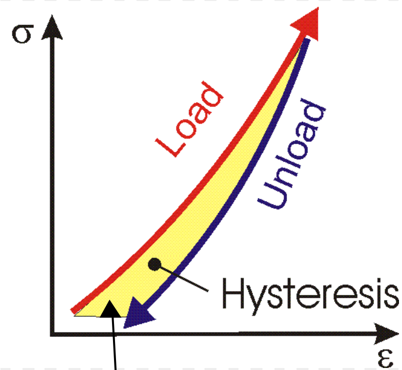
Mechanisms of attenuation

- Three processes lead to reduction of elastic amplitude as the wave propagate away from the source:
  - *Geometrical spreading* - total mechanical energy is conserved but distributed over larger wavefronts
    - In fact, not so easy to define mathematically
  - *Scattering* (elastic attenuation) – mechanical energy is scattered out of the seismic phase of interest
    - In practice, this phenomenon can be hard to differentiate from geometrical spreading
  - *Anelastic* (intrinsic) attenuation, or *absorption* – elastic energy is converted to heat
    - Frequency dependence is viewed as the key distinction between these mechanisms

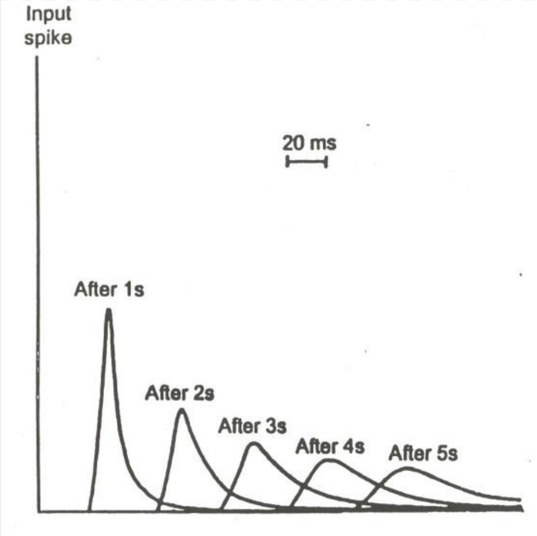
Absorption

- When an elastic wave travels through any medium, its *mechanical energy* is progressively converted into *heat* (through friction and viscosity)
  - On grain boundaries, pores, cracks, pore water, gas, *etc*.
  - Loss of elastic energy causes the amplitude to *decrease* and the pulse to *broaden:*

Mechanical-energy absorption (and release of heat) occurs during each period of cyclic loading and unloading of the material:



Area of the hysteresis curve is a measure of absorption



Scattering

- Wavelength-dependent;
- *Scattering regime* is controlled by the ratio of the *characteristic scale length* of the *heterogeneity* of the medium, a, to the wavelength.
- Described in terms of *wavenumber*, $k = 2\pi/wavelength$:
  - $ka << 0.01$ (quasi-homogeneous medium) - no significant scattering;
  - $ka < 0.1$ (*Rayleigh scattering*) - produces *apparent Q* and anisotropy;
  - $0.1 < ka < 10$ (*Mie scattering*) - introduces strong attenuation and discernible scattering noise in the signal.
    - typical for high-resolution seismic studies (boulder clay with 0.5-1 m boulders, $V_P \approx 2000$ m/s, $f \approx 500$ Hz

Quality Factor, *Q*

- Attenuation is measured in terms of *quality factor*, *Q:*
  - The logarithmic decrement of amplitude α is generally proportional to frequency

$$A(t) = A(0)e^{-\alpha x} = A(0)e^{\frac{-\pi ft}{Q}}$$

$\alpha \propto f$

$x=Vt$

Therefore, *Q* here is approximately frequency-independent

  - Amplitude and energy loss per cycle (wavelength):

$$\ln\left(\frac{A(t+T)}{A(t)}\right) = \frac{-\pi fT}{Q} = \frac{-\pi}{Q}$$

This value, in *dB*, is also often used to characterize attenuation

$$\ln\left(\frac{E(t+T)}{E(t)}\right) = \ln\left(\frac{E(t)-\delta E}{E(t)}\right) = \frac{-\delta E}{E(t)} = \frac{-2\pi}{Q}$$

  - Thus, *Q* measures the relative mechanical-energy loss per cycle:

$$Q = 2\pi\frac{E}{\delta E}$$

- Typical values:
  - $Q \approx 30$ for weathered sedimentary rocks;
  - $Q \approx 1000$ for granite.

