

Internal Friction in Moon and Earth Rocks

RECENT measurements of the quality factor, Q , for mechanical vibrations generated by dropping parts of the Apollo lunar module and by moonquakes¹ indicate that the Q of the Moon for these vibrations is of the order of 3,000, but seismological measurements of the Earth's rocks indicate Q s of 110 for the outer mantle², with higher values for the inner mantle. It is the purpose of this report to indicate that these lunar results are consistent with a dislocation source for internal friction.

A dislocation mechanism has been discussed³ which gives an internal friction independent of the frequency for low frequencies and a Granato-Lücke type performance at higher frequencies. The complete normalized internal friction curve is shown in Fig. 1. The normalized frequency ratio (ω/ω_0) is determined by the angular measuring frequency ω divided by $\omega_0 = \mu b^2 / B l_A^2$ where μ is the shear stiffness, b the Burger's distance, B the drag coefficient which measures the force proportional to the velocity exerted on dislocations by phonons and electrons, and l_A is the average distance between impurity pinning points for an exponential distribution. The normalized internal friction ratio determines the ratio of Q^{-1} to $\bar{N} R l_A^2$ where \bar{N} is the total length of dislocation per cm^3 , and R is an orientation factor which determines the ratio of the shearing stress in the glide plane to the stress in the mode of vibration. For a Young's modulus type of vibration, a reasonable value is $R = 0.25$. Assuming, as will be shown likely, that Moon rocks are similar to impure metals and alloys, which have $l_A \doteq 3 \times 10^{-5}$ cm, gives a Q of 3,000 when $\bar{N} = 5 \times 10^7$, a value consistent with alloys and impure metals.

Two measurements⁴ have been made on Westerley granite and Pennsylvania slate which agree with the form of the theoretical curve of Fig. 1. A third rock Solenhofen limestone has recently been completed with the results shown in Fig. 2. The lower frequency part of this curve and the squares of the high frequency measurements were made by a continuous wave measurement with two equal transducers cemented to a half wave specimen of rock. The resonant frequency determines the elastic modulus while the width of the resonant curve determines the internal friction Q^{-1} . The crosses show measurements by a pulsing method which also gives the velocity as 5.65×10^5 cm s^{-1} . The triangles and circles show other measurements of the same limestone⁵ (triangles) and a different limestone⁶. The agreement is good up to 5 MHz. Above this frequency, scattering losses become large enough to make measurements questionable. As can be seen the dislocation loop length is in the order of 1.9×10^{-4} cm while the number of dislocations is about 10^7 cm^{-2} . As will be shown here, these values are associated with the numbers expected for free surfaces where l_A is effectively larger and \bar{N} smaller than for a continuous medium.

For a rock most of the internal friction is connected with the grain boundaries as can be seen from the fact that hydrostatic pressure reduces the internal friction by a factor of 10

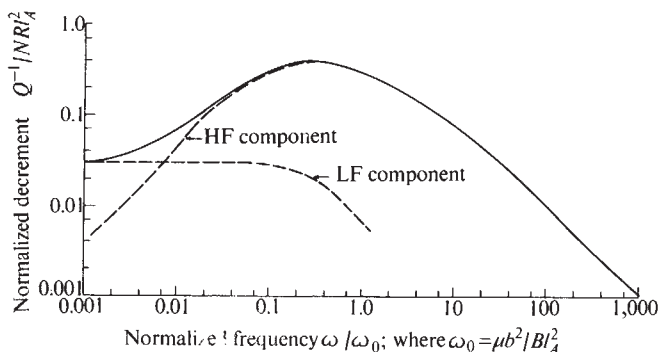


Fig. 1 Normalized internal friction curve against normalized frequency for an exponential distribution of pinning points.

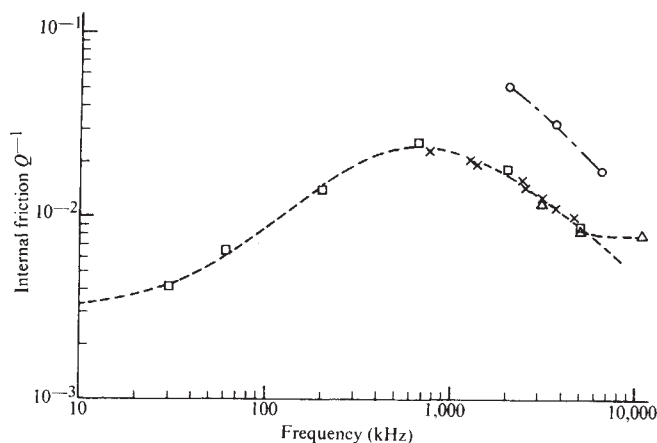


Fig. 2 Internal friction measurements for Solenhofen limestone. \circ , Krishnamurthi and Balakrishna; \triangle , Peselnick and Zeitz; \square , continuous wave measurement; \times , pulsing measurement; ---, theoretical.

