Formula reminder sheet for GEOL483.3

You are expected to be able to give and *explain* the following equations. Those marked with asterisks (*) do not need to be memorized exactly, but still need to be understood and interpreted. What physical quantities do they relate? What physics are they related to and what are their consequences?

General:

Relations between wave length, period, velocity, frequency, and wavenumber: $\lambda = VT$, $f = \frac{1}{T}$,

$\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}$.

Phase velocity: $V_{phase} = \frac{\omega}{k}$.

Group velocity: $U_{group} = \left[\frac{dk}{d\omega}\right]^{-1} = \frac{d\omega}{dk}$.

Amplitude (absolute value, A) and phase (argument, ϕ) of a complex number (e.g., spectrum): $Z = Ae^{i\Phi}$

Decibel expression of relative amplitudes (powers): $\left(\frac{A_1}{A_2}\right)_{dB} = 20\log_{10}\left(\frac{A_1}{A_2}\right)$

Nyquist frequency: $f_N = \frac{1}{2\Delta t}$

Frequency resolution (sampling interval): $f_N = \frac{1}{N\Delta t}$

Z-transform: $U_z(z) = u_0 + u_1 z^1 + u_2 z^2 + u_3 z^3 + \ldots = \sum_{i=0}^{\infty} u_i z^i$

Fourier transform*: $U_F(f_j) = U_z(e^{-2i\pi f_j}). = \sum_{k=0}^{N} u_k e^{-2i\pi f_j t_k}$.

Inverse Fourier transform*: $u(t_k) = \frac{1}{N}\sum_{j=0}^{N} U(f_k) e^{2i\pi f_j t_k}$.

Wave equation (general): $\left[\frac{\partial^2}{\partial t^2} - \nabla^2\right] f(t, \boldsymbol{r}) = 0.$

Linear inverse:

Travel-time tomography problem: $t_i = \sum_j L_{ij} \frac{1}{V_j} = \sum_j L_{ij} s_j$.

In matrix form: $d = Lm$.

Generalized inverse : $m = L_g^{-1} d^{observed.}$

Least-Squares Inverse*: $L_g^{-1} = (L^T L)^{-1} L^T$.

Data misfit minimized in Least Squares Inverse: $Misfit(m) = (d^{observed} - Lm)^T (d^{observed} - Lm)$.

Damped Least Squares: $m = (L^T L + \varepsilon I)^{-1} L^T d^{observed}$.

Resolution matrix: $R = L_g^{-1} L$

Theory of elasticity:

Elementary force acting on a surface within elastic medium: $dF_i = dS \sum_{j=1,2,3} \sigma_{ij} n_{j.}$

Elementary force acting on a volume within elastic medium: $dF_i = dV \sum_{j=1,2,3} \frac{\partial \sigma_{ij}}{\partial x_j}$.

Strain: $\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$.

Dilatational strain: $\Delta = \frac{\delta V}{V} = tr\, \varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial U_j}{\partial x_i}$

Hooke's Law for isotropic medium: $\sigma_{ij} = \lambda' \Delta + 2\mu\varepsilon_{ij}$ for $i=j$, and $\sigma_{ij} = 2\mu\varepsilon_{ij}$ for $i \neq$j. Combined, this can also be written as: $\sigma_{ij} = \lambda' \Delta\, \delta_{ij} + 2\mu\varepsilon_{ij}$ (the "Kronecker symbol", $\delta_{ij} = 1$ if $i=j$ and 0 otherwise).

Elastic moduli:

- Young's modulus*: $E = \frac{2\mu(3\lambda + 2\mu)}{\lambda + \mu}$.
- Poisson's ratio (modulus)*: $\nu = \frac{\lambda}{2(\lambda + \mu)}$.
- Bulk modulus*: $K = \lambda + \frac{2}{3}\mu$

- Rigidify modulus = μ.

Newton's law for elastic medium: $\rho \frac{\partial^2 U_i}{\partial^2 t} = \sum_{j=1,2,3} \frac{\partial \sigma_{ij}}{\partial x_j}$.

P-wave velocity: $V_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{K - \frac{4}{3}\mu}{\rho}}$.