Relations between Q_P and Q_S

- *P*- and *S*-waves have different *Q*-factors
- Q_P and Q_S are thought to be related to the quality factors associated with the *K* (bulk) and μ (shear) moduli of the medium:

$$Q_P^{-1} = LQ_\mu^{-1} + (1-L)Q_\kappa^{-1}$$

Bulk attenuation

Shear attenuation

$$Q_S^{-1} = Q_\mu^{-1}$$

where: $$L = \frac{4}{3}\left(\frac{V_S}{V_P}\right)^2$$

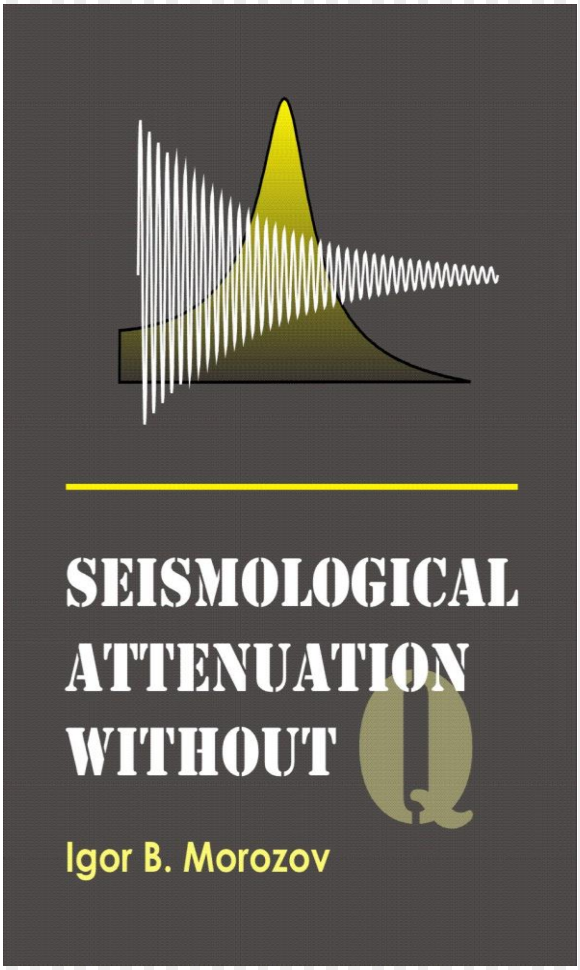
- Q_κ is usually very high (assumed infinite)
- Because the S- to P-wave velocity ratio $\frac{V_S}{V_P} \approx \frac{1}{\sqrt{3}} \cdots \frac{1}{2}$

the P- and S-wave attenuation factors are related as:

$$Q_P^{-1} \approx \left(\frac{1}{3} \cdots \frac{1}{2}\right) Q_S^{-1}$$

Note that P-wave attenuation is weaker, which is often seen in seismic records

This is not that simple though...



- *Q* is not really a property of the medium
- It is still a property of selected waves
  - Therefore, comparing different rocks in terms of their Q_P, Q_S is actually tricky and ambiguous

Typical values of Q_P

Table 6.1 *Absorption constants for rocks*

	Q	δ (dB) = $\eta\lambda$
Sedimentary rocks	20–200	0.16–0.02
Sandstone	70–130	0.04–0.02
Shale	20–70	0.16–0.05
Limestone	50–200	0.06–0.02
Chalk	135	0.02
Dolomite	190	0.02
Rocks with gas in pore space	5–50	0.63–0.06
Metamorphic rocks	200–400	0.02–0.01
Igneous rocks	75–300	0.04–0.01