or more^{7,8}. A model is used here for which a grain boundary is represented by a series of spherical surfaces having a shearing modulus $\mu(1 + j/Q_1)$ pressing on each other, while the interior is represented by a solid having a stiffness $\mu[1 + j/Q_2]$. Making use of the Hertz theory of contacts and Mindlin's⁹ calculations of the displacement of a set of convex surfaces, then for an applied alternating force T we find, for the sidewise displacement dx_2 ,

$$dx_2 = \frac{(2 - \sigma)T}{8a\mu(1 + j/Q_1)} \tag{1}$$

The tangential force T is related to the applied shearing stress T_6 by

$$T_6 = T(\pi a^2)n_0 \tag{2}$$

where a is the Hertz radius of contact, and n_0 the total number of contacts per cm^2 . The Hertz radius of contact is determined by

$$a = \sqrt[3]{\frac{3}{4} Nr \left(\frac{1 - \sigma}{\mu} \right)} \tag{3}$$

where r is the radius of the spherical contact surfaces and N the normal force on each contact. The product Nr can be eliminated by considering the thickness t_1 of the layer $r\theta^2$ and the area $\pi r^2\theta^2$ where θ is half the angle determined by the radius of curvature r . Then

$$Nr = T_1/n_0\pi t_1 \tag{4}$$

where T_1 is the sum of the pressure holding the rock together, T_1 , plus the applied hydrostatic pressure. Introducing all these values the sidewise displacement is

$$dx_2 = \frac{(2 - \sigma)}{6(1 - \sigma)} \frac{T_6 t_1}{T_1(1 + j/Q_1)} \tag{5}$$

To the displacement of the intermediate layer we have to add that in the two outside layers which result in

$$dx_1 = \frac{T_6 t_2}{\mu(1 + j/Q_2)} \tag{6}$$

Making use of the fact that Q_1^{-1} and Q_2^{-1} are small, the ratio between the applied shearing stress T_6 and the shearing strain

$$S_6 = \frac{2(dx_1 + dx_2)}{t_1 + t_2}$$

is

$$\mu_T = \mu[1 + t_1/t_2]/(1 + A); Q_T^{-1} = Q_2^{-1} - \frac{A(Q_2^{-1} - Q_1^{-1})}{1 + A} \tag{7}$$

where

$$A = \frac{(2 - \sigma)}{6(1 - \sigma)} \frac{\mu}{T_1} \frac{t_1}{t_2}$$

The fact that lunar rocks studied on Earth¹⁰ have a Q of only 10 to 100 is not in contradiction with the high Q measured on the Moon. As shown by equation (7) the Q measured for rocks in the laboratory is mostly determined by grain boundaries and cracks between various parts. It is not until the pressure is high enough to eliminate these that the Q is determined by the internal structure of the rock. Hence the measurements of Warren *et al.*¹⁰ in no way contradict the ideas expressed here.

Using measurements¹¹ of the shear stiffness of Westerley granite (Fig. 3) it is found that $T_{10} = 3 \times 10^8$ dyne cm^{-2} ; $t_1/t_2 = 1.7 \times 10^{-3}$ and $\mu = 3.57 \times 10^{11}$ dyne cm^{-2} . Hence for no external pressure the internal friction is

$$Q_T^{-1} = 0.545 Q_2^{-1} + 0.455 Q_1^{-1} \quad (8)$$

while for high pressures it is equal to Q_2^{-1} , the value for the medium. Using the above values and Q_1^{-1} equal to 1.1×10^{-2} to agree with the torsional wave measurements of Birch and Bancroft⁷ on Rockport granite (550×10^{-5} at 200 bar and 60×10^{-5} at 4 kbar) the value due to the boundary is less than 2×10^{-4} at 10 kbar while Q_2^{-1} , the internal friction of rocks without flaws, is less than 2×10^{-4} . Hence granite under pressure has internal friction values of the order of that found for Moon rocks. Because all the seismological measurements and the Moon rock measurements are for hydrostatic pressures large enough to eliminate the effect of the boundary, we are dealing with the internal friction of the medium itself. The question then is why the internal friction of the Moon rocks is so much lower.