S-wave velocity: $V_S = \sqrt{\frac{\mu}{\rho}}$

For $\nu = 0.25$, $\frac{V_P}{V_S} = \sqrt{3}$

Relation of displacements to wave potentials*:

General*: $\vec{U} = \vec{\nabla}\phi + \vec{\nabla} \times \vec{\psi}$.

P-waves*: $\vec{u}_P(\vec{x}, \vec{z}) = \left(\frac{\partial \phi}{\partial x}, 0, \frac{\partial \phi}{\partial z}\right)$,

SV-waves*: $\vec{u}_S(\vec{x}, \vec{z}) = \left(\frac{-\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x}\right)$,

Elastic energy density*:

General*: $E = \frac{1}{2}\sigma_{ij}\epsilon_{ij}$

In a P- or S-wave*: $E_{elastic} = \frac{1}{2}\rho \dot{\vec{u}}^2 = E_{kinetic}$

Average energy flux (for harmonic wave): $J = c\,E_{kinetic} = \frac{1}{2} Z \omega^2 u^2 = \frac{1}{2}\rho c \omega^2 u^2$, where c is the wave speed, and Z is the impedance

Attenuation

Quality factor: $Q = 2\pi \frac{E}{\delta E}$

Measurement using spectral ratios*: $\ln\left(\frac{A(f, t_2)}{A(f, t_1)}\right) = const - \frac{\pi(t_2 - t_1)}{Q} f$.

Refraction seismics:

Snell's law of refraction: $\frac{\sin i_1}{V_1} = \frac{\sin i_2}{i_2} = ... = const = p$

Headwave travel time (linear moveout) equation, zero dip: $t(x) = t_{Intercept} + \frac{x}{V}$

Intercept time: $t_{Intercept} = \frac{2\mathrm{h} \cos i_c}{V_{1.}}$

Refraction time, down dip: $t(x) = \frac{2\mathrm{h}_d \cos i_c}{V_{1.}} + \frac{x}{V_1} \cos(i_c + \alpha)$ up dip: $t(x) = \frac{2\mathrm{h}_u \cos i_c}{V_{1.}} + \frac{x}{V_1} \cos(i_c - \alpha)$

Inversion for refractor dip (small-dip approximation)*: $\sin i_c \approx \frac{V_1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right)$.

and refractor velocity*: $\frac{1}{V_2} \approx \frac{1}{2}\left(\frac{1}{V_d} + \frac{1}{V_u}\right)$.

Delay time: $t_{S,R\,Delay} = \frac{h_{s,r} \cos i_c}{V_{1.}}$

Diving waves*:

$$x(p) = 2 \int_0^{h_{max}} \frac{p V(z) dz}{\sqrt{1 - (pV(z))^2}},$$

$$t(p) = 2 \int_0^{h_{max}} \frac{dz}{V(z)\sqrt{1 - (pV(z))^2}},$$

Reflection seismics:

Vertical reflection resolution: $\delta z = \frac{\lambda}{4}$.

Horizontal reflection resolution: $\delta x \approx \sqrt{\frac{1}{2} H \lambda}$.

Reflection travel time (normal moveout) equation $t(x) = \sqrt{t_0^2 + (\frac{x}{V})^2} \approx t_0 + \frac{1}{2t_0}\left(\frac{x}{V}\right)^2$

Dip moveout: $t(x) \approx t_0 + \frac{x^2}{2t_0 V^2} + \frac{x \sin\theta}{V}$.

RMS velocity: $V_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} t_i V_i^2}{\sum_{i=1}^{n} t_i}}$.

NMO correction: $t(x) \rightarrow t_0 \approx t(x) - \frac{1}{2t(x)}\left(\frac{x}{V}\right)^2$

Stacking velocity in the presence of dip: $V_{Stacking} = \frac{V_{true}}{\cos\theta}$.

Sampling theorem: $\textit{Geophone Spacing}_{max} < \frac{\lambda_{apparent}}{2} = \frac{\lambda_{min}}{2\sin\theta} = \frac{V_{min}}{2 f_{max} \sin\theta}$

Or in terms of event moveout*: $\textit{Geophone Spacing}_{max} < \frac{V_{min}}{2 f_{max} \frac{dt}{dx}}$

Signal-to-noise improvement by stacking: $\frac{Signal}{Noise} = \sqrt{N} \frac{S}{\sigma_n}$.