- For sandstones with porosity ϕ% and clay content C %, at 1 MHz and 40 MPa:

$$Q_P = 179C^{-0.84\phi}$$

General model for *Q* measurement

- The following model of seismic amplitudes is commonly used in attenuation measurements:

$$A_{\mathrm{recorded}}(t,f) = \underbrace{A_{\mathrm{Source}}(f)}_{\text{Source spectrum}}\,\underbrace{G(t)}_{\text{Geometrical spreading}}\,\underbrace{A_{\mathrm{Receiver}}(f)}_{\text{Receiver response}}\,\underbrace{\exp\left(\frac{-\pi ft}{Q}\right)}_{\text{Attenuation}}$$

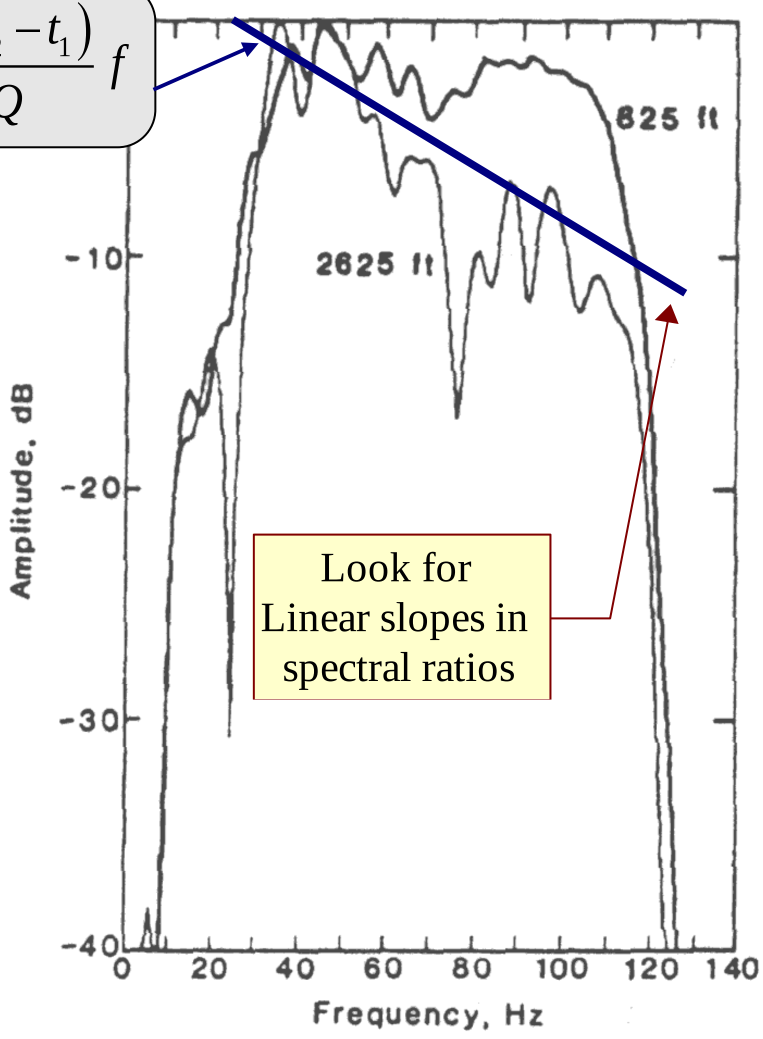
- Therefore, there are two basic approaches to measurement of *Q*:
  1) Model-based correction for geometrical spreading $G(t)$ and $A_{source}(f)$
  2) Using ratios of spectral amplitudes measured at different *t*.
     - The poorly known source and receiver factors cancel in these ratios, and $G(t)$ is unimportant

Spectral ratios

- Take logarithms of *spectral ratios* of seismic spectra measured at two propagation times
  - The signal in the two windows must be the same in all other respects