The answer seems to be that the shearing stress associated with the ellipticity of the Moon (about 10^7 dyne cm^{-2} , ref. 12) is insufficient to generate new dislocations by the Frank-Read mechanism whereas the stress in the Earth's crust (from 6×10^7 to 10^8 dyne cm^{-2}) is large enough to generate new dislocations. A Frank-Read mechanism starts to function when the applied shearing stress—perhaps tectonic in origin—is

$$T_6 = \frac{\mu b}{l_N} \quad (9)$$

where l_N is the network length, that is, the distance between dislocation joins. The Frank-Read source will operate until

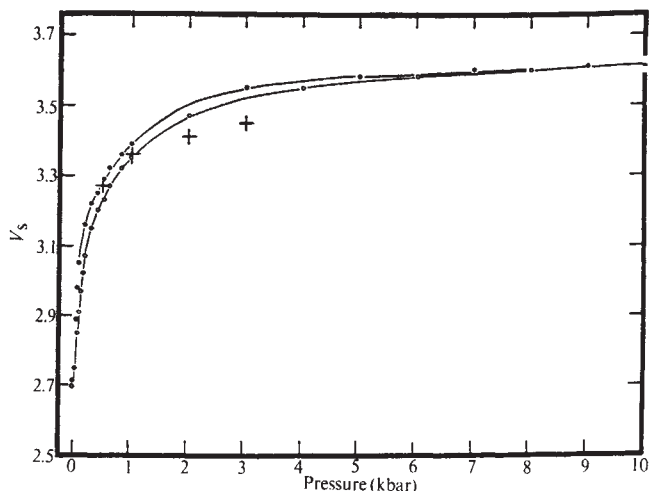


Fig. 3 Shear velocity of Westerley granite as a function of hydrostatic pressure. The lower curve was obtained with increasing pressure. Crosses represent data from Birch and Bancroft (from ref. 11).

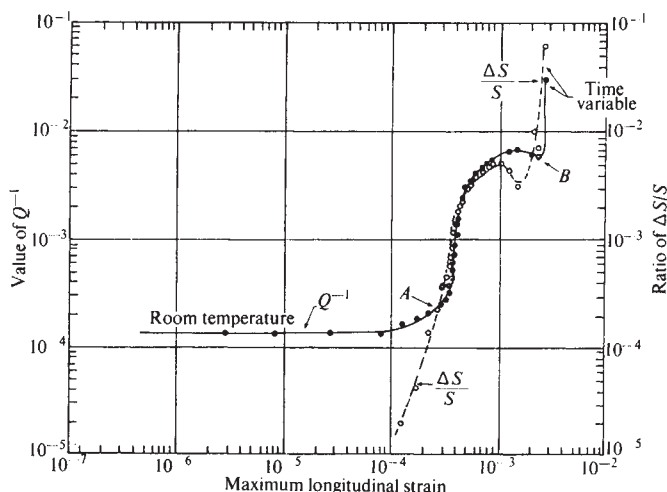


Fig. 4 Internal friction plotted as a function of the maximum longitudinal strain for brass. Slip bands begin to form at a strain of 3×10^{-4} .

the length l_N is equal to $\mu b/T_6$. If some of these loops anneal out, the mill is started again until l_N is determined by $\mu b/T_6$. Some idea of l_N can be obtained for brass from the data of Fig. 4, which shows the internal friction as a function of the longitudinal strain. At about 3×10^{-4} , the dislocation mill starts to move as can be seen by the presence of slip bands¹³. This requires a longitudinal stress of 3×10^8 dyne cm^{-2} or, with an orientation factor of 0.25, a shearing stress of 7.5×10^7 dyne cm^{-2} which from (9) results in a network length of 1.25×10^{-4} cm. For the rock it might be somewhat larger but would not be large enough to cause the mill to operate at a stress of 10^7 dyne cm^{-2} . An even stronger argument¹² is that the ellipticity of the Moon has not changed during geological times even though it keeps the same face to the Earth.