Relation of zero-offset (apparent) and true reflector dips: $\sin\theta_{true} = \tan\theta_{apparent}$

Reflectivity and AVO:

Scattering matrix: $\begin{pmatrix} \acute{P}_1 \\ \acute{S}_1 \\ \grave{P}_2 \\ \grave{S}_2 \end{pmatrix} = \boldsymbol{S} \begin{pmatrix} \grave{P}_1 \\ \grave{S}_1 \\ \acute{P}_2 \\ \acute{S}_2 \end{pmatrix}$, where $\boldsymbol{S} = \begin{pmatrix} \grave{P}\acute{P} & \grave{S}\acute{P} & \acute{P}\acute{P} & \acute{S}\acute{P} \\ \grave{P}\acute{S} & \grave{S}\acute{S} & \acute{P}\acute{S} & \acute{S}\acute{S} \\ \grave{P}\grave{P} & \grave{S}\grave{P} & \acute{P}\grave{P} & \acute{S}\grave{P} \\ \grave{P}\grave{S} & \grave{S}\grave{S} & \acute{P}\grave{S} & \acute{S}\grave{S} \end{pmatrix}$

Its relation to boundary conditions*: $\boldsymbol{S} = \boldsymbol{M}^{-1}\boldsymbol{N}$, where **M** and **N** are from: $\begin{pmatrix} u_x \\ u_y \\ \sigma_{xz} \\ \sigma_{zz} \end{pmatrix} = \boldsymbol{M} \begin{pmatrix} \acute{P}_1 \\ \acute{S}_1 \\ \grave{P}_2 \\ \grave{S}_2 \end{pmatrix} = \boldsymbol{N} \begin{pmatrix} \grave{P}_1 \\ \grave{S}_1 \\ \acute{P}_2 \\ \acute{S}_2 \end{pmatrix}$.

Seismic Impedance: $Z = \rho V$

Normal-incidence reflection coefficient: $R = \dfrac{Z_2 - Z_1}{Z_1 + Z_2} \approx \dfrac{1}{2}\Delta(\ln Z) \approx \dfrac{1}{2}\left(\dfrac{\Delta V_P}{V_P} + \dfrac{\Delta\rho}{\rho}\right)$

Transmission coefficient: $T = 1 - R = \dfrac{2Z_1}{Z_1 + Z_2}$.

Energy reflection coefficient: $E_R = R^2$

Energy transmission coefficient: $E_T = 1 - E_R = \dfrac{4Z_1 Z_2}{Z_1 + Z_2}$.

Shuey's AVA formula*: $\dfrac{R(\theta)}{R(0)} \approx 1 + P\sin^2\theta + Q(\tan^2\theta - \sin^2\theta)$

AVA for angles <~30°: $\dfrac{R(\theta)}{R(0)} \approx 1 + P\sin^2\theta$

Surface waves:

General form for potential: $\phi = e^{-mz} e^{i(kx-\omega t)}$,

where for Rayleigh wave $m = \sqrt{k^2 - \frac{\omega^2}{V_P^2}}$ for P-wave component,

$m = \sqrt{k^2 - \frac{\omega^2}{V_S^2}}$. for SV-wave component.

Wavelets, time series:

Convolution $(u * w)_k = \sum_i u_{k-i} w_i$

Representation of the Maximum, Minimum, and mixed-delay wavelets: $W(z) = \prod_{i=0}^{N} (1 + a_i z)$

Ricker wavelet*: $u(t) = (1 - 2\pi^2 f_M^2 t^2) e^{-(\pi f_M t)^2}$

Ricker wavelet spectrum*: $U(f) = \frac{2f^2}{\sqrt{\pi} f_M^2} e^{-\left(\frac{f}{f_M}\right)^2}$

Moveout filtering:

τ-p (slant-stack) transform: $S(p, \tau) = \sum_x u(x, \tau + px)$