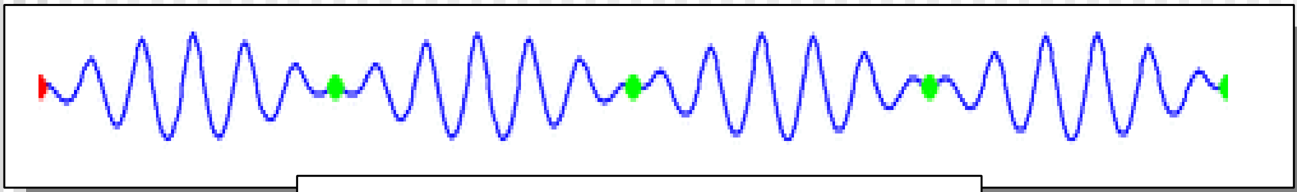
$$\ln\left(\frac{A(f,t_2)}{A(f,t_1)}\right)=\ln\frac{G(t_2)}{G(t_1)}-\frac{\pi(t_2-t_1)}{Q}f$$

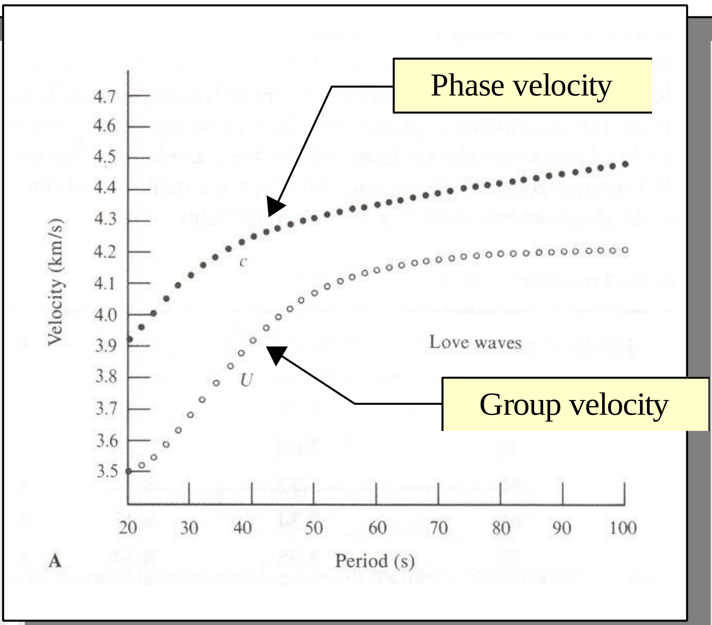
Ratio of spectral amplitudes does not depend on the source spectrum



Phase-Velocity dispersion

- When phase velocity is dependent on frequency, the wave is called *dispersive*. There are two manifestations of this phenomenon:
  - Waveforms change shapes and spread out ("disperse") when traveling
  - *Signal or Group velocity* (velocity of a wave packet or pulse, U) differs from phase velocity (V):
    - $U < V$ – "*Normal dispersion*";
    - $U > V$ – "*Inverse dispersion*".





Phase and Group velocities

- Consider a plane harmonic wave:

$$u(x,t) = Ae^{i\varphi(x,t)} = Ae^{i\left[k(\omega)x-\omega t\right]}$$

  where $k=\omega/V$ is the *wavenumber*.

  - Note that k is dependent on ω.

- *Phase velocity* is the velocity of propagation of the constant-phase plane ($\varphi(x,t)$=const):

$$\boxed{V_{phase} = \frac{\omega}{k}}$$

- *Group velocity* is the velocity of propagation of the amplitude peak in the wavelet
  - this is the point where the phase is *stationary* (independent of ω):

$$\frac{d\left[k(\omega)x-\omega t\right]}{d\omega} = \frac{dk(\omega)}{d\omega}x - t = 0$$

  - hence:

$$\boxed{U_{group} = \left(\frac{dk}{d\omega}\right)^{-1} = \frac{d\omega}{dk}}$$

Group velocity

- Example: two cosine waves with

$$\omega_1 = \omega_0 - \Delta\omega, \quad k_1 = k_0 - \Delta k$$
$$\omega_2 = \omega_0 + \Delta\omega, \quad k_2 = k_0 + \Delta k$$