For the Earth, the stresses are larger and they occur every 12 h. Thus new dislocations can be generated which will probably be pinned eventually by impurities giving the same pinning distance l_A as in the unstrained rock. The number of dislocations will continue to increase until the joins of the network reduce the network length l_N to a length such that the Frank-Read mechanism can no longer operate. A rough idea of the relation between \bar{N} and l_N can be obtained from the equation

$$\bar{N} l_N^2 = 3$$

as can be seen from a figure showing the network as blocks with dislocations going straight through. For $\bar{N} = 5 \times 10^7$; l_N is 2.3×10^{-4} cm which is not long enough to start a dislocation mill on the Moon. For a Q of 110, the number of dislocations has increased by a factor 27.2 which results in a value of l_N of 4.7×10^{-5} cm. This gives a shearing stress of 1.9×10^8 dyne cm^{-2} to cut the mill off. Although all these figures are approximate, they are close to what is expected.

WARREN P. MASON

Henry Krumb School of Mines,
Columbia University,
New York, NY 10027

Received July 14; revised November 8, 1971.

- ¹ Latham, G., Ewing, M., Dorman, J., Press, F., Tokusoz, N., Sutton, G., Meissner, Rolf, Duennebie, F., Nakamura, Y., Kovach, R., and Yates, M., *Science*, **170**, 620 (1970).
- ² Knopoff, L., *Physical Acoustics* (edit. by Mason, W. P.), **3**, B, ch. 7 (Academic Press, New York, 1965).
- ³ Mason, W. P., *J. Geophys. Res.*, **74**, 4963 (1969).
- ⁴ Mason, W. P., Beshers, D. N., and Kuo, J. T., *J. Appl. Phys.*, **41**, 5206 (1970); Mason, W. P., and Kuo, J. T., *J. Geophys. Res.*, **76**, 2084 (1971).
- ⁵ Peselnick, L., and Zeitz, I., *Geophysics*, **24**, 285 (1959).
- ⁶ Krishnamurthi, M., and Balakrishna, S., *Geophysics*, **22**, 268 (1957).

- ⁷ Birch, F., *Handbook of Physical Constants*, 92, No. 36 (Geophysical Society, 1942).
⁸ Gordon, R. B., and Davis, L. A., *J. Geophys. Res.*, 73, 3917 (1968).
⁹ Mindlin, R. D., *Trans. Amer. Soc. Mech. Eng.*, 71, 259 (1949).
¹⁰ Warren, N., *et al.*, *Proc. Apollo 12 Conf.* (MIT, 1971).
¹¹ Simmons, G., *J. Geophys. Res.*, 69, 1117 (1964).
¹² Jeffreys, H., *The Earth*, ch. 12 (Cambridge Press, 1929).
¹³ Mason, W. P., and Wood, W. A., *J. Appl. Phys.*, 39, 5581 (1968).

Time of Formation of the Earth's Core

ACCORDING to several theories¹⁻³, the Earth was formed by accretion from a mixture of silicate particles and metal particles, something like those in chondritic meteorites. The accreting material may also have contained relatively small quantities of other components such as iron meteorites⁴ and carbonaceous chondrites⁵. Most advocates of these theories have maintained that accretion occurred relatively slowly (over a period of about 10^8 yr), so that the gravitational potential energy was efficiently radiated away and the Earth formed in a relatively cool and unmelted condition. Subsequently, heating by long lived radioactive elements occurred, leading to melting of the metal phase and segregation of the core. Differentiation and formation of the core may thus have occurred much later than the primary accretion of the Earth. It has even been argued that core formation might not yet have proceeded to completion^{1,6,7}. On the other hand, alternative theories of origin of the Earth^{8-10,11} have maintained that much of the accretion of the Earth occurred in high temperature conditions and that core formation occurred either during accretion or very soon afterward. Clearly, an independent determination of the time interval ΔT_c between accretion and core formation would have a crucial bearing on these contrasting theories.

The age of the Earth, T_0 , is believed to be close to 4.55×10^9 yr (refs. 12 and 13). This age is based on the discovery that the isotopic composition of modern terrestrial lead falls on the meteoritic isochron, which records a differentiation of lead relative to uranium within meteorites at this time. The mean growth curve for lead ore bodies indicates an age of 4.58×10^9 or 4.55×10^9 yr for the Earth depending on the choice of ores included in the calculation^{14,15}. Both the meteorite and ore body ages assume that the lead/uranium ratio of the upper mantle-crust system had been established at T_0 , and since then had remained constant to a first approximation.

In 1960, Ringwood⁸ argued that formation of the core would probably alter the Pb/U ratio of the mantle because the molten iron would be expected to carry down substantial amounts of lead but not uranium. He concluded, therefore, that the 4.55×10^9 yr terrestrial event recorded by Pb/U geochronology actually referred to the date of core formation, which must have occurred very soon after formation of the Earth.