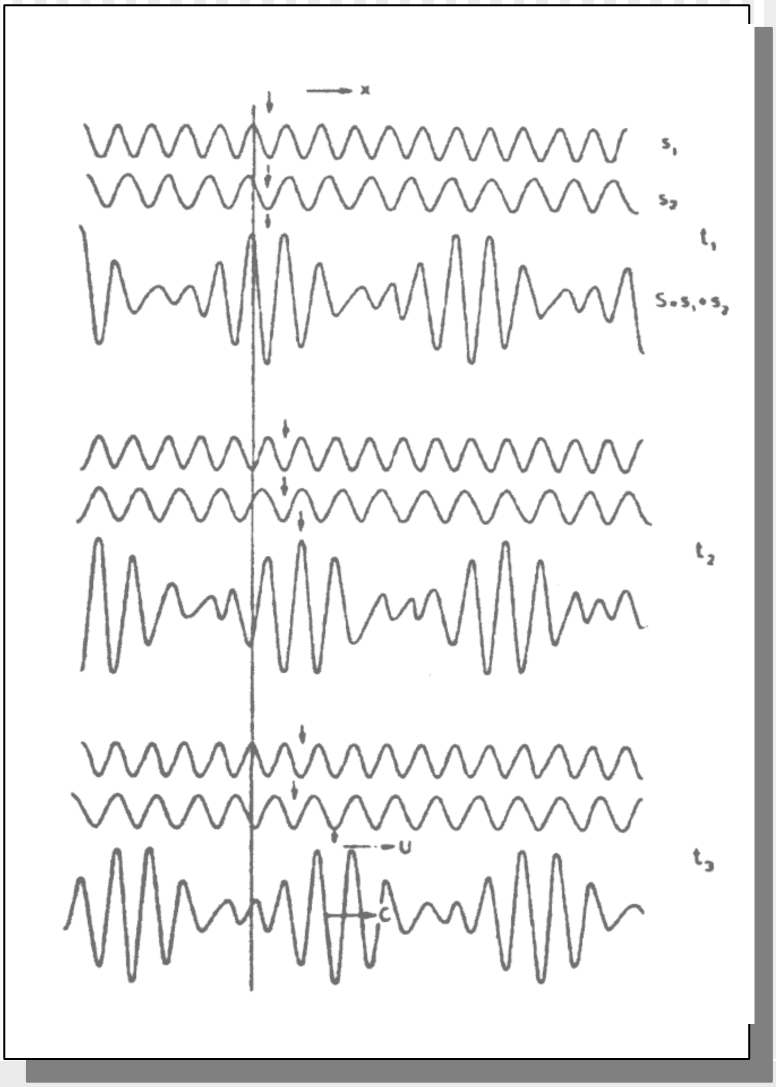
superimpose to form beats shown in the picture:

Show that the envelope of these beats travels with group velocity:

$$U = \frac{\Delta\omega}{\Delta k}$$

...while within the beats, peaks and troughs propagate at approximately:

$$V = \frac{\omega}{k}$$



Normal and Inverse dispersion

- When phase velocity is frequency-dependent, group velocity differs from it:

$$U = \frac{d\omega}{dk} = \frac{d(kV)}{dk} = V + k\frac{dV}{dk} = V - \lambda\frac{dV}{d\lambda} \approx V + \omega\frac{dV}{d\omega}$$

because $k = 2\pi/\lambda = \omega/V$

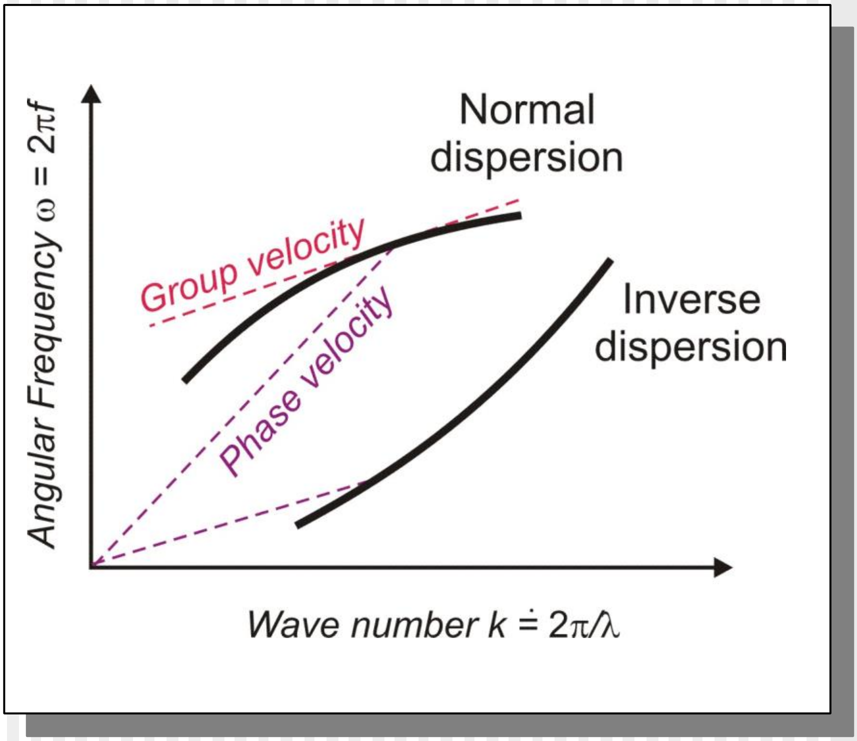
- Therefore, two types of dispersion are differentiated:

$$\frac{dV}{d\omega} < 0$$

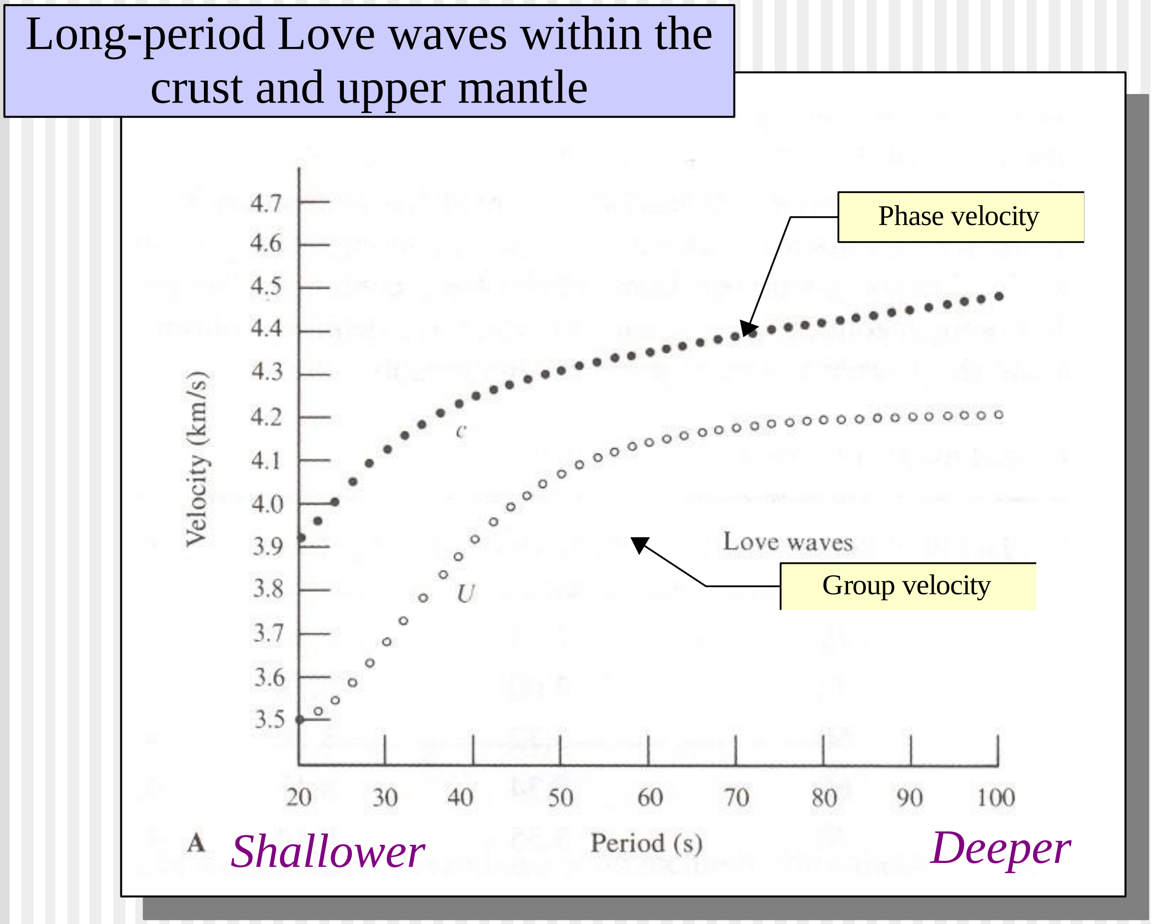
Normal dispersion
(typically observed in seismic ground roll)

$$\frac{dV}{d\omega} > 0$$

Inverse dispersion



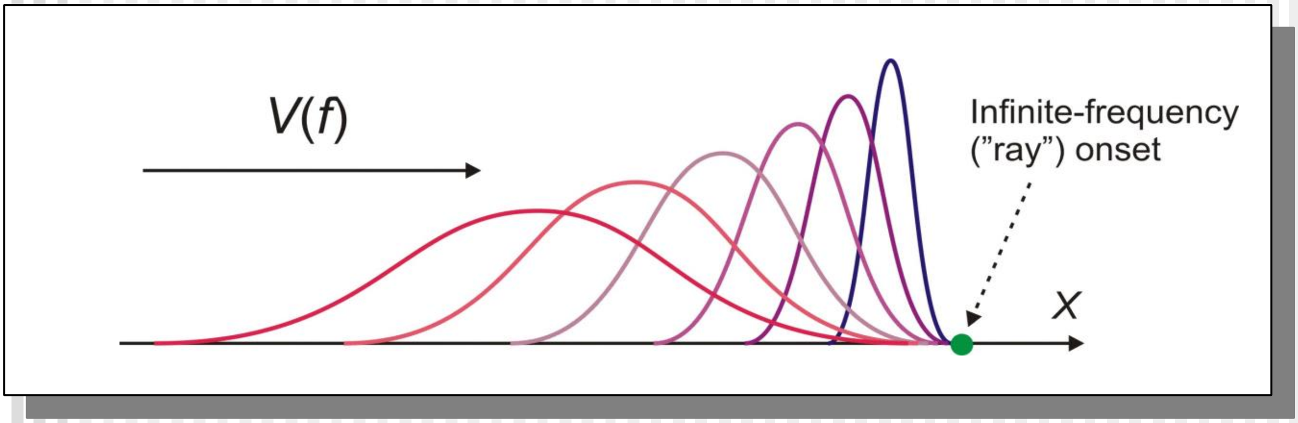
Example:
normal dispersion of surface waves



- Normal dispersion occurs in layered media, because the deeper layers are usually faster
- Lower-frequency waves have longer wavelengths and sample deeper (faster) layers

Attenuation and Dispersion

- Attenuating medium is **always** dispersive
  - Example: ground roll is quickly attenuated and shows strong normal dispersion.
- *Causality* requires that lower-frequency wave components travel slower (*i.e.*, *inverse dispersion*):



- Mathematically, causality is expressed by the so-called "Kramers-Krönig relations" relating the $c(\omega)$ to *Q*
- For example, in a homogenous constant-*Q* medium:

$$c(\omega) = c(\omega_0)\left[1 + \frac{1}{\pi Q}\ln\frac{\omega}{\omega_0}\right]$$

Inverse dispersion

If $Q(\omega) = \text{const}$, $c(\omega)$ increases with frequency