The principal assumption behind this conclusion was that during core formation the Pb/U ratio of the metal phase would be much higher than that of the silicate phase and that a substantial proportion of the total lead in the Earth would enter the core. This assumption was not accepted by Patterson and Tatsumoto¹⁶, however, who maintained that lead would be preferentially partitioned into the silicate phase. Their calculations of mantle evolution implicitly require that either trivial amounts of lead are partitioned into the core, or that core formation occurred instantaneously 4.55×10^9 yr ago. It is not obvious that either of these implicit assumptions is necessarily true. Clearly, experimental measurements of the distribution coefficient of lead between relevant metal and silicate systems are necessary. We have carried out a series of experiments of this nature. It was difficult to perform these experiments at atmospheric pressure because lead tends to

volatilize at temperatures at which the metallic iron-rich phase is molten. Accordingly, the experiments were performed at high pressures under closed system conditions, thereby preventing volatilization.

The experiments were designed as analogues of the core formation process in the Earth, as envisaged in various theories. The Earth's core is believed to contain 10-30% of one or more light elements (such as S, Si, C) in addition to iron¹⁷⁻¹⁹. According to a widely discussed current theory the Earth accreted mainly from material resembling ordinary chondrites^{3,20-22} together with a small proportion of carbonaceous chondrite like material, which was the "carrier" of volatile components⁵. On this hypothesis the principal light-element component of the core would consist of sulphur with a smaller amount of carbon. We represented this model with a synthetic metal phase of composition (weight per cent) $\text{Fe}_{83}\text{S}_{15}\text{C}_2$. Another model^{8,19} of core formation regards silicon as the most important light element. To test this model, a metal phase of composition $\text{Fe}_{89}\text{Si}_{11}$ was used. Finally, according to a third hypothesis²³ the Earth's core contains the primordial abundance of sulphur. In the relevant experiment, a metal phase of composition $\text{Fe}_{70}\text{S}_{30}$ was used.

The silicate phase used in the experiments consisted of simplified synthetic basaltic compositions, doped with 1,000 p.p.m. of lead (as oxide) in some runs and 100 p.p.m. Pb in others. For experiments on the $\text{Fe}_{89}\text{Si}_{11}$ -basalt system, the simplified basalt composition contained no iron, whereas the other basaltic compositions used contained normal amounts of oxidized iron.

In the experiments, finely powdered samples of metal phase were mixed with silicate in the ratio 1 part (by weight) of metal to 2 parts silicate. The mixture (400 mg) was then charged into a closed aluminum capsule which was placed in an internally heated piston cylinder high pressure apparatus²⁴. In a given run, the sample was first subjected to the desired pressure. Temperature was then increased to 1,100° C and held for 30 min, to recrystallize and equilibrate the charge under sub-solidus conditions. The temperature was then slowly increased over a period of 1.5 h to a maximum temperature at which both metal and silicate phase were known to be molten. The temperature was held steady for 10 min, and the system was then switched off. Pressure was released after cooling and the samples removed from the apparatus. The metal was usually found to have sunk to the bottom of the capsule as one or more globules. Metal and silicate phases were crushed and separated magnetically. The concentrations of lead in coexisting metal and silicate phases were then determined by isotope dilution with Pb runs as PbI_2^+ (ref. 25). Blanks were approximately 20 ng per analysis.

Distribution coefficients for lead between metal and silicate phases (K^m 's) were obtained for each experiment. The distribution coefficient is the ratio of Pb concentration in the metal to Pb concentration in the silicate phase. Results are shown in Table 1. They are sufficiently consistent to support the belief that equilibrium was approached. The fact that all lead was initially in the silicate phase means that our distribution coefficients represent minimum values.

A series of runs was made at 10 kbar. These illustrate the relation between distribution coefficients and metal phase

Table 1 Distribution Coefficients for Lead between Metal and Silicate Phases

Metal phase	Pressure (kbar)	Maximum temperature (°C)	K metal/silicate
$\text{Fe}_{70}\text{S}_{30}$	10	1,400	9.8, 12.4
$\text{Fe}_{89}\text{Si}_{11}$	10	1,400	3.7, 3.8
$\text{Fe}_{83}\text{S}_{15}\text{C}_2$	10	1,400	1.9, 2.0, 2.1
$\text{Fe}_{83}\text{S}_{15}\text{C}_2$	5-40	1,350-1,650	1.4 to 2.9
	12 runs over 5 kbar intervals including repeats		Most probable value 2.5 ± 0